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FOURTH MEETING OF
INTER-PROGRAMME EXPERT TEAM ON
DATA REPRESENTATION MAINTENANCE AND
MONITORING

ITEM 2.2

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Encoding fields from Limited Area models combining conformal projections and bi-periodic spectral geometry.

Submitted by Philippe Marguinaud, Toulouse (France)

Summary and Purpose of Document

This document proposes some new GRIB templates and tables entries to support specific issues of limited area models used in ALADIN, LACE and HIRLAM consortia.

ACTION PROPOSED

The meeting is requested to review the proposed new templates and tables and approve them for validation.

Introduction

This document explains the technical aspects which are specific to AROME and ALADIN limited area models and proposes new templates to encode fields produced by these model.

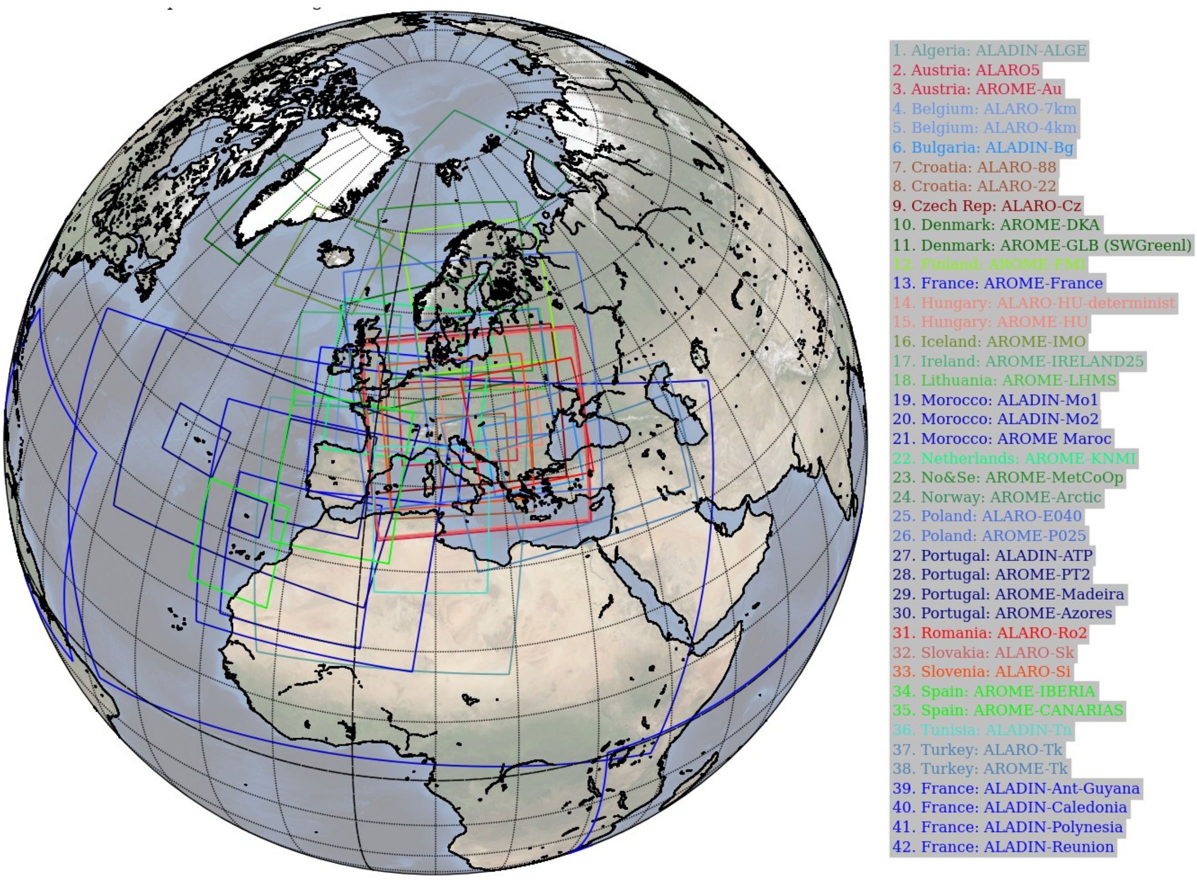
This document is organized as follows:

- The first section describes the theoretical background necessary to understand the details that may be important to justify the evolution of the standard requested in this document.
- The second section describes the evolutions required to encode limited area grid-point fields. These evolutions are only slight modifications of already existing templates.
- The third section describes the evolutions required to encode limited area spectral fields; this is the most complicated part of this document. Readers familiar with templates describing spherical harmonic fields (already part of the GRIB standard) will find similarities between these two topics.

Readers interested only in the technical changes affecting the templates can skip the first section, and look directly to the second and third sections.

1 Geometry of limited area models AROME and ALADIN

AROME and ALADIN are limited area models used in operations by about 25 European countries; their code development is organized in three consortia : ALADIN, LACE and HIRLAM.



Picture 1: Operational configurations used by ALADIN and HIRLAM consortia

These models are spectral models; that is, a field in AROME or ALADIN may have both a grid-point and a spectral representation. The concept is very similar to global models like IFS (used at ECMWF): a grid-point field can be projected onto a basis of harmonics, and spectral coefficients calculated thereby can be used to reconstruct a grid-point field. For global models, the basis of functions is a set of spherical harmonics, while for limited area models (LAM in the rest of this document), bi-Fourier harmonics are used.

LAM grid-point fields also have some specific features: they are bi-periodic, and some of their values should be interpreted with caution.

Eventually, it is worth noting that LAM models are forced by lateral boundary conditions (LBC in the rest of this document); these LBC are typically provided by a global model such as IFS or ARPEGE.

1.1 LAM projections

LAM models currently use three kinds projections (which already exist in GRIB):

- Lambert conformal (for mid-latitudes domains)

- Polar stereographic (for domains near or over the poles)
- Mercator (for domains near the equator)

The ability to choose between these three kinds of projections makes it possible to keep a mapping factor close to 1.

1.2 LAM domain decomposition

Because of their dual representation (both grid-point and spectral), LAM fields are required to be bi-periodic. Moreover, LBC coupling occurs in a part of the domain.

The following diagram shows the different parts of LAM fields (the outer frame encompasses the whole LAM domain) :

- I is the area where coupling with LBC takes place
- C is the useful area where the model produces a forecast
- E is the extension area, used to make fields bi-periodic

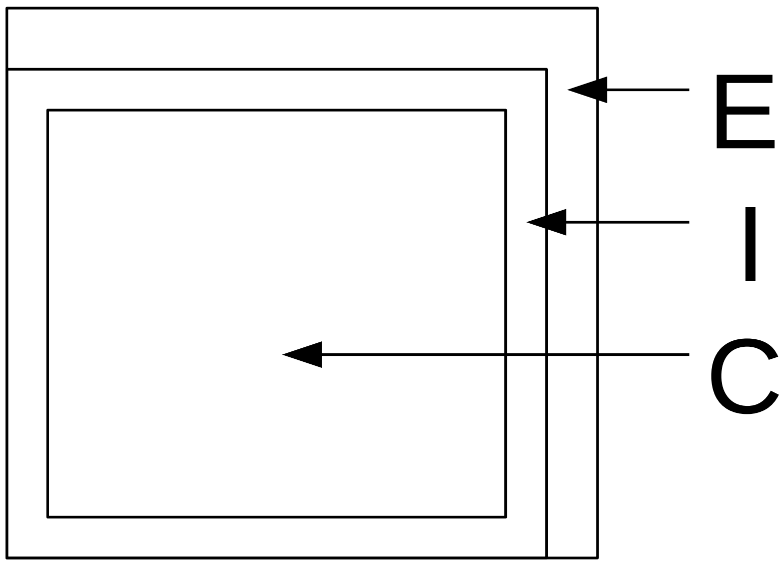


Diagram 1: AROME and ALADIN geometry

Note that only values in C and I are sensible. Values in E do not have a physical interpretation, but are required by the model. Note also that the bi-periodicisation is not a trivial process: in order to avoid spurious waves and enforce numerical stability, fields have to be well-balanced, and the 3D state of the model has to be consistent. Therefore, values in E zone are part of the field, and should be kept at all time with values located in C and I zones.

For Météo-France operational AROME running using a 1.3km mesh over France, the width of E is 11 grid points, the width of I is 16 grid points, while the size of the whole domain is 1440 x 1536 grid points.

Eventually, please note that the concept of C, I and E zones apply equally to both grid-point and spectral fields.

1.3 LAM spectral fields

1.3.1 Notations

We take here the notations from "ARPEGE/ALADIN, adiabatic model equations and algorithm, A. Joly, 1992".

Let $Q(x,y)$ be a two dimensional field on a rectangular domain of size L_x , L_y . It is well known, that such a field can be approximated by a sum of complex exponentials :

$$Q(x)=\sum_{m=-M}^{+M}\sum_{n=-N}^{+N} Q_m^n e^{i\cdot m(\frac{2\pi}{L_x})\cdot x} e^{i\cdot n(\frac{2\pi}{L_y})\cdot y}$$

Where bi-Fourier coefficients are calculated using a discrete Fourier transform:

$$Q_m^n=\frac{1}{J\times K}\sum_{j=0}^{J-1}\sum_{k=0}^{K-1} Q(x_j, y_k) e^{-i\cdot m(\frac{2\pi}{L_x})\cdot x_j} e^{-i\cdot n(\frac{2\pi}{L_y})\cdot y_k}$$

With :

$$x_j=\frac{j\cdot L_x}{J}, y_k=\frac{k\cdot L_y}{K}$$

The formulas above approximate Q by a sum of complex exponentials over a rectangular bi-Fourier truncation: that is, m varies from -M to +M and n varies to -N to +N, the couple (n, m) spawning a rectangle.

But it is possible (and this what is actually done in ALADIN and AROME) to restrict (n, m) to an ellipse, thus giving a spatial isotropic spectral representation of the field :

$$\frac{m^2}{M^2}+\frac{n^2}{N^2}\leqslant 1$$

1.3.2 Computational perspectives

When computing the Fourier transforms of a sequence of 2N real numbers, fast Fourier calculation software usually yields a sequence of N complex numbers.

Computing the two-dimensional FFT of the real (that is not complex) field $Q(x, y)$ involves therefore two steps:

- a FFT over the x dimension :

$$Q(x, y) \xrightarrow{FFT_x} (Q_{mr}(y), Q_{mi}(y))_{m=0..M}$$

- a FFT over the y dimension of the pair of coefficients obtained in previous step:

$$Q_{mr}(y) \xrightarrow{FFT_y} (Q_{mr}^{nr}, Q_{mr}^{ni})_{m=0..M, n=0..N}$$

$$Q_{mi}(y) \xrightarrow{FFT_y} (Q_{mi}^{nr}, Q_{mi}^{ni})_{m=0..M, n=0..N}$$

Hence, for each $m=0..M, n=0..N$, we obtain a quadruplet of coefficients :

$$(Q_{mr}^{nr}, Q_{mr}^{ni}, Q_{mi}^{nr}, Q_{mi}^{ni})$$

Going back to the grid-point space involves applying inverse FFTs to the quadruplets of spectral coefficients.

As the result of a two-dimensional FFT applied to a real field is such a list of quadruplets, it is more convenient to work with $(Q_{mr}^{nr}, Q_{mr}^{ni}, Q_{mi}^{nr}, Q_{mi}^{ni})$ than with (Q_m^n) , and this is what is done in AROME and ALADIN.

1.3.3 The Laplacian operator and its role in data packing algorithms

Let $Q(x, y)$ be a two-dimensional field, and $(Q_{mr}^{nr}, Q_{mr}^{ni}, Q_{mi}^{nr}, Q_{mi}^{ni})$, its spectral representation as defined in the previous sections.

Reckoning the spectral coefficients of the Laplacian of $Q(x, y)$ only requires computing the Laplacian of basis functions $e^{i \cdot m (\frac{2\pi}{L_x}) \cdot x} e^{i \cdot n (\frac{2\pi}{L_y}) \cdot y}$; applying $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ to this product of exponentials yields :

$$4\pi^2 ((\frac{m}{L_x})^2 + (\frac{n}{L_y})^2) \times e^{i \cdot m (\frac{2\pi}{L_x}) \cdot x} e^{i \cdot n (\frac{2\pi}{L_y}) \cdot y}$$

Therefore, the spectral coefficients of $\Delta Q(x, y)$ are :

$$4\pi^2\left(\left(\frac{m}{L_x}\right)^2+\left(\frac{n}{L_y}\right)^2\right)\times\left(Q_{mr}^{nr},Q_{mr}^{ni},Q_{mi}^{nr},Q_{mi}^{ni}\right)$$

Assuming the size of the domain is a square (in real life, it is generally very close to a square, and this will be considered good enough for packing purposes) and forgetting about the scaling by $4\frac{\pi^2}{L_x^2}$, applying a power L of the Laplacian operator to the spectral representation of a field is equivalent to multiplying its spectral coefficients by : $(m^2+n^2)^L$. This simplified Laplacian operator will be used to minimize the range of values taken by the spectral coefficients before packing.

2 New templates for grid-point fields geometry (section 3)

We request three new templates:

- Lambert conformal LAM with explicit boundary for bi-periodic field (3.33); this will be an extension of template 3.30
- Polar stereographic LAM with explicit boundary for bi-periodic field (3.23); this will be an extension of template 3.20
- Mercator LAM with explicit boundary for bi-periodic field (3.13); this will be an extension of template 3.10

These three templates are mere extensions of already existing templates; only the following parameters have to be appended:

Type	Description
unsigned[4]	Nux - Size of model forecast area C in X direction (number of grid points)
unsigned[4]	Ncx - Width of coupling area I in X direction (number of grid points)
unsigned[4]	Nuy - Size of model forecast area C in Y direction (number of grid points)
unsigned[4]	Ncy - Width of coupling area I in Y direction (number of grid points)

We detail here Lambert conformal LAM with explicit boundary for bi-periodic field (3.33); new parameters are at the end of the table:

Octet No.	Contents
15	Shape of the Earth (See Code Table 3.2)

16	Scale factor of radius of spherical Earth
17-20	Scale value of radius of spherical Earth
21	Scale factor of major axis of oblate spheroid Earth
22-25	Scaled value of major axis of oblate spheroid Earth
26	Scale factor of minor axis of oblate spheroid Earth
27-30	Scaled value of minor axis of oblate spheroid Earth
31-34	Nx — number of points along the x-axis
35-38	Ny — number of points along the y-axis
39-42	La_1— latitude of first grid point
43-46	Lo_1— longitude of first grid point
47	Resolution and component flags (see Flag Table 3.3)
48-51	LaD — latitude where Dx and Dy are specified
52-55	LoV — longitude of meridian parallel to y-axis along which latitude increases as the y-coordinate increases
56-59	Dx — x-direction grid length (see Note 1)
60-63	Dy — y-direction grid length (see Note 1)
64	Projection centre flag (see Flag Table 3.5)
65	Scanning mode (see Flag Table 3.4)
66-69	Latin 1 — first latitude from the pole at which the secant cone cuts the sphere
70-73	Latin 2 — second latitude from the pole at which the secant cone cuts the sphere
74-77	Latitude of the southern pole of projection
78-81	Longitude of the southern pole of projection

82-85	Nux – Size of model forecast area C in X direction (number of grid points)
86-89	Ncx – Width of coupling area I in X direction (number of grid points)
90-93	Nuy – Size of model forecast area C in Y direction (number of grid points)
94-97	Ncy – Width of coupling area I in Y direction (number of grid points)

3 New templates for LAM spectral fields (section 3, 5, 7)

3.1 Geometry (section 3)

As explained in the previous sections, LAM fields are defined on either Lambert conformal, Mercator or Polar stereographic projections. In the case of spectral fields, however, existing templates for these projections cannot be used as such, because they involve parameters which are strongly associated with a grid point representation (Dx, Dy, Nx, Ny). Let us recall that a spectral field is defined everywhere and not at particular grid points (the same applies to spectral fields on the sphere).

As it is not possible to describe any dimension in terms of number of grid points, Nx, Ny, Nux, Ncx, Nuy, Ncy cannot be used and have to be replaced by the following parameters :

Type	Description
unsigned[8]	Lx - Size in meters of the domain along X axis
unsigned[8]	Lux - Size in meters of model forecast area C along X axis
unsigned[8]	Lcx - Width in meters of coupling area I along X axis
unsigned[8]	Ly - Size in meters of the domain along Y axis
unsigned[8]	Luy - Size in meters of model forecast area C along Y axis
unsigned[8]	Lcy - Width in meters of coupling area I along Y axis

For spectral coefficients on the sphere, it is necessary to define several parameters to describe the truncation (triangular, rhomboidal, etc...); the same applies to bi-Fourier coefficients, and the following parameters are required to fully describe the bi-Fourier truncation:

We need to define three kinds of truncations :

- Rectangular $m=0..M, n=0..N$

- Elliptic $\frac{m^2}{M^2} + \frac{n^2}{N^2} \leqslant 1$

- Diamond $\frac{m}{M} + \frac{n}{N} \leqslant 1$

This will eventually yield three new templates (parameters taken from Lambert conformal 3.30, Mercator 3.10 and polar stereographic 3.20 are highlighted):

- Spectral Lambert conformal LAM with explicit boundary for bi-periodic field 3.63

Octet No.	Contents
15	Spectral Representation type (see Code table 3.6)
16-19	N – bi-Fourier Resolution Parameter
20-23	M – Bi-Fourier Resolution Parameter
24	Bi-Fourier Truncation Type (see Code table 3.25)
25-32	Lx - Size in meters of the domain along X axis
33-40	Lux - Size in meters of model forecast area C along X axis
41-48	Lcx - Width in meters of coupling area I along X axis
49-56	Ly - Size in meters of the domain along Y axis
57-64	Luy - Size in meters of model forecast area C along Y axis
65-72	Lcy - Width in meters of coupling area I along Y axis
73	Shape of the Earth (see Code table 3.2)
74	Scale factor of radius of spherical Earth
75-78	Scaled value of radius of spherical Earth
79	Scale factor of major axis of oblate spheroid Earth
80-83	Scaled value of major axis of oblate spheroid Earth
84	Scale factor of minor axis of oblate spheroid Earth
85-88	Scaled value of minor axis of oblate spheroid Earth

89-92	La1 - latitude of first grid point
93-96	Lo1 - longitude of first grid point
97-100	LaD - latitude where Dx and Dy are specified
101-104	LoV - longitude of meridian parallel to y-axis along which latitude increases as the y-coordinate increases
105	Projection centre flag (see Flag table 3.5)
106-109	Latin 1 - first latitude from the pole at which the secant cone cuts the sphere
110-113	Latin 2 - second latitude from the pole at which the secant cone cuts the sphere
114-117	Latitude of the southern pole of projection
118-121	Longitude of the southern pole of projection

- Spectral Mercator LAM with explicit boundary for bi-periodic field 3.61

Octet No.	Contents
15	Spectral Representation type (see Code table 3.6)
16-19	N – bi-Fourier Resolution Parameter
20-23	M – Bi-Fourier Resolution Parameter
24	Bi-Fourier Truncation Type (see Code table 3.25)
25-32	Lx - Size in meters of the domain along X axis
33-40	Lux - Size in meters of model forecast area C along X axis
41-48	Lcx - Width in meters of coupling area I along X axis
49-56	Ly - Size in meters of the domain along Y axis
57-64	Luy - Size in meters of model forecast area C along Y axis
65-72	Lcy - Width in meters of coupling area I along Y axis
73	Shape of the Earth (see Code table 3.2)

74	Scale factor of radius of spherical Earth
75-78	Scaled value of radius of spherical Earth
79	Scale factor of major axis of oblate spheroid Earth
80-83	Scaled value of major axis of oblate spheroid Earth
84	Scale factor of minor axis of oblate spheroid Earth
85-88	Scaled value of minor axis of oblate spheroid Earth
89-92	La1 - latitude of first grid point
93-96	Lo1 - longitude of first grid point
97-100	LaD - latitude(s) at which the Mercator projection intersects the Earth (Latitude(s) where Di and Dj are specified)
101-104	La2 - latitude of last grid point
105-108	Lo2 - longitude of last grid point
109-112	Orientation of the grid, angle between i direction on the map and the Equator (see Note 1)

Notes:

(1) Limited to the range of 0 to 90 degrees.

- Spectral polar stereographic LAM with explicit boundary for bi-periodic field 3.62

Octet No.	Contents
15	Spectral Representation type (see Code table 3.6)
16-19	N – bi-Fourier Resolution Parameter
20-23	M – Bi-Fourier Resolution Parameter
24	Bi-Fourier Truncation Type (see Code table 3.25)
25-32	Lx - Size in meters of the domain along X axis
33-40	Lux - Size in meters of model forecast area C along X axis

41-48	Lcx - Width in meters of coupling area I along X axis
49-56	Ly - Size in meters of the domain along Y axis
57-64	Luy - Size in meters of model forecast area C along Y axis
65-72	Lcy - Width in meters of coupling area I along Y axis
73	Shape of the Earth (see Code table 3.2)
74	Scale factor of radius of spherical Earth
75-78	Scaled value of radius of spherical Earth
79	Scale factor of major axis of oblate spheroid Earth
80-83	Scaled value of major axis of oblate spheroid Earth
84	Scale factor of minor axis of oblate spheroid Earth
85-88	Scaled value of minor axis of oblate spheroid Earth
89-92	La1 - latitude of first grid point
93-96	Lo1 - longitude of first grid point
97	Resolution and component flags (see Flag table 3.3)
98-101	LaD - latitude where Dx and Dy are specified
102-105	LoV - orientation of the grid
106	Projection centre flag (see Flag table 3.5)

3.2 Data representation (section 5)

The meta-data required here is very similar to those needed for global spectral data: the spectra are multiplied by a power of the Laplacian operator before being packed. A sub-truncation may be left unpacked either in IEEE-32 or IEEE-64 precision, as well as spectral coefficients located on the M and N axes.

This yields the following template:

Spectral LAM data - complex packing 5.53

Octet No.	Contents
12-15	Reference value (R) (IEEE 32-bit floating-point value)

16-17	Binary scale factor (E)
18-19	Decimal scale factor (D)
20	Number of bits used for each packed value (field width)
21	Bi-Fourier Sub-Truncation Type (see Code table 5.25)
22	Packing mode for Axes (see Code table 5.26)
23-26	P - Laplacian scaling factor (expressed in 10-6 units)
27-28	NS - Bi-Fourier resolution parameter of the unpacked subset (see Note 1)
29-30	MS - Bi-Fourier resolution parameter of the unpacked subset (see Note 1)
31-34	TS – total number of values in the unpacked subset (see Note 1)
35	Precision of the unpacked subset (see Code table 5.7)

Notes:

(1) The unpacked subset is a set of values defined in the same way as the full set of values (on a spectrum limited to NS and MS), but on which scaling and packing are not applied. Associated values are stored in octets 6 onwards of Section 7.

(2) The remaining coefficients are multiplied by $(n^2 + m^2)^P$, scaled and packed. The operator associated with this multiplication is derived from the Laplacian operator.

(3) The retrieval formula for a coefficient of wave number n is then: $Y = (R + X \times 2^E) \times 10^{-D} \times (m^2 + n^2)^{-P}$ where X is the packed scaled value associated with the coefficient.

3.3 Packed data (section 7)

For spherical harmonics coefficients, unpacked data is stored at the beginning of section 7, and are followed by packed coefficients. We choose to do that for bi-Fourier coefficients as well.

Let us recall that the unpacked coefficients may be located on M and N axes (if Packing mode for Axes is true) as well as in a sub-truncation. Packed coefficients are applied a power of the Laplacian operator before being packed. The calculation of this power of Laplacian is not meant to be defined by the standard; instead, finding its optimal value is left to encoding software developers.

The existence of the “Packing mode for Axes” parameter is justified, because half of the coefficients located on axes are equal to zero. Removing them from the set of packed coefficients may have a positive impact on the packing error.

Similarly to spherical harmonics coefficients, coefficients are stored N-wise (that is, when going through an array of coefficients, N is the dimension which changes fastest), but we store here quadruplets $\left(Q_{mr}^{nr}, Q_{mr}^{ni}, Q_{mi}^{nr}, Q_{mi}^{ni}\right)$, as explained in the previous sections.

This yelds to the following template:

Spectral LAM data - complex packing 7.53

Octet No.	Contents
6-(5+IxTS)	Data values from the unpacked subset (IEEE floating-point values on I octets)
(6+IxTS)–nn	Binary data values – binary string, with each (scaled) data value out of the unpacked subset

4 New tables entries in table 3.1, table 3.6 and table 3.??

- We need to add 6 entries in table 3.1 :

GRIB code	Meaning
13	Mercator LAM with explicit boundary for bi-periodic field
23	Polar stereographic LAM with explicit boundary for bi-periodic field
33	Lambert conformal LAM with explicit boundary for bi-periodic field
61	Spectral Mercator LAM with explicit boundary for bi-periodic field
62	Spectral polar stereographic LAM with explicit boundary for bi-periodic field
63	Spectral Lambert conformal LAM with explicit boundary for bi-periodic field

- The bi-Fourier spectral type requires an extra entry to be appended to the table 3.6 :

GRIB code	Meaning
2	Bi-Fourier representation

- These three kinds of truncations will be associated to GRIB codes through the new table 3.25:

GRIB code	Meaning
77	Rectangular
88	Elliptic
99	Diamond

- We need to add the following entry in table 5,1 :

• GRIB code	• Meaning
• 53	Spectral LAM data - complex packing

- These three kinds of sub-truncations will be associated to GRIB codes through the new table 5.25 similar to table 3.25 :

GRIB code	Meaning
77	Rectangular
88	Elliptic
99	Diamond

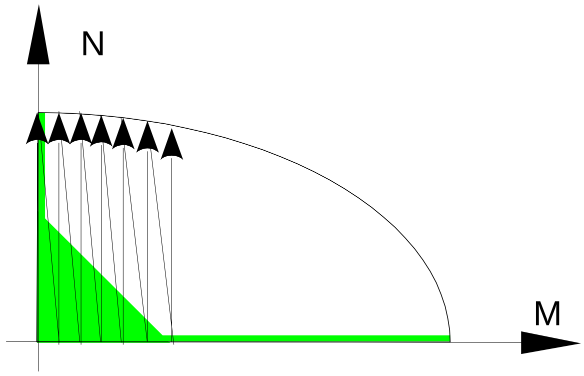
- The packing mode for axes will be associated to GRIB codes through the new table 5.26 :

GRIB code	Meaning
0	Spectral coefficients for axes are packed
1	Spectral coefficients for axes included in the unpacked subset

Annex : An example of encoding bi-Fourier coefficients in section 7

Let us take an example; on the following diagram, a spectrum truncated to an ellipse is drawn; in green are marked coefficients which should be kept with IEEE precision, and are located in:

- M and N axis (Packing mode for Axes=1)
- a diamond sub-truncation (in green)



Let us assume that :

- M = 7
- N = 4
- Resolution parameter of unpacked subset MS = 2
- Resolution parameter of unpacked subset NS = 2

The number of (m,n) , such that $\frac{m^2}{M^2}+\frac{n^2}{N^2}\leqslant 1$ and $m\geq 0, n\geq 0$ is 28. We list below the possible values for (m,n) ; we also have highlighted (m,n) which belong to either IEEE precision truncation or axis:

	m	n	Belong to IEEE precision sub-sub-truncation ?	Belong to M or N axis ?
0	0	0	X	X
1	0	1	X	X
2	0	2	X	X
3	0	3		X

4	0	4		X
5	1	0	X	X
6	1	1	X	
7	1	2		
8	1	3		
9	2	0	X	X
10	2	1		
11	2	2		
12	2	3		
13	3	0		X
14	3	1		
15	3	2		
16	3	3		
17	4	0		X
18	4	1		
19	4	2		
20	4	3		
21	5	0		X
22	5	1		
23	5	2		
24	6	0		X
25	6	1		
26	6	2		
27	7	0		X

Note that in the table above, (m,n) are listed in canonical order, that is, n varies fastest.

As explained before, unpacked coefficients will be stored at the beginning of section 7, either in IEEE-32 or IEEE-64 representation; in the case of our example, these numbers are :

$$
\begin{gathered}
(Q_{0r}^{0r}, Q_{0r}^{0i}, Q_{0i}^{0r}, Q_{0i}^{0i})(Q_{0r}^{1r}, Q_{0r}^{1i}, Q_{0i}^{1r}, Q_{0i}^{1i}) \\
(Q_{0r}^{2r}, Q_{0r}^{2i}, Q_{0i}^{2r}, Q_{0i}^{2i})(Q_{0r}^{3r}, Q_{0r}^{3i}, Q_{0i}^{3r}, Q_{0i}^{3i}) \\
(Q_{0r}^{4r}, Q_{0r}^{4i}, Q_{0i}^{4r}, Q_{0i}^{4i})(Q_{1r}^{0r}, Q_{1r}^{0i}, Q_{1i}^{0r}, Q_{1i}^{0i}) \\
(Q_{1r}^{1r}, Q_{1r}^{1i}, Q_{1i}^{1r}, Q_{1i}^{1i})(Q_{2r}^{0r}, Q_{2r}^{0i}, Q_{2i}^{0r}, Q_{2i}^{0i}) \\
(Q_{3r}^{0r}, Q_{3r}^{0i}, Q_{3i}^{0r}, Q_{3i}^{0i})(Q_{4r}^{0r}, Q_{4r}^{0i}, Q_{4i}^{0r}, Q_{4i}^{0i}) \\
(Q_{5r}^{0r}, Q_{5r}^{0i}, Q_{5i}^{0r}, Q_{5i}^{0i})(Q_{6r}^{0r}, Q_{6r}^{0i}, Q_{6i}^{0r}, Q_{6i}^{0i}) \\
(Q_{7r}^{0r}, Q_{7r}^{0i}, Q_{7i}^{0r}, Q_{7i}^{0i})
\end{gathered}
$$

Other coefficients are stored in the following order, after being applied a power of the Laplacian operator, and a packing operation (using a reference value and binary/decimal scaling factors, similarly to what is done for spherical harmonics).

Again, in our example, L being the power of the Laplacian operator, this yields :

$$
\begin{gathered}
(Q_{1r}^{2r}\times(1^2+1^2)^L, Q_{1r}^{2i}\times(1^2+1^2)^L, Q_{1i}^{2r}\times(1^2+1^2)^L, Q_{1i}^{2i}\times(1^2+1^2)^L) \\
(Q_{1r}^{3r}\times(1^2+1^2)^L, Q_{1r}^{3i}\times(1^2+1^2)^L, Q_{1i}^{3r}\times(1^2+1^2)^L, Q_{1i}^{3i}\times(1^2+1^2)^L) \\
(Q_{2r}^{1r}\times(2^2+2^2)^L, Q_{2r}^{1i}\times(2^2+2^2)^L, Q_{2i}^{1r}\times(2^2+2^2)^L, Q_{2i}^{1i}\times(2^2+2^2)^L) \\
(Q_{2r}^{2r}\times(2^2+2^2)^L, Q_{2r}^{2i}\times(2^2+2^2)^L, Q_{2i}^{2r}\times(2^2+2^2)^L, Q_{2i}^{2i}\times(2^2+2^2)^L) \\
(Q_{2r}^{3r}\times(2^2+2^2)^L, Q_{2r}^{3i}\times(2^2+2^2)^L, Q_{2i}^{3r}\times(2^2+2^2)^L, Q_{2i}^{3i}\times(2^2+2^2)^L) \\
(Q_{3r}^{1r}\times(3^2+3^2)^L, Q_{3r}^{1i}\times(3^2+3^2)^L, Q_{3i}^{1r}\times(3^2+3^2)^L, Q_{3i}^{1i}\times(3^2+3^2)^L) \\
(Q_{3r}^{2r}\times(3^2+3^2)^L, Q_{3r}^{2i}\times(3^2+3^2)^L, Q_{3i}^{2r}\times(3^2+3^2)^L, Q_{3i}^{2i}\times(3^2+3^2)^L) \\
(Q_{3r}^{3r}\times(3^2+3^2)^L, Q_{3r}^{3i}\times(3^2+3^2)^L, Q_{3i}^{3r}\times(3^2+3^2)^L, Q_{3i}^{3i}\times(3^2+3^2)^L) \\
(Q_{4r}^{1r}\times(4^2+4^2)^L, Q_{4r}^{1i}\times(4^2+4^2)^L, Q_{4i}^{1r}\times(4^2+4^2)^L, Q_{4i}^{1i}\times(4^2+4^2)^L) \\
(Q_{4r}^{2r}\times(4^2+4^2)^L, Q_{4r}^{2i}\times(4^2+4^2)^L, Q_{4i}^{2r}\times(4^2+4^2)^L, Q_{4i}^{2i}\times(4^2+4^2)^L) \\
(Q_{4r}^{3r}\times(4^2+4^2)^L, Q_{4r}^{3i}\times(4^2+4^2)^L, Q_{4i}^{3r}\times(4^2+4^2)^L, Q_{4i}^{3i}\times(4^2+4^2)^L) \\
(Q_{5r}^{1r}\times(5^2+5^2)^L, Q_{5r}^{1i}\times(5^2+5^2)^L, Q_{5i}^{1r}\times(5^2+5^2)^L, Q_{5i}^{1i}\times(5^2+5^2)^L) \\
(Q_{5r}^{2r}\times(5^2+5^2)^L, Q_{5r}^{2i}\times(5^2+5^2)^L, Q_{5i}^{2r}\times(5^2+5^2)^L, Q_{5i}^{2i}\times(5^2+5^2)^L) \\
(Q_{6r}^{1r}\times(6^2+6^2)^L, Q_{6r}^{1i}\times(6^2+6^2)^L, Q_{6i}^{1r}\times(6^2+6^2)^L, Q_{6i}^{1i}\times(6^2+6^2)^L) \\
(Q_{6r}^{2r}\times(6^2+6^2)^L, Q_{6r}^{2i}\times(6^2+6^2)^L, Q_{6i}^{2r}\times(6^2+6^2)^L, Q_{6i}^{2i}\times(6^2+6^2)^L)
\end{gathered}
$$

Eventually, we an example with numeric values :

m	n	Q_{mr}^{nr}	Q_{mr}^{ni}	Q_{mi}^{nr}	Q_{mi}^{ni}
0	0	2.7518129e+02	0.0000000e+00	0.0000000e+00	0.0000000e+00
0	1	2.0003144e-02	2.2098626e-01	0.0000000e+00	0.0000000e+00
0	2	6.5083158e-02	9.5691591e-02	0.0000000e+00	0.0000000e+00
0	3	7.6995000e-02	2.1512657e-02	0.0000000e+00	0.0000000e+00
0	4	2.1519178e-02	-1.7775195e-02	0.0000000e+00	0.0000000e+00

1	0	4.3116591e-02	0.0000000e+00	3.8033992e-01	0.0000000e+00
1	1	6.2610784e-03	1.6461671e-03	7.1552945e-03	-1.8621094e-02
1	2	5.3677237e-03	-3.3721561e-03	-6.2792737e-03	1.5941080e-02
1	3	1.1037082e-02	-6.8403990e-04	-7.3511754e-03	-4.8763524e-03
2	0	7.0401413e-02	0.0000000e+00	9.6275693e-02	0.0000000e+00
2	1	5.0130899e-02	1.9087472e-02	-2.8095387e-02	9.9476468e-03
2	2	2.7851647e-02	-3.0661852e-02	7.5807849e-03	-4.1879753e-03
2	3	9.2813170e-03	-1.4642329e-02	-1.1298243e-02	-4.4516319e-03
3	0	5.8110362e-02	0.0000000e+00	-6.1161322e-03	0.0000000e+00
3	1	4.4314082e-03	-2.6099993e-03	-9.4066831e-03	-1.4866123e-02
3	2	1.3650213e-02	-2.1479628e-02	-4.2952013e-03	-1.2692347e-02
3	3	7.8613585e-03	1.2872591e-03	-6.8223337e-03	4.3968672e-03
4	0	7.1075296e-02	0.0000000e+00	-2.8152529e-02	0.0000000e+00
4	1	8.8950277e-03	3.6632427e-03	-1.5046246e-02	-2.6391650e-03
4	2	6.7965217e-03	-1.6552945e-02	-1.7892818e-02	-3.1430372e-03
4	3	7.1100625e-03	5.3468447e-03	-1.1632748e-02	-8.0546855e-03
5	0	1.2137912e-02	0.0000000e+00	-3.4446277e-02	0.0000000e+00
5	1	2.1031688e-03	-7.4796438e-03	1.4063181e-03	-4.5603980e-03
5	2	9.3483157e-03	2.1565529e-03	-1.0611783e-02	-1.7067935e-02
6	0	1.5505006e-02	0.0000000e+00	1.0888269e-02	0.0000000e+00
6	1	4.7101184e-03	-5.8772050e-03	1.0504778e-02	4.5805960e-04
6	2	2.0050440e-03	-2.9542472e-03	4.8779936e-03	8.3513518e-03
7	0	7.1985800e-03	0.0000000e+00	3.3231129e-03	0.0000000e+00

Below are listed values whose precision will be restricted to IEEE:

m	n	Q_{mr}^{nr}	Q_{mr}^{ni}	Q_{mi}^{nr}	Q_{mi}^{ni}
0	0	2.7518129e+02	0.0000000e+00	0.0000000e+00	0.0000000e+00
0	1	2.0003144e-02	2.2098626e-01	0.0000000e+00	0.0000000e+00
0	2	6.5083158e-02	9.5691591e-02	0.0000000e+00	0.0000000e+00

0	3	7.6995000e-02	2.1512657e-02	0.0000000e+00	0.0000000e+00
0	4	2.1519178e-02	-1.7775195e-02	0.0000000e+00	0.0000000e+00
1	0	4.3116591e-02	0.0000000e+00	3.8033992e-01	0.0000000e+00
1	1	6.2610784e-03	1.6461671e-03	7.1552945e-03	-1.8621094e-02
2	0	7.0401413e-02	0.0000000e+00	9.6275693e-02	0.0000000e+00
3	0	5.8110362e-02	0.0000000e+00	-6.1161322e-03	0.0000000e+00
4	0	7.1075296e-02	0.0000000e+00	-2.8152529e-02	0.0000000e+00
5	0	1.2137912e-02	0.0000000e+00	-3.4446277e-02	0.0000000e+00
6	0	1.5505006e-02	0.0000000e+00	1.0888269e-02	0.0000000e+00
7	0	7.1985800e-03	0.0000000e+00	3.3231129e-03	0.0000000e+00

And below, values outside axis and IEEE sub-truncation; we have used the following packing parameters :

- Power of the Laplacian operator = 8.93785419541825665e-01
- Decimal scale factor = -1
- Binary scale factor = -20
- Reference value = -3.46138887107372284e-02
- Bits per value = 16

m	n	Spectral coefficients $(Q_{mr}^{nr}, Q_{mr}^{ni}, Q_{mi}^{nr}, Q_{mi}^{ni})$	Laplacian scaling	Spectral coefficients x Laplacian scaling $(m^2+n^2)^L$	Packed values
1	2	5.3677237e-03	4.2143335e+00	2.2621378e-02	38667
		-3.3721561e-03		-1.4211391e-02	34805
		-6.2792737e-03		-2.6462954e-02	33520
		1.5941080e-02		6.7181028e-02	43340
1	3	1.1037082e-02	7.8304190e+00	8.6424976e-02	45358
		-6.8403990e-04		-5.3563190e-03	35734
		-7.3511754e-03		-5.7562783e-02	30259
		-4.8763524e-03		-3.8183882e-02	32291
2	1	5.0130899e-02	4.2143335e+00	2.1126833e-01	58448

		1.9087472e-02		8.0440974e-02	44730
		-2.8095387e-02		-1.1840333e-01	23880
		9.9476468e-03		4.1922702e-02	40691
2	2	2.7851647e-02	6.4145809e+00	1.7865664e-01	55029
		-3.0661852e-02		-1.9668293e-01	15672
		7.5807849e-03		4.8627558e-02	41394
		-4.1879753e-03		-2.6864106e-02	33478
2	3	9.2813170e-03	9.8997872e+00	9.1883063e-02	45930
		-1.4642329e-02		-1.4495594e-01	21096
		-1.1298243e-02		-1.1185020e-01	24567
		-4.4516319e-03		-4.4070208e-02	31674
3	1	4.4314082e-03	7.8304190e+00	3.4699783e-02	39934
		-2.6099993e-03		-2.0437388e-02	34152
		-9.4066831e-03		-7.3658270e-02	28572
		-1.4866123e-02		-1.1640797e-01	24089
3	2	1.3650213e-02	9.8997872e+00	1.3513420e-01	50465
		-2.1479628e-02		-2.1264375e-01	13998
		-4.2952013e-03		-4.2521579e-02	31837
		-1.2692347e-02		-1.2565153e-01	23120
3	3	7.8613585e-03	1.3241700e+01	1.0409775e-01	47211
		1.2872591e-03		1.7045498e-02	38083
		-6.8223337e-03		-9.0339294e-02	26823
		4.3968672e-03		5.8221995e-02	42400
4	1	8.8950277e-03	1.2582206e+01	1.1191907e-01	48031
		3.6632427e-03		4.6091674e-02	41128
		-1.5046246e-02		-1.8931497e-01	16444
		-2.6391650e-03		-3.3206518e-02	32813
4	2	6.7965217e-03	1.4549266e+01	9.8884405e-02	46664
		-1.6552945e-02		-2.4083321e-01	11042

		-1.7892818e-02		-2.6032737e-01	8998
		-3.1430372e-03		-4.5728885e-02	31500
4	3	7.1100625e-03	1.7760607e+01	1.2627903e-01	49537
		5.3468447e-03		9.4963209e-02	46253
		-1.1632748e-02		-2.0660467e-01	14631
		-8.0546855e-03		-1.4305611e-01	21295
5	1	2.1031688e-03	1.8394244e+01	3.8686201e-02	40352
		-7.4796438e-03		-1.3758240e-01	21869
		1.4063181e-03		2.5868159e-02	39008
		-4.5603980e-03		-8.3885076e-02	27499
5	2	9.3483157e-03	2.0280067e+01	1.8958447e-01	56175
		2.1565529e-03		4.3735038e-02	40881
		-1.0611783e-02		-2.1520767e-01	13729
		-1.7067935e-02		-3.4613887e-01	0
6	1	4.7101184e-03	2.5213619e+01	1.1875913e-01	48748
		-5.8772050e-03		-1.4818560e-01	20757
		1.0504778e-02		2.6486347e-01	64068
		4.5805960e-04		1.1549340e-02	37506
6	2	2.0050440e-03	2.7033183e+01	5.4202722e-02	41979
		-2.9542472e-03		-7.9862706e-02	27921
		4.8779936e-03		1.3186770e-01	50123
		8.3513518e-03		2.2576362e-01	59968

Section 7 has therefore the following contents (we use the hexadecimal format) :

0000021d07407132e69057d1780000000000000000000000000000000000000000000000003f947bb44
5304f983fcc49471bc85d7c000000000000000000000000000000003fb0a94a332161f23fb87f3e7dd8
e29f000000000000000000000000000000003fb3b5f1bef49cf53f960769f90f6d53000000000000000
000000000000000003f96091f96fcb9d8bf9233a924d1f17a000000000000000000000000000000003f
a61360b8851fc300000000000000003fd8577d3f70d93100000000000000003f79a5376f7379763f5af
88676ed96483f7d4edebdee9629bf93116876fba3bf3fb205d3b66d5c4600000000000000003fb8a586
18d5b14400000000000000003fadc0a430b28b940000000000000000bf790d3abc6ea4b100000000000
000003fb231fd97df9efb0000000000000000bf9cd4043d6b344500000000000000003f88dbc2f8a75f
f00000000000000000bfa1a2f1425e2f9c00000000000000003f8fc116ad8de4a700000000000000003

f864c96ba1ed2dd00000000000000003f7d7c421ad79cb500000000000000003f6b3912a73ecc2d0000000000000000970b87f582f0a94cb12e8b9676337e23e450aeba5d489ef3d6f53d38a1b282c6b36a52685ff77bba9bfe85686f9c5e19c52136ae7c5d5a50b86b94c368c7a5a0bb9fa0a8403c802db6482b2223267b0cc181b4ad3927532f9da0556d98606b6bdb6f9fb135a10000be6c5115fa449282a3fb6d11c3cbea40

Where colours have the following meaning:

- section length
- section number
- IEE64 values
- packed values