

# Estimation of linear observation impact and its applications

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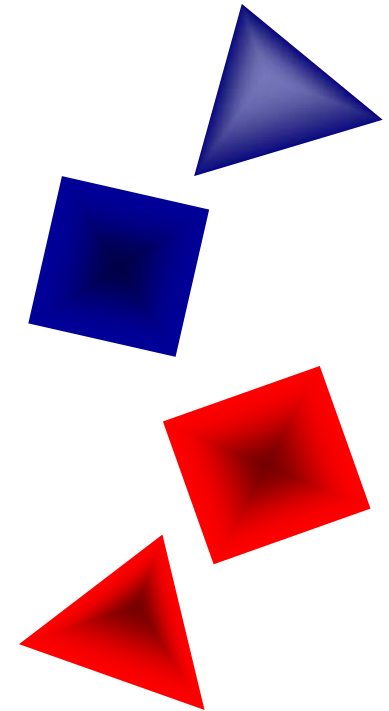
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## 4. Summary



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# 1. Observation impact estimation

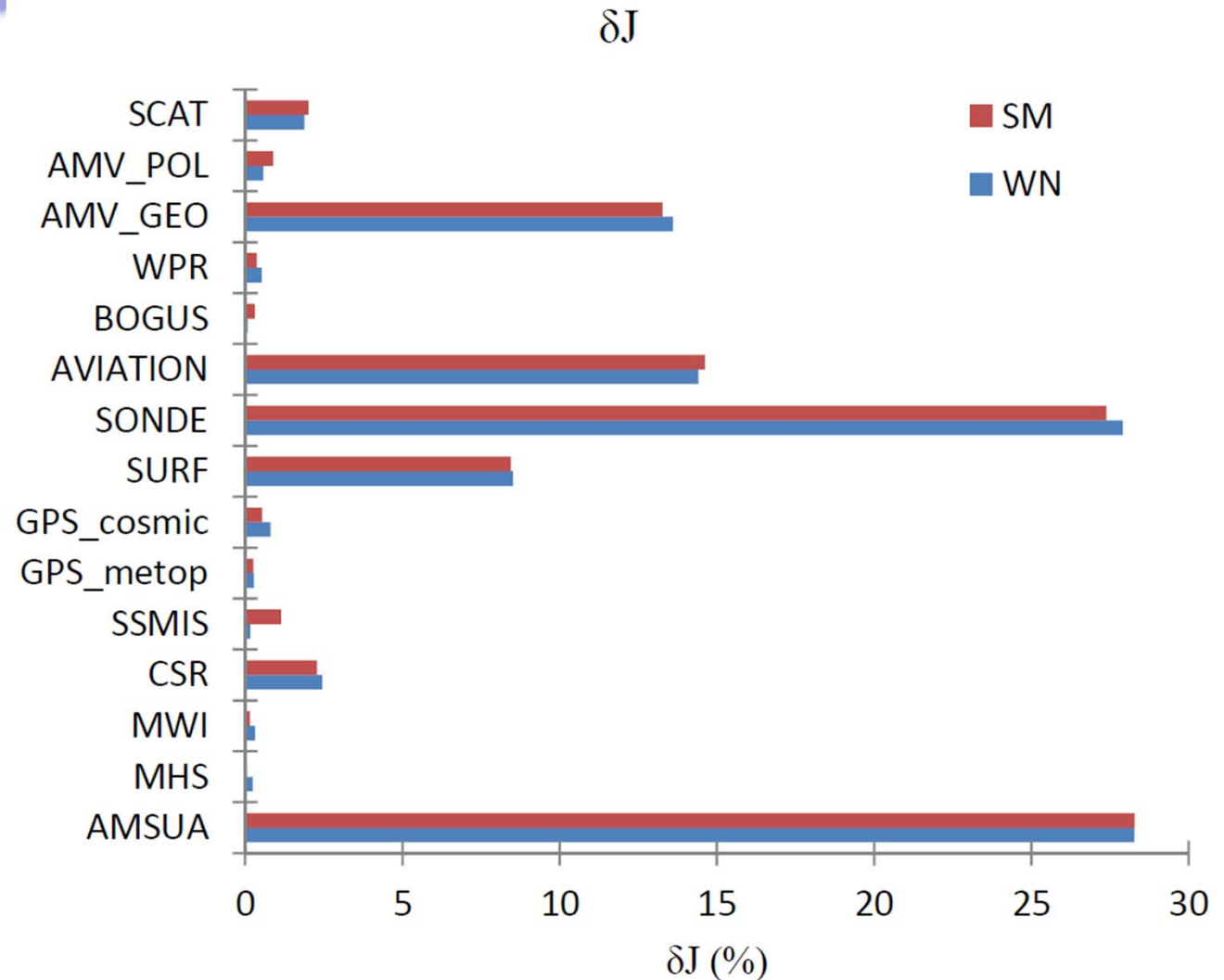
# Two types of observation impact

- Basic definition
  - The observation impact is defined as “the variations of analyses and forecasts caused by **changes of observation data**”.
- Non-linear observation impact
  - This is the observation impact that has no limitations on the **changes of observation data**, so the changes includes perturbations in observation data values, and additions of observation datasets.
  - Estimation methods:
    - ✓ OSE (observing system experiment).
- Linear observation impact
  - This is the observation impact that has an limitation on the **changes of observation data, which is Kalman gain is invariant**.
  - Estimation methods are
    - ✓ ADJ-based method (FSO)
      - Langland and Baker (2004), Errico (2007), Cardinali (2009), Tremolet (2008)
    - ✓ TL-based method
      - Ishibashi (2011)
    - ✓ DFS
      - Cardinali et al (2004), Desroziers et al (2005)

*These two observation impacts are different quantities, so, in general, they cannot work as a proxy for each other.*

# ADJ-based estimation in JMA global 4D-Var

- JMA global 4D-Var
  - ✓ Using Low resolution system (TI319/T106).
- Evaluation periods are:
  - Summer: Aug 2010,
  - Winter: Jan 2010.
  - \* 00UTC analyses only.
- Using dry total energy norm.
- Forecast error evaluation time is 15hours.



# Formulation of TL-based method

- The analysis increment vector can be written as a superposition of partial increment vectors (PIVs).

$$\delta \mathbf{x} = \delta \mathbf{x}^{*P} + \delta \mathbf{x}^{*Q} + \dots \quad \delta x_i^{*P} \equiv \sum_{r \in P} K_{i,r} d_r ; \quad \delta x_i^{*Q} \equiv \sum_{r \in Q} K_{i,r} d_r$$

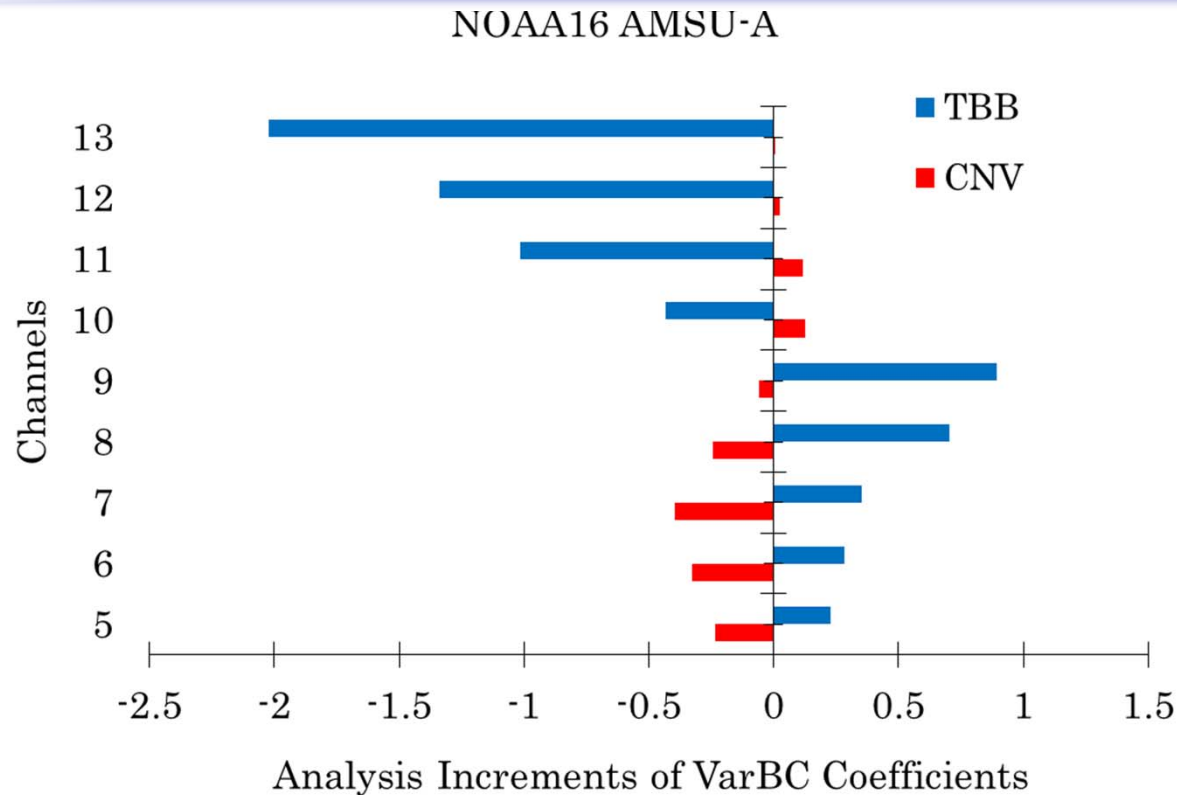
- The PIV represents a linear observation impact of each dataset.
- The departure vector (observations minus background) can be written a superposition of partial departure vectors (PDVs).

$$\mathbf{d} = \mathbf{d}^{*P} + \mathbf{d}^{*Q} + \dots \quad d_r^{*P} \equiv \begin{cases} d_r & r \in P \\ 0 & r \notin P \end{cases} ; \quad d_r^{*Q} \equiv \begin{cases} d_r & r \in Q \\ 0 & r \notin Q \end{cases}$$

- PIVs can be written in terms of PDVs

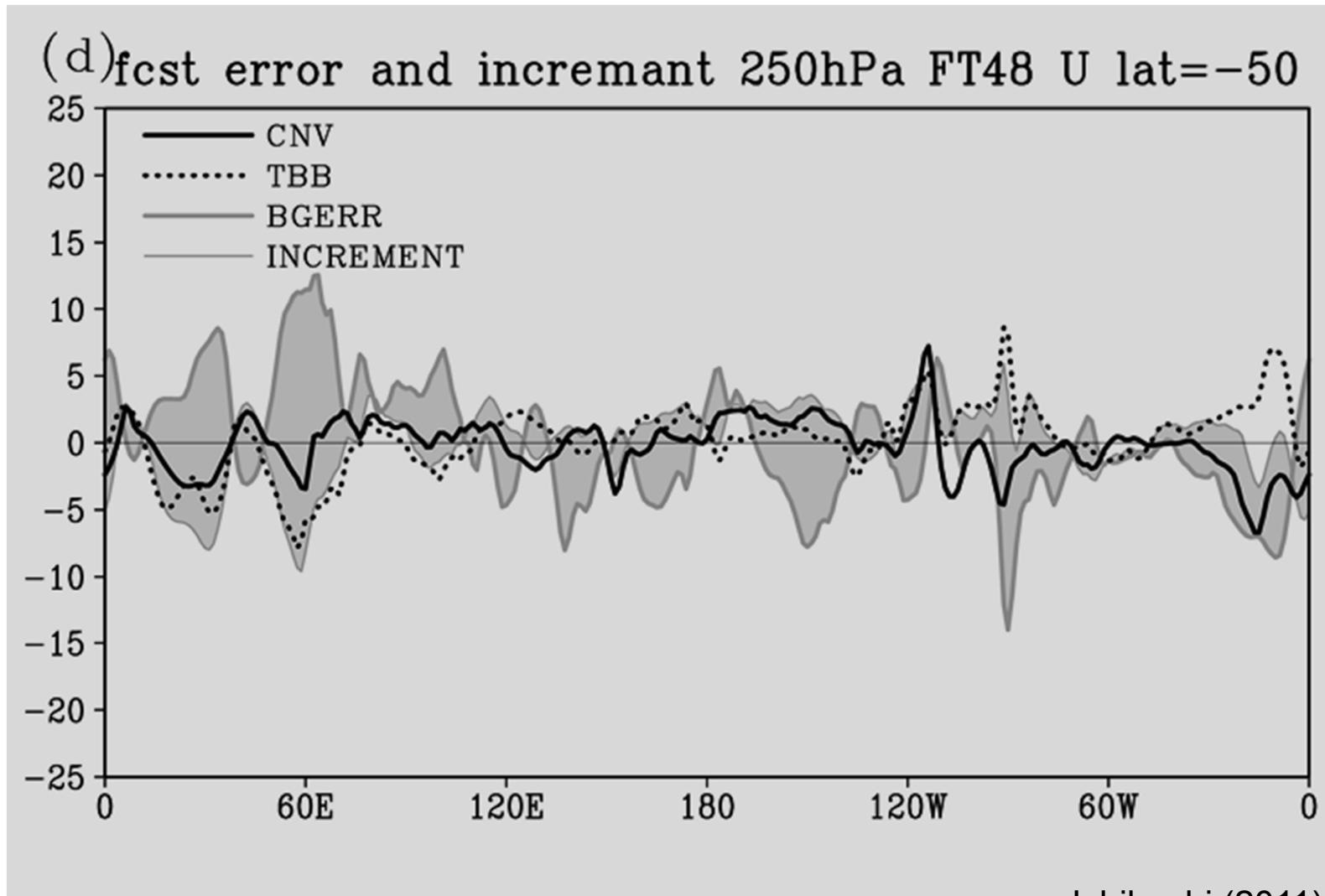
$$\delta \mathbf{x}^{*P} = \mathbf{K} \mathbf{d}^{*P}, \quad \delta \mathbf{x}^{*Q} = \mathbf{K} \mathbf{d}^{*Q}, \quad \dots$$

# TL-based method



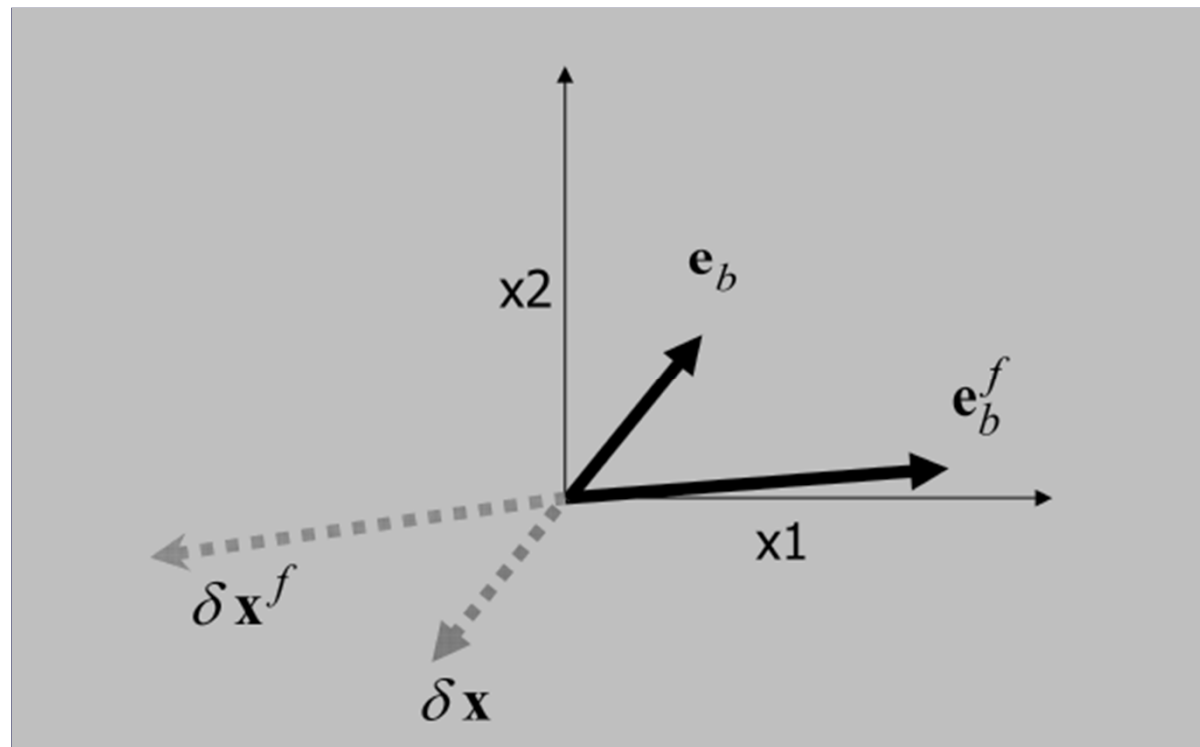
- This figure shows the CNV-PIV and the TBB-PIV for VarBC (variational bias correction) variables of the AMSU-A sensor of the NOAA16 satellite.
- We can find finite contribution from the CNV.
- This result suggests the existence of a stability effect of the CNV for the VarBC variables (Auligné et al., 2007) at least qualitatively.

# TL-based method

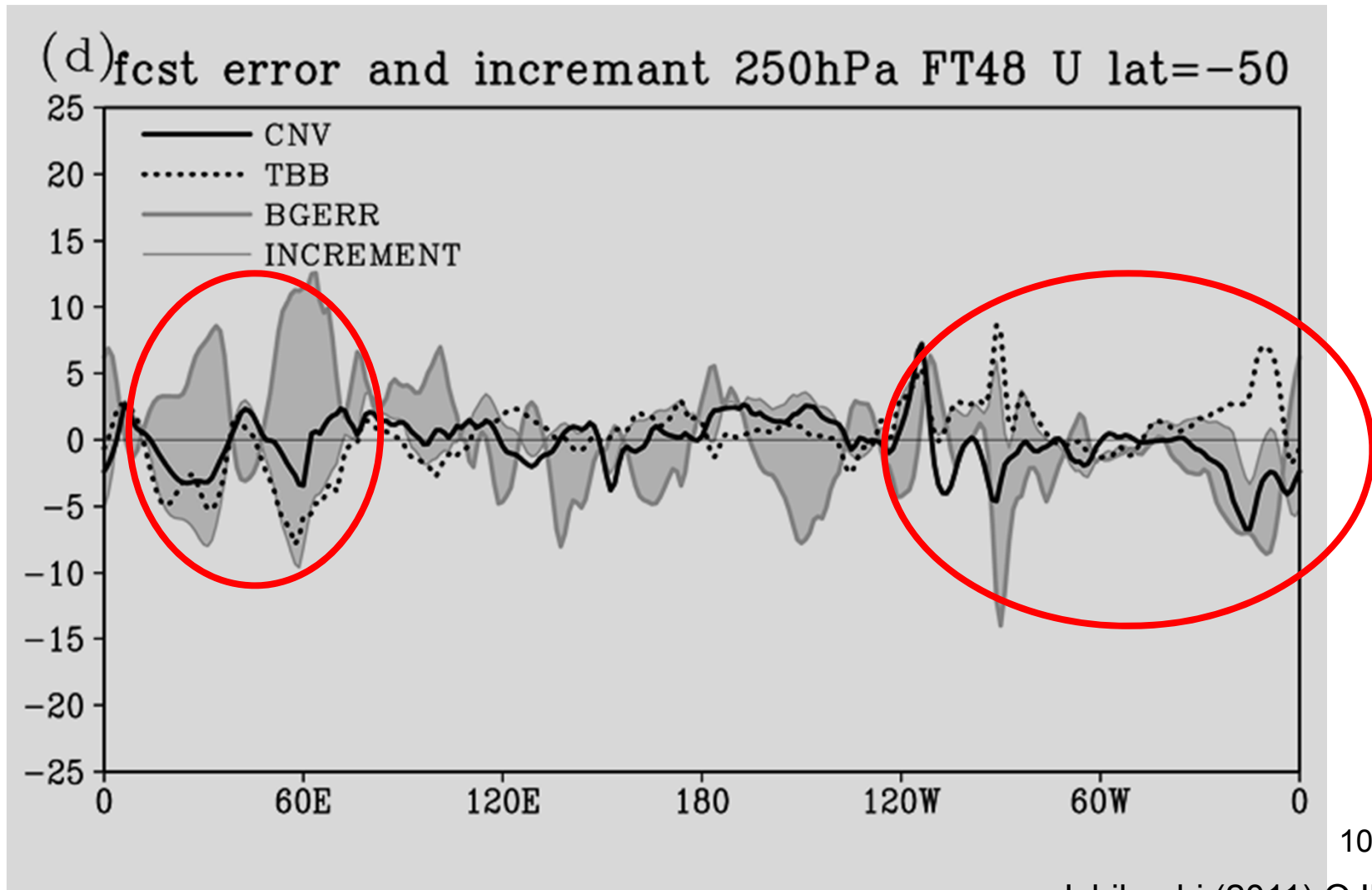




# TL-based method



# TL-based method



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## 2. Covariance matrix optimization

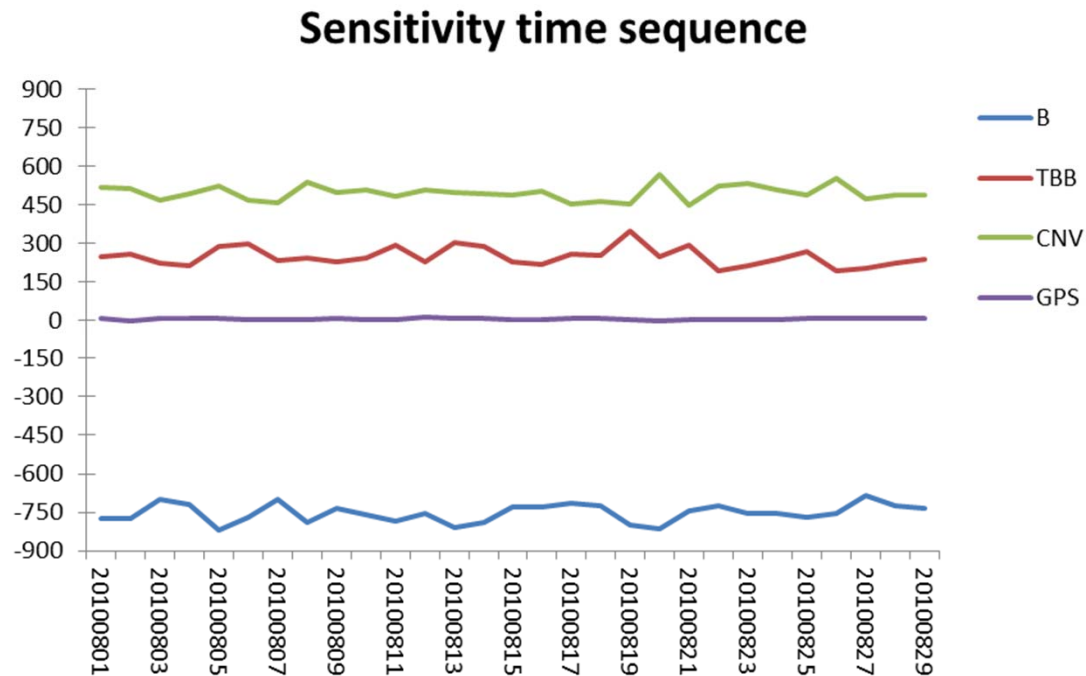
# Relationships between observation impact estimations and covariance optimizations

- Two types of error covariance matrix optimization methods.
  1. Expectation-based method
    - ▣ This method optimizes error covariance matrices based on the theoretical relation ships;
$$2E[J_o] = Tr[\mathbf{I} - \mathbf{H}\mathbf{K}] \quad 2E[J_b] = Tr[\mathbf{K}\mathbf{H}]$$
    - ▣ Desroziers and Ivanov (2001), Desroziers et al (2005), and Chapnik et al (2004, 2006)
  2. Sensitivity-based method
    - ▣ This method uses sensitivity of forecast errors with respect to covariance matrices;
$$\frac{\partial J}{\partial \mathbf{R}}, \frac{\partial J}{\partial \mathbf{B}}$$
    - ▣ Daescu (2008) , Daescu and Todling (2010).
- Each optimization method include a linear observation impact estimation.
  1. Expectation-based method includes DFS calculation.
  2. Sensitivity-based method includes ADJ-based estimation.



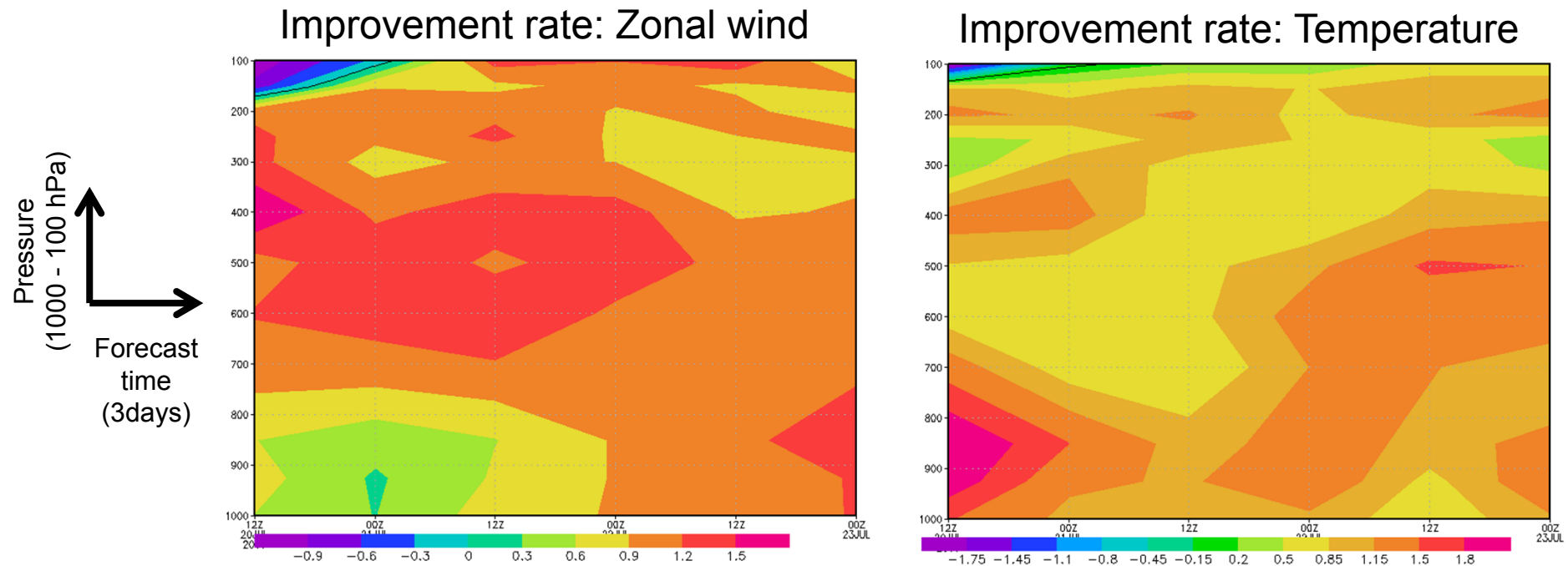
Here, Let's see the sensitivity-based method

# Diagnoses of the JMA global 4D-Var



- Sensitivity calculation results in August 2010.
- Using dry total energy norm with 15hr forecasts.
- The results show that **B** is too small and **R** is too large in average.

# Impact of error covariance optimization on forecast accuracy



Results of a single case experiment of covariance optimization using the sensitivity method.

- TEST uses optimized  $R$ , CNTL uses original  $R$  (operational setting).
- The figure shows normalized forecast RMSE differences between TEST and CNTL:  $(CNTL - TEST)/CNTL$ .
- Warm (cold) color areas are forecast error decrease (increase) areas.

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## 3. Analysis error estimation

# 3. Analysis error estimation

## Background

- We want to know analysis errors of a DAS because the analysis error information is useful to improve current DASs and to design future observational systems which can detect the analysis errors.
- Analysis error estimation is the same with construction of more accurate analysis than current DASs. Such analyses can be used as “pseudo truth”.

## Previous studies

- “Key analysis error” (Rabier et al 1996, Klinker et al 1998, Isaksen et al 2005) can generate more accurate forecasts than current DASs.
- However, there are inconsistency between key analysis errors and observation information. SOSE (Marseille 2007) can partly reduce this problem.

## Our approach

1. We construct the pseudo truth based on the data assimilation theory.
2. We construct the pseudo truth based on the ADJ-based method.



# Data assimilation theory based method

- Conditional PDF

$$P(\mathbf{x}|\mathbf{y}, \mathbf{x}_b) \propto P(\mathbf{y}|\mathbf{x})P(\mathbf{x}_b|\mathbf{x})$$

$$P(\mathbf{x}|\mathbf{y}, \mathbf{x}_b, \mathbf{x}_{ref}) \propto P(\mathbf{y}|\mathbf{x})P(\mathbf{x}_b|\mathbf{x})P(\mathbf{x}_{ref}|\mathbf{x})$$

Ordinary 4D-Var

Extended 4D-Var  
with reference  
analyses  
information

- Add reference analysis fields information

$$J = J_{org} + 1/2(\mathbf{x}_{ref} - M(\mathbf{x}_b + \delta\mathbf{x}))^T \mathbf{A}^{-1}(\mathbf{x}_{ref} - M(\mathbf{x}_b + \delta\mathbf{x}))$$

$$= J_{org} + 1/2(\mathbf{e}^t + \mathbf{M}\delta\mathbf{x})^T \mathbf{A}^{-1}(\mathbf{e}^t + \mathbf{M}\delta\mathbf{x})$$

$$J_{org} \equiv 1/2\delta\mathbf{x}^T \mathbf{B}^{-1}\delta\mathbf{x} + 1/2(\mathbf{d} - \mathbf{H}\delta\mathbf{x})^T \mathbf{R}^{-1}(\mathbf{d} - \mathbf{H}\delta\mathbf{x})$$

- Analytical solution has an error covariance matrix  $\mathbf{A}$  of reference information in Kalman gain, and forecast error in input data, as follows,

$$\delta\mathbf{x} = (\mathbf{B}^{-1} + \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} + \mathbf{M}^T \mathbf{A}^{-1} \mathbf{M})^{-1} \{ \mathbf{H}^T \mathbf{R}^{-1} \mathbf{d} - \mathbf{M}^T \mathbf{A}^{-1} \mathbf{e}^t \}$$

$\mathbf{X}$ : analysis,

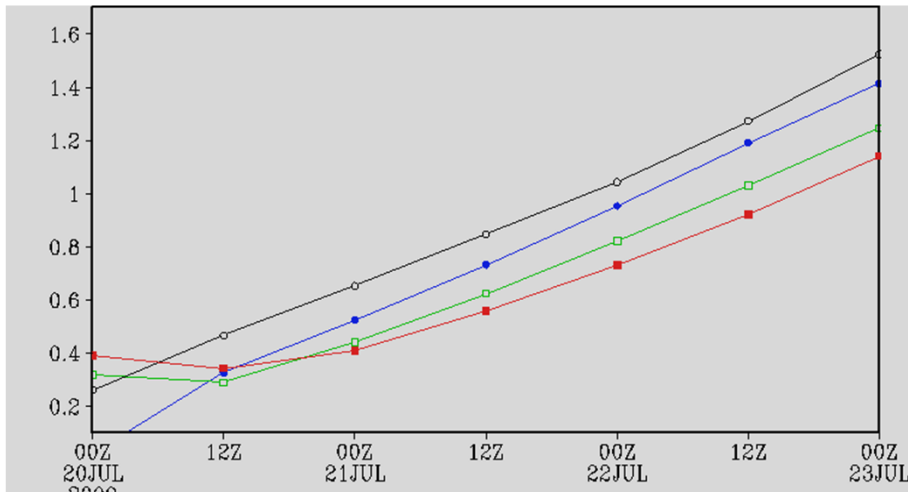
$\mathbf{Y}$ : observations

$\mathbf{X}_b$ : background field

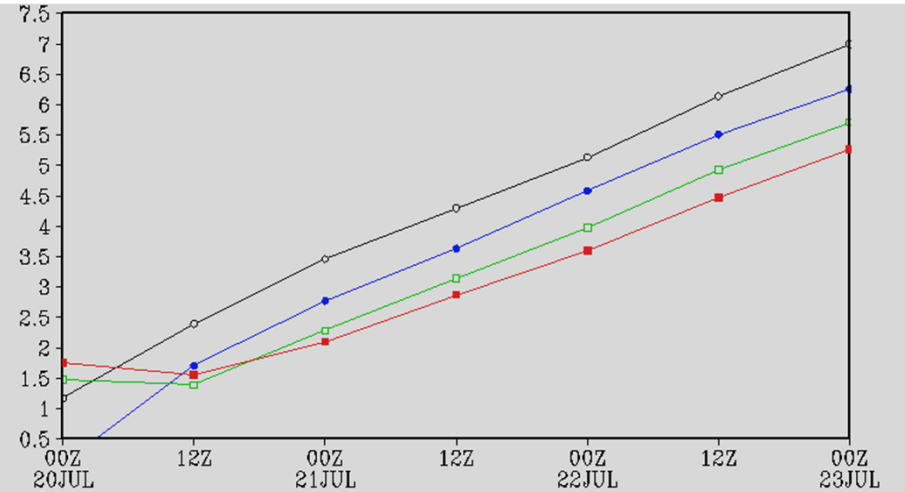
$\mathbf{X}_{ref}$ : reference  
analyses

# Accuracy of optimized forecasts





500hPa Temperature



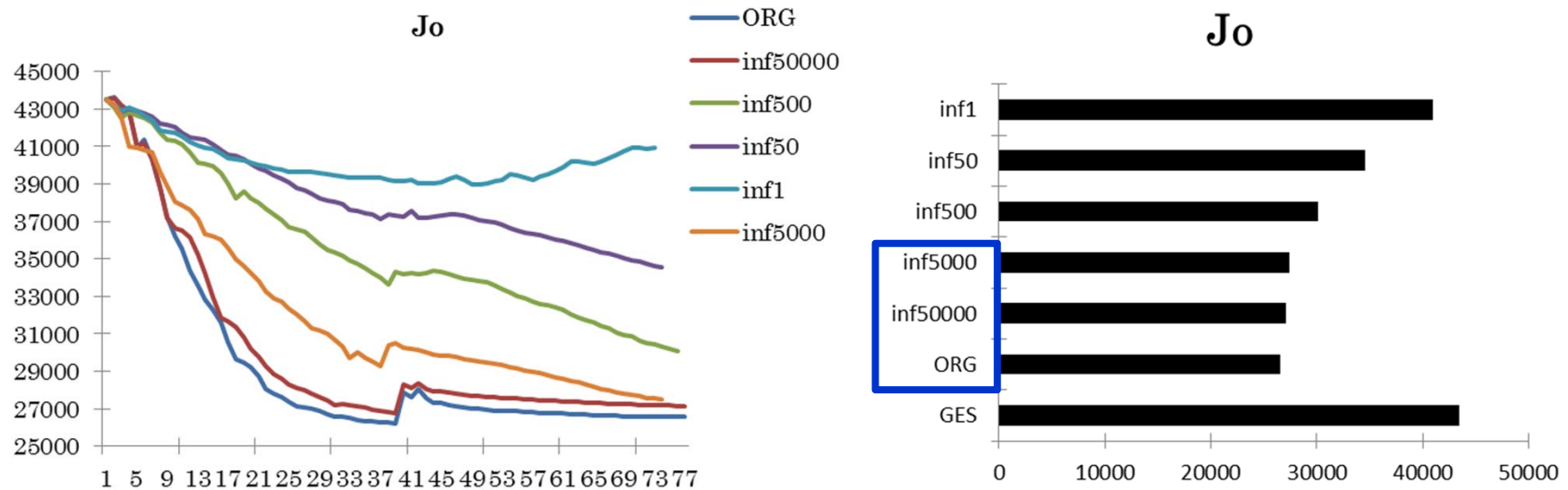
250hPa Zonal wind



**\*Inflation factor is one.**

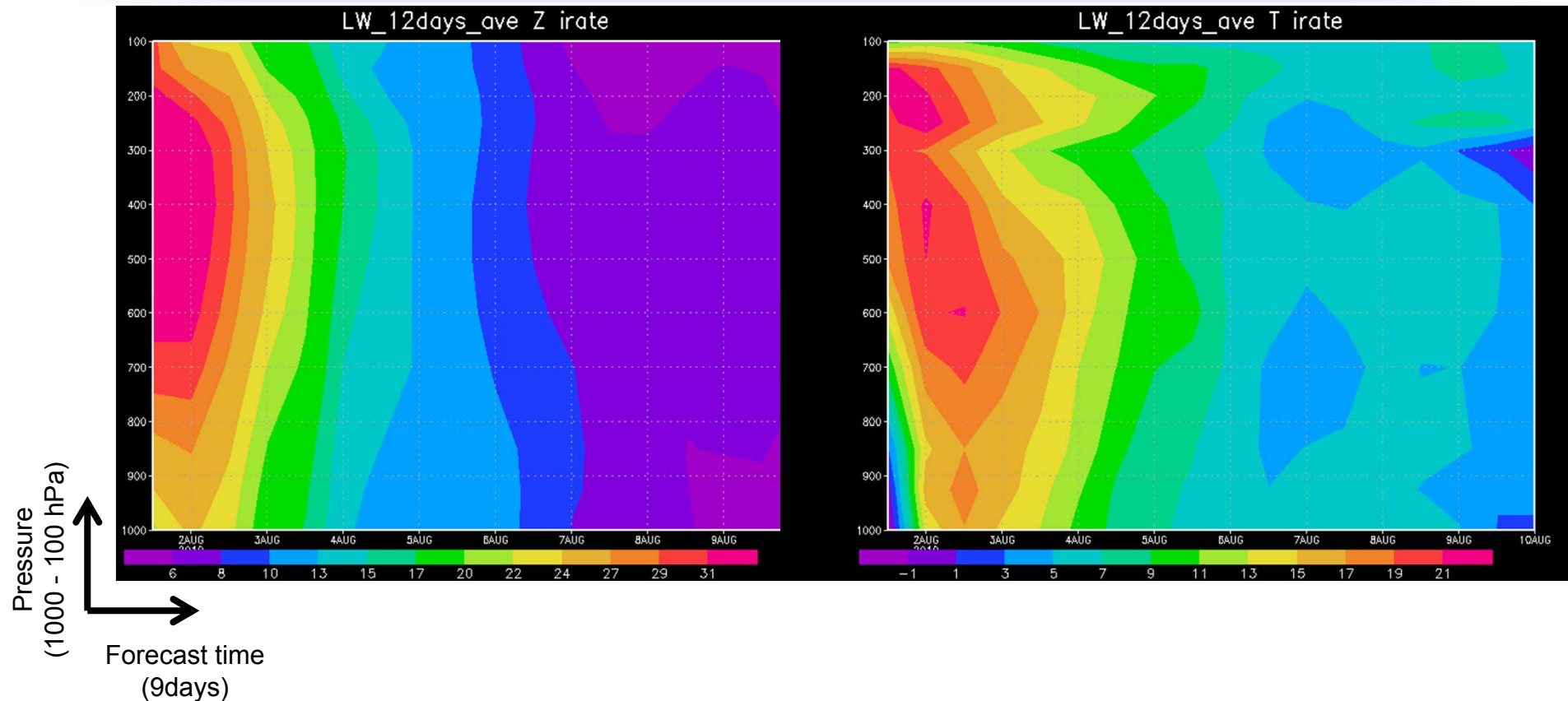
-  Optimized forecast with four reference analyses of every 6 hours.
-  Optimized forecast with only two reference analyses
-  Original forecast
-  Original forecast from 6 hours after initial.

# Fitting of the optimized analysis to observations



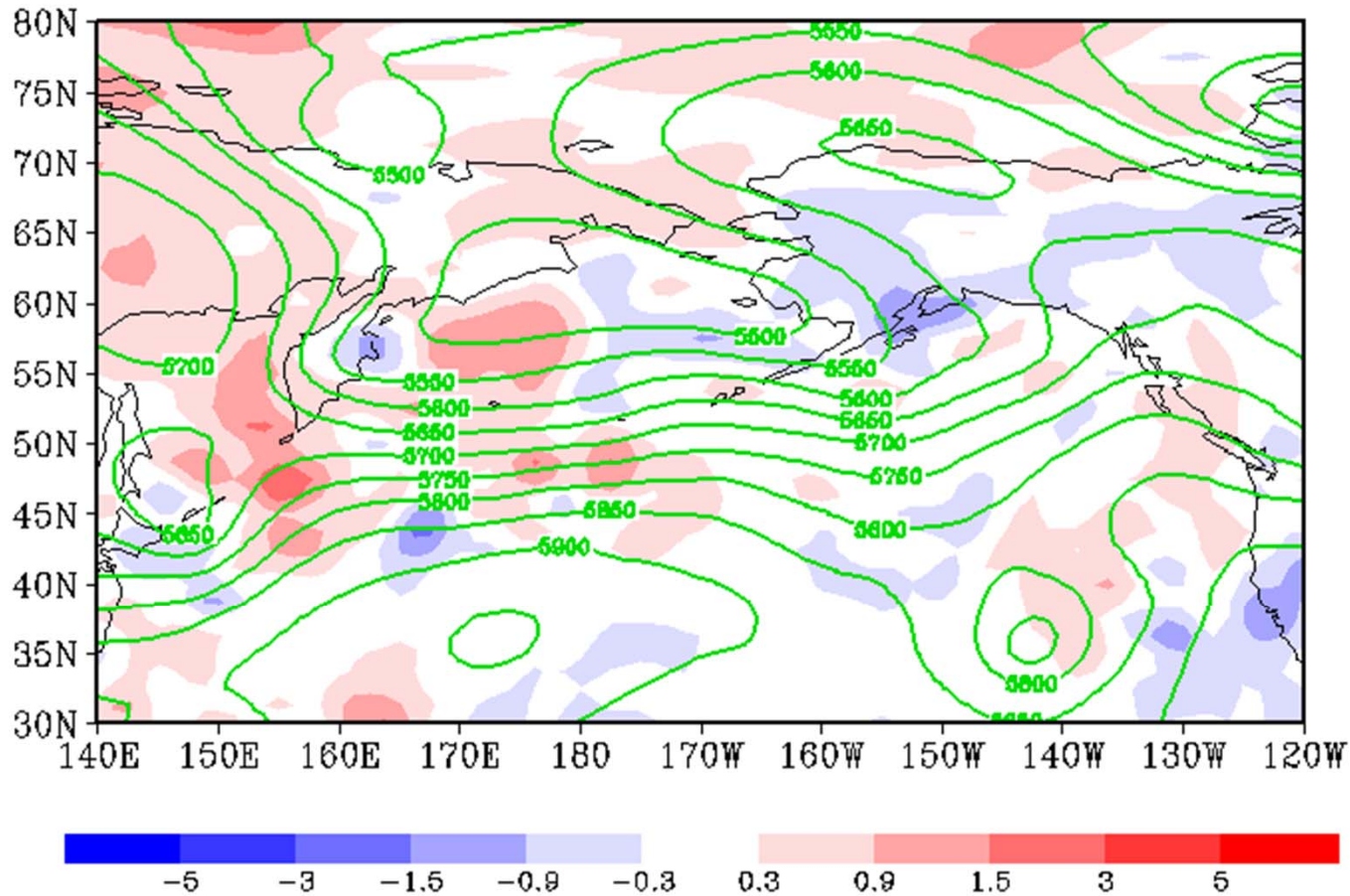
- The inflation factor dominates the fittings of analysis to observations.
- The inflation factors larger than 500 achieve good fitting to observations.

# Two weeks statistics



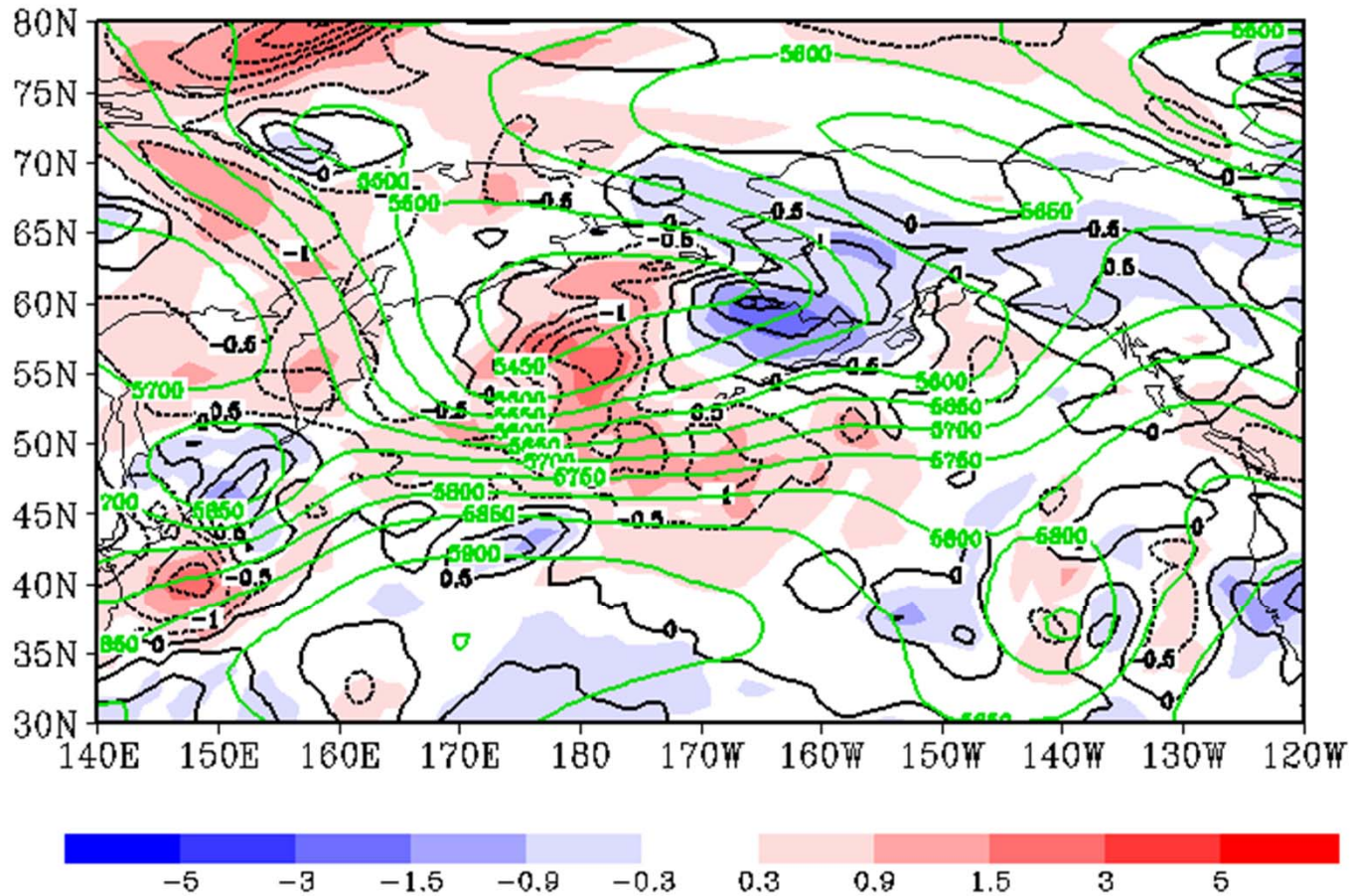
- Forecast accuracy improvement rate of the optimized forecasts against the original forecasts.
- Forecast accuracy are kept 9days with 95% statistical significance until 6 or 7days.
- The inflation factor is 5000.

# Optimized analysis increments and background error 072000



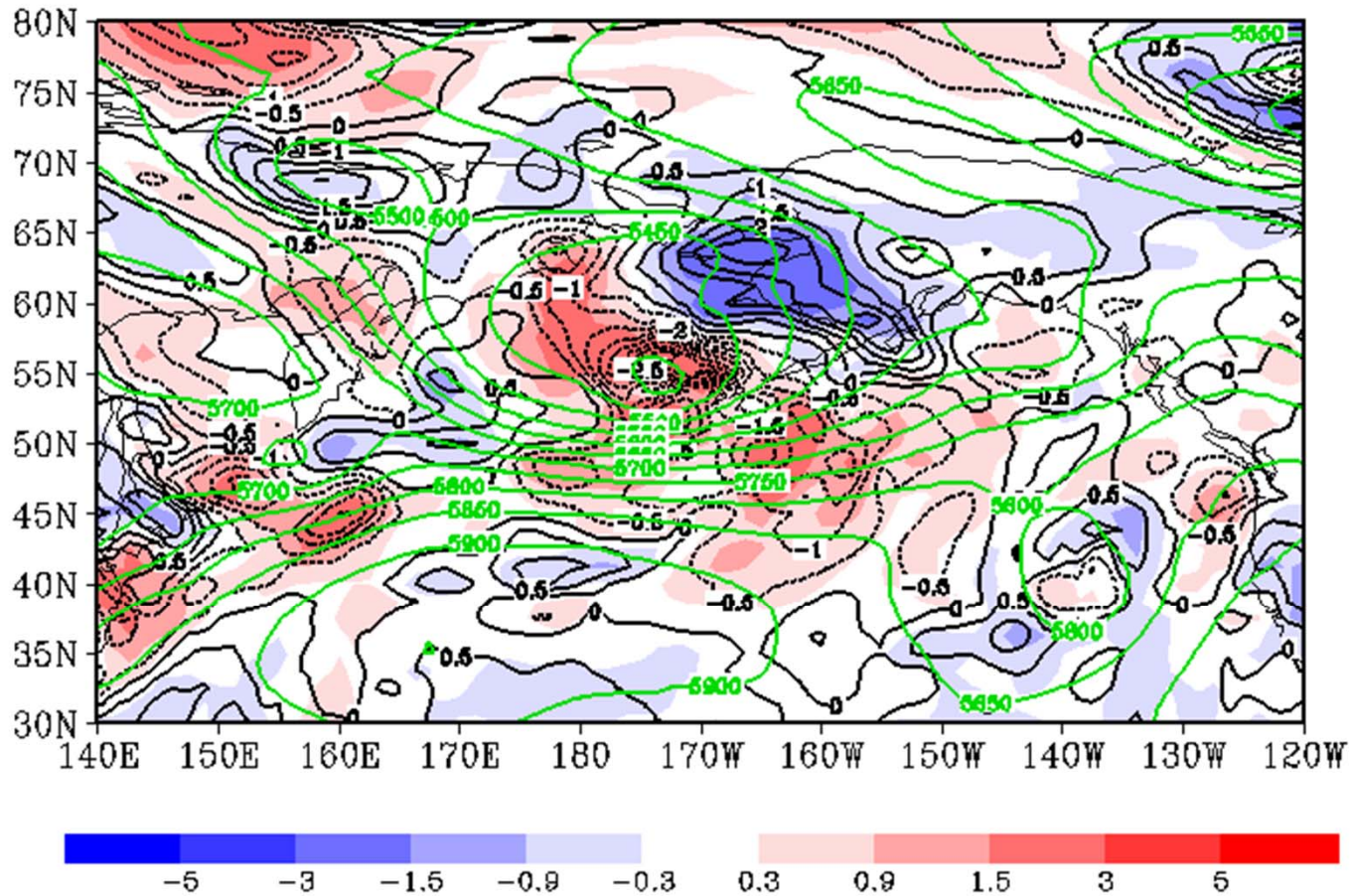
- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

# Optimized analysis increments and background error 072012



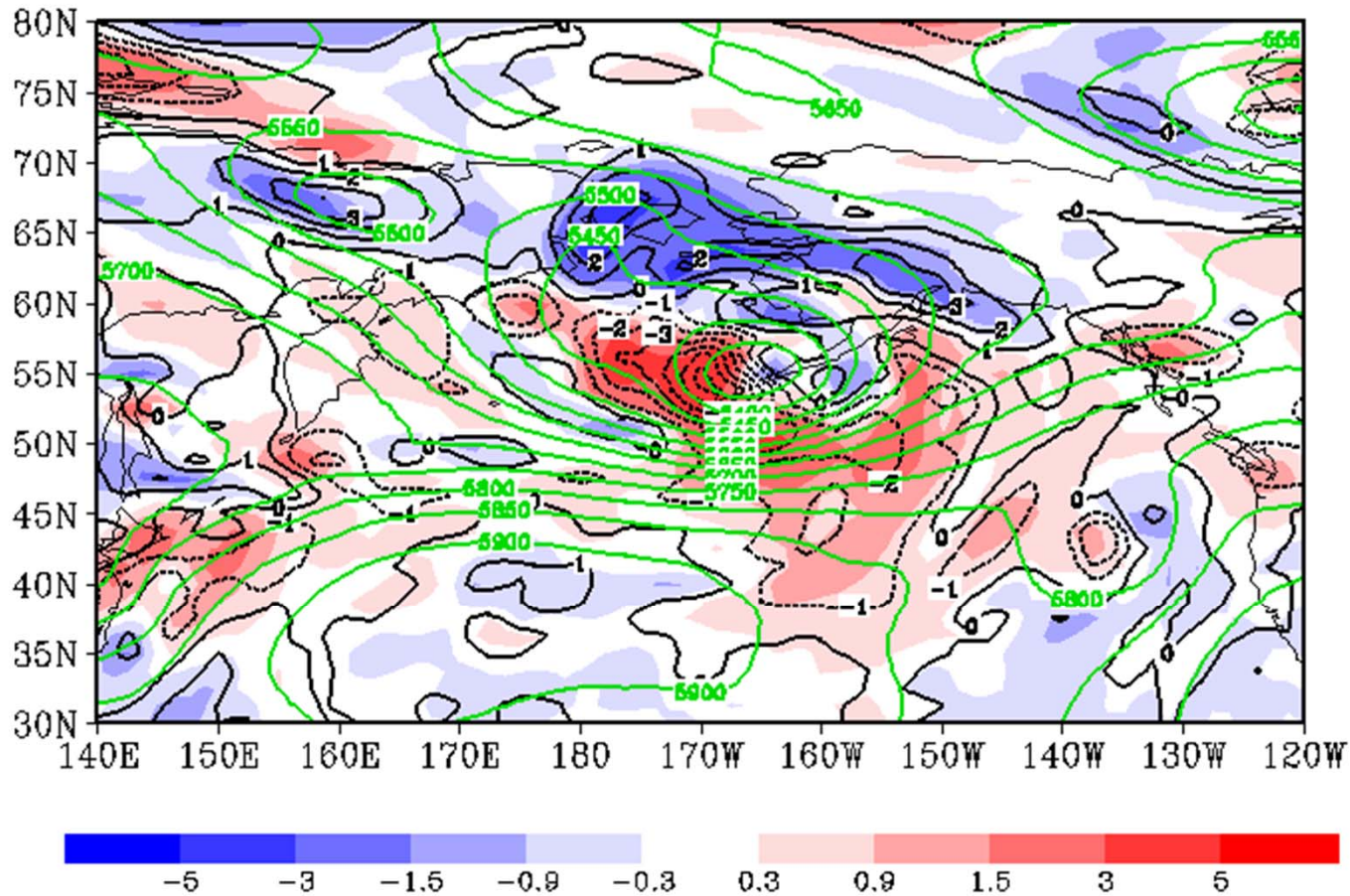
- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

# Optimized analysis increments and background error 072100



- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

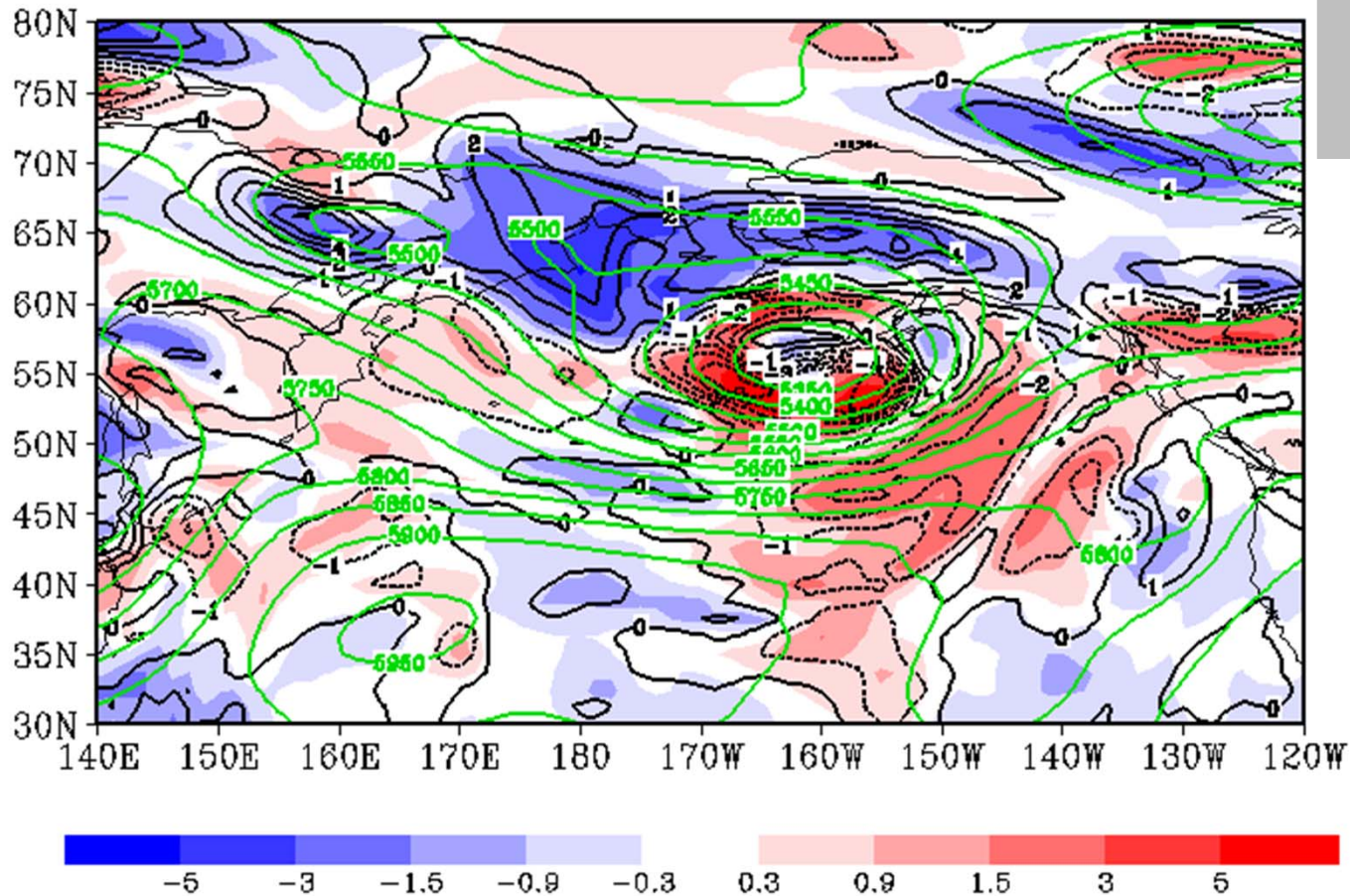
# Optimized analysis increments and background error 072112



- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

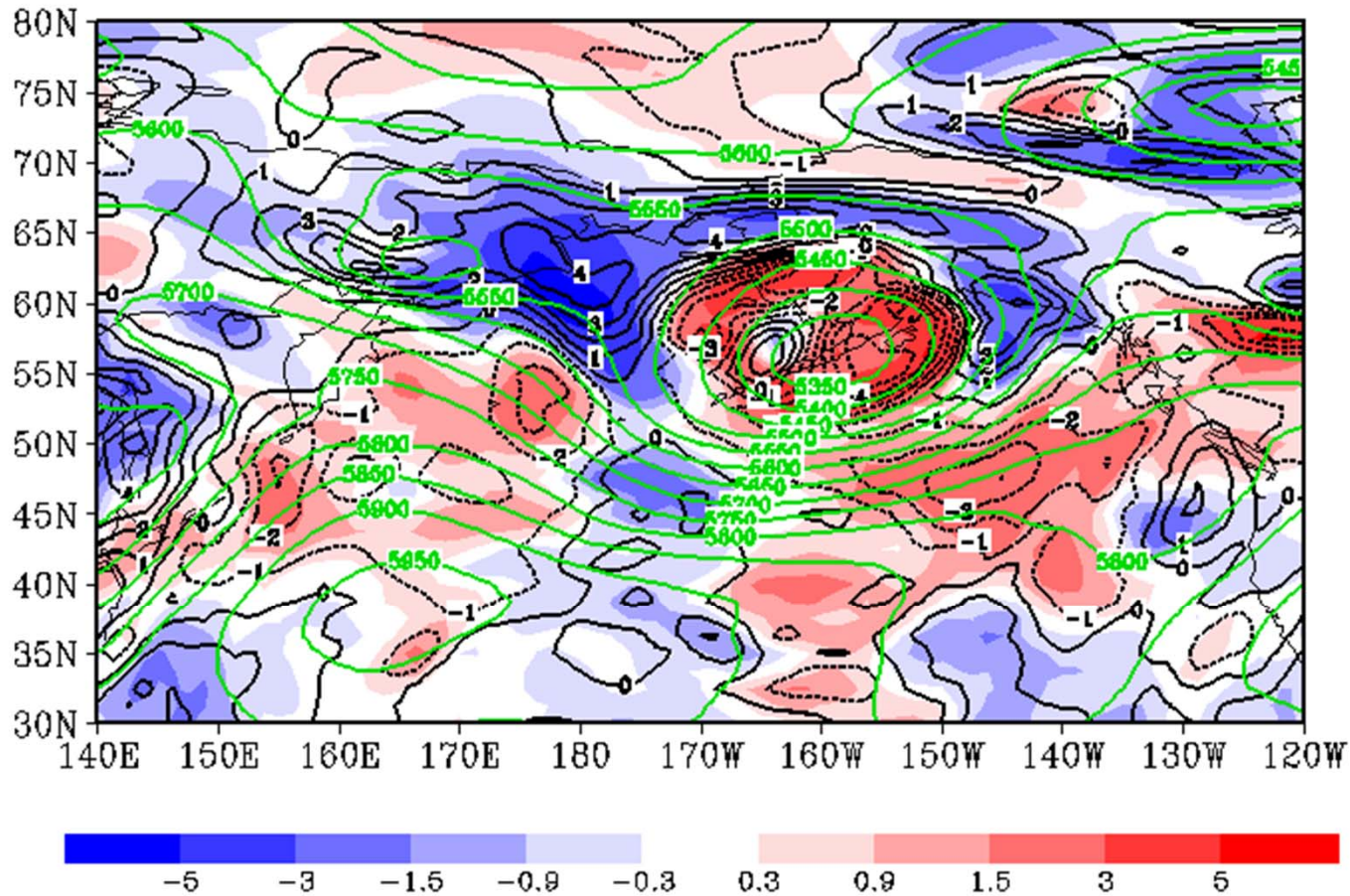


# Optimized analysis increments and background error 072200



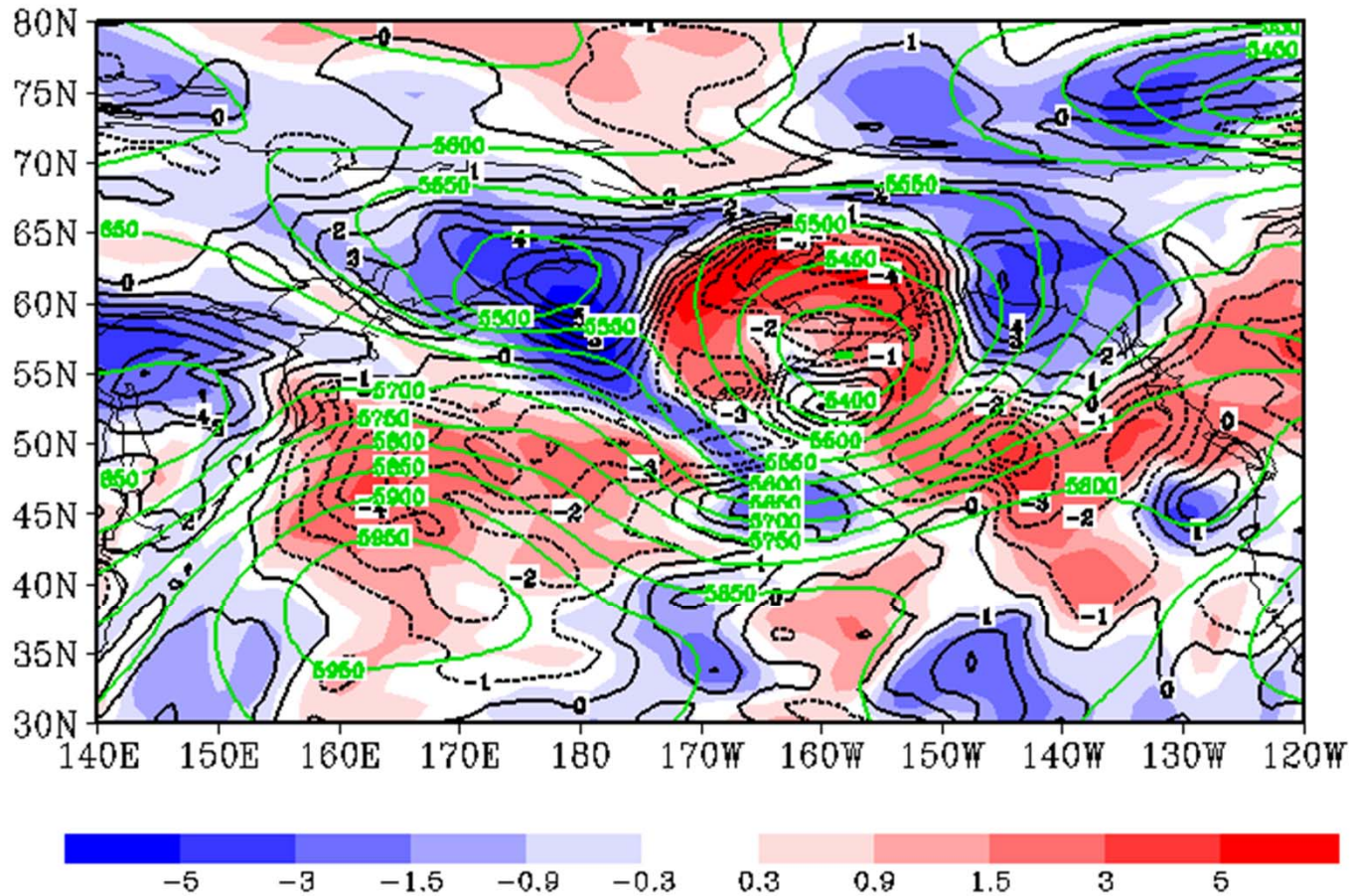
- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

# Optimized analysis increments and background error 072212



- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

# Optimized analysis increments and background error 072300



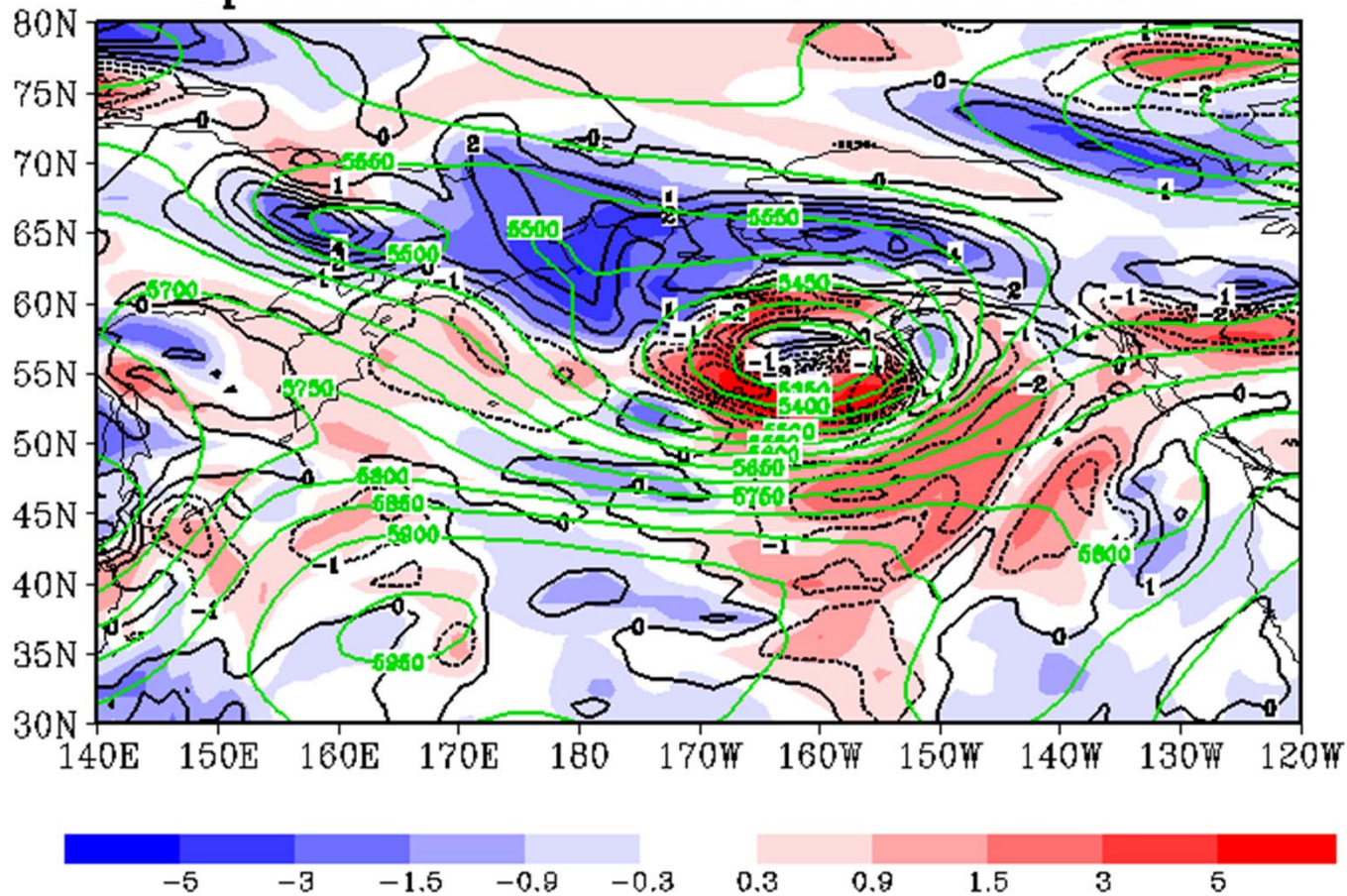
- Color shade: Optimized increments, red=plus, blue=minus.
- Green contour: 500hPa height.
- Black contour: Integrated background error, solid lines=plus, dotted lines=minus.

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# Comparison between original analysis and optimized analysis

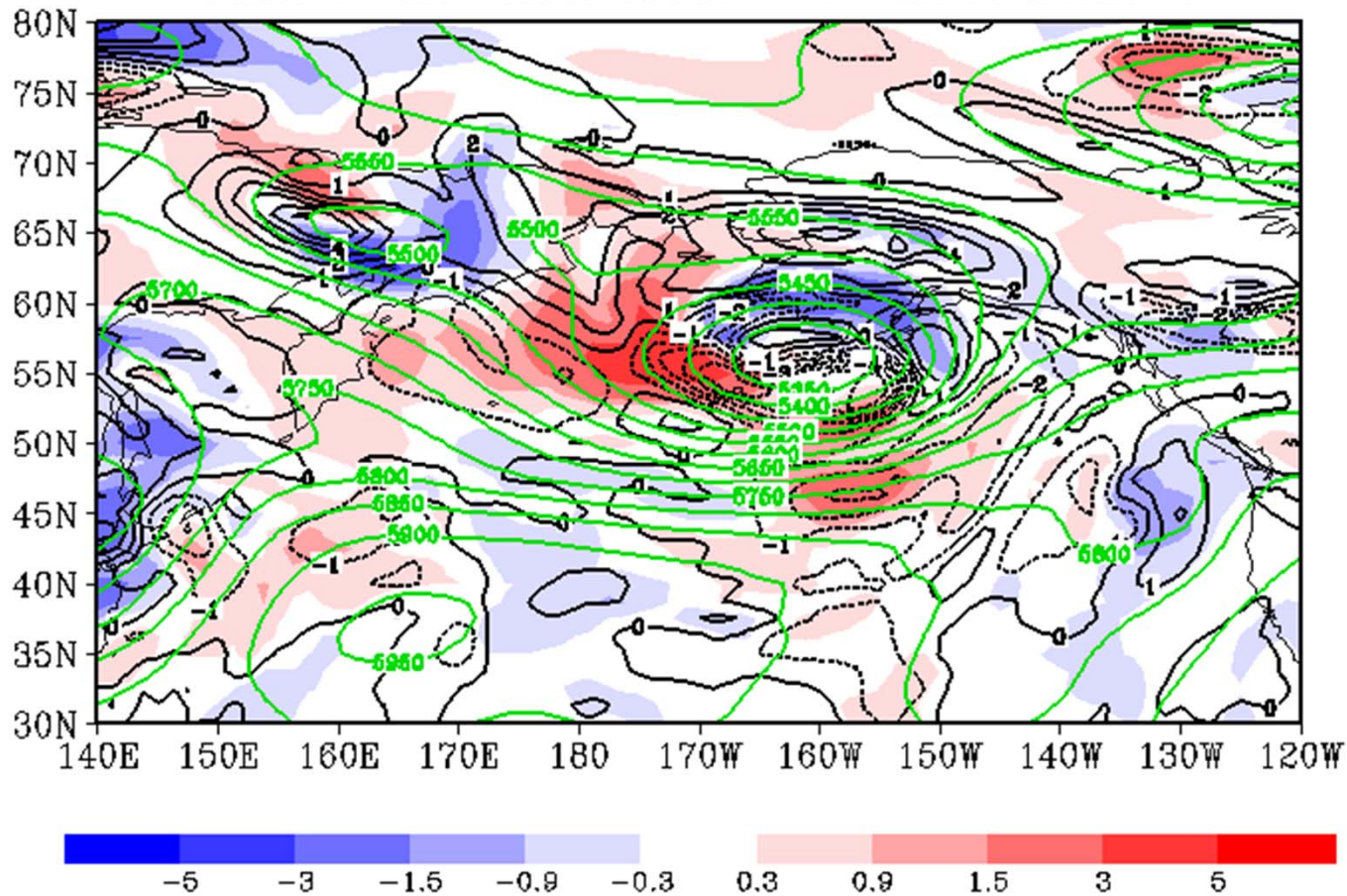
# Optimized :FT48 500hPa T

optimal inc and error t 500hPa 072200



# Original: FT48 500hPa T

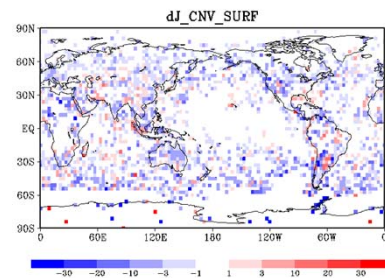
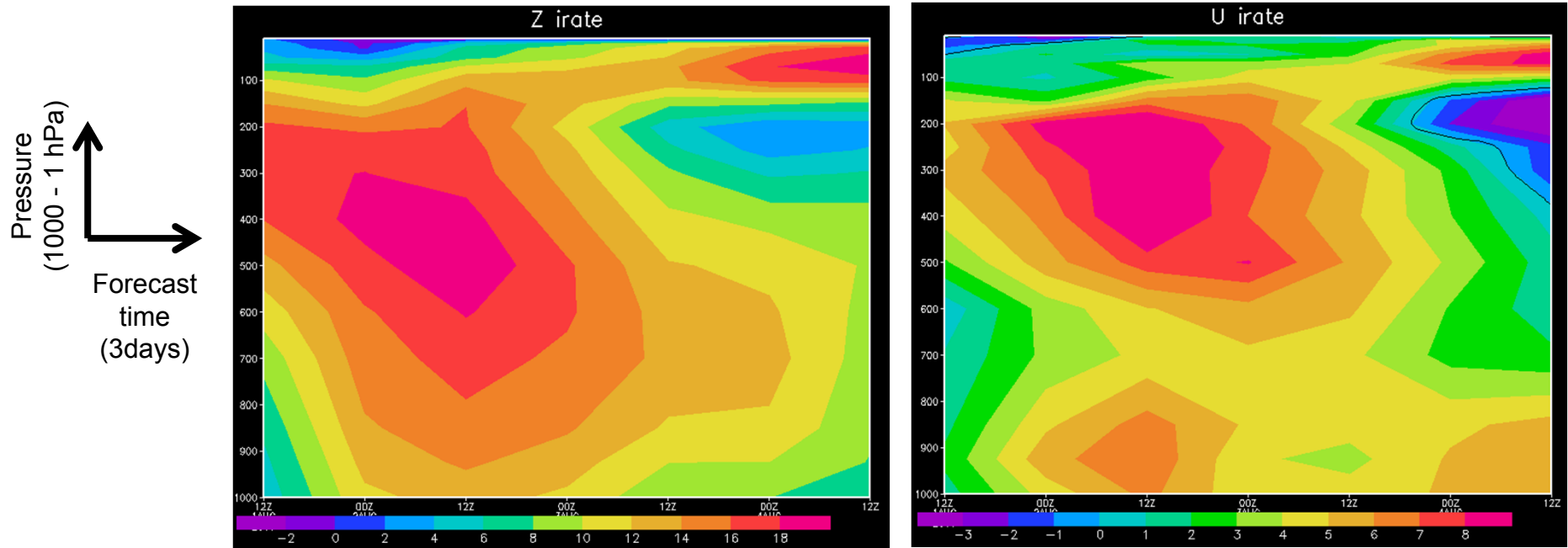
alldata inc and error t 500hPa 072200



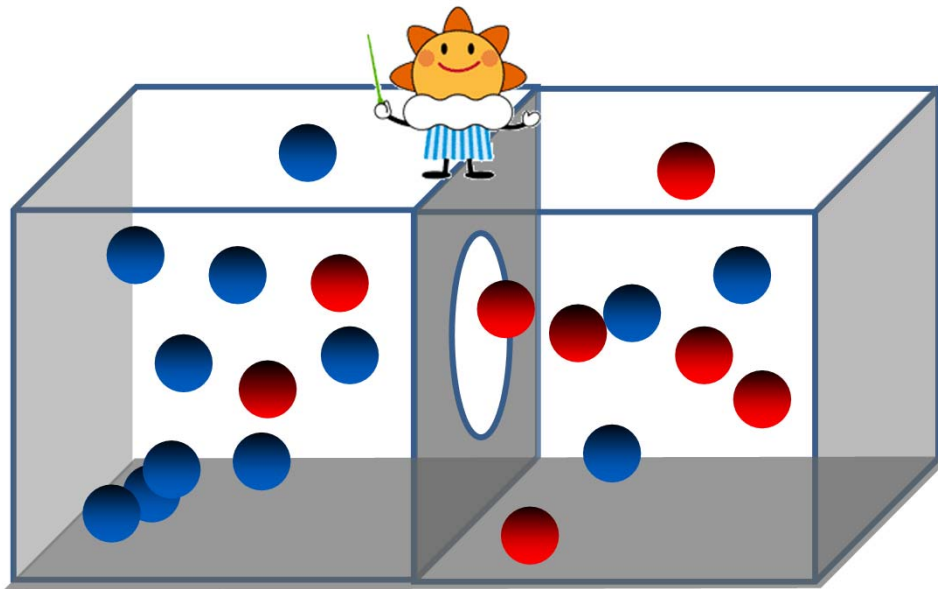
# Pseudo truth with ADJ-based method

Improvement rate: Height

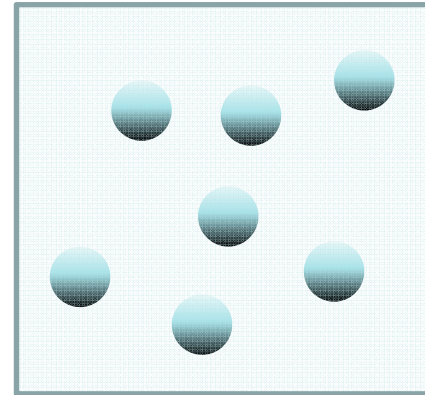
Improvement rate: Zonal wind



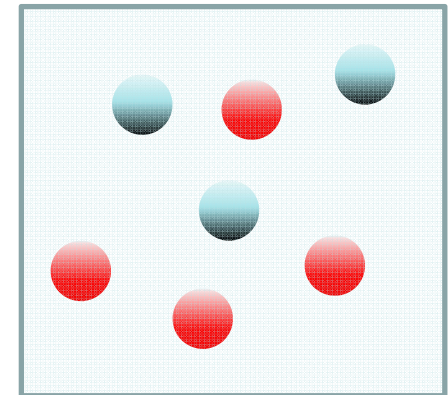
# Maxwell's demon ?



Lets think about a system on thermal equilibrium at temperature  $T$ . We know only statistical property of the system, temperature  $T$ . While, if one can know velocity of each particle, the one can get usable energy from this max entropy state, This is the Maxwell's demon.



We know only statistical property of data, R and B.



We know property of each observation and can use this information.



# Summary

## *Observation impact*

- We defined two types of observation impact; the linear impact and the non-linear impact.
- Diagnoses of the JMA global 4D-Var shows almost all observation data types contribute forecast error reduction in monthly average.
- The diagnoses imply that it is possible to derive more information from radiance data by improving usage of these data and operators.
- The TL-based method was introduced.
- We can see time evolution and space distribution of linear observation impacts, and evaluate them by comparison with those of integrated background errors.

## *Covariance matrix optimization*

- Optimization methods include observation impact estimations.
- Sensitivity based method diagnosed the JMA GDAS has too large (small)  $\mathbf{R}$  ( $\mathbf{B}$ ).
- The single case experiment of optimization showed the explicit forecast error reductions.

## *Analysis error estimation*

- We constructed new method based on data assimilation theory. The method assimilate reference analysis fields.
- The method reduce forecast error and also consistent with observations, if adequate inflation factor is given.
- ADJ-based method can be used to generate improved forecasts, so it may be possible to be used as pseudo truth.