Distribution functions in GRIB2

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Goal: representation of fields, that depend not only from space and time, but also from an additional continuous parameter, e.g. diameter d or particle mass m. Such fields at the end are (density) distribution functions $f(x, y, z, t; d) \equiv f(\mathbf{r}, t; d)$. They describe e.g. the distribution of particles with different particle sizes in the air. For simplicity, the time variable t is omitted in the following; in GRIB, times are noted in the PDS, anyway.

Furthermore, this is a try to describe unimodal and multimodal distribution functions in a common GRIB2-framework.

In a GRIB-file one or several fields are contained, which describe the distribution function (concentrations, number densities, ...). The purpose of this GRIB-template is to enable the user to calculate additional interesting variables (these are mostly integrals) from these fields, if he knows the underlying distribution function. Examples are the mass density of cloud droplets

$$\rho(\mathbf{r}) = \int_0^\infty \frac{1}{6}\pi d^3 \rho_w \, f(\mathbf{r}, d)\, dd \tag{1.1}$$

(with the density of water $\rho_w = 1000$ kg/m^3) or the radar reflectivity of rain droplet distributions

$$Z(\mathbf{r}) = const. \int_0^\infty d^6 \, f(\mathbf{r}, d)\, dd \tag{1.2}$$

In the following some examples of distribution functions are listed:

1. bin-model with concentrations $c_l(\mathbf{r})$ in the class (or mode) l. A concentration distribution function is described by

   $$f(\mathbf{r}; d) = \sum_{l=1}^{N} c_l(\mathbf{r})\delta(d - D_l). \tag{1.3}$$

   In this model, the numbers D_l for the diameter in these N classes are fixed and prescribed.
   ($p1 = D_l$)

   Area of application: bin-models in the cloud microphysics, volcanic ash, ...

2. N-modal concentration distribution function, composed by Gaussian functions

   $$f(\mathbf{r}; d) = \sum_{l=1}^{N} c_l(\mathbf{r}) \frac{1}{\sqrt{2\pi\sigma_l}} e^{-\left(\frac{d-D_l}{\sigma_l}\right)^2}. \tag{1.4}$$

   Again, N concentrations $c_l(\mathbf{r})$ must be stored. The N modes are defined by fixed values for diameter D_l and width σ_l.
   (therefore, $p1 = D_l$ and $p2 = \sigma_l$)

3. N-modal concentration distribution function, composed by Gaussian function, whose diameter and width can vary from grid point to grid point:

   $$f(\mathbf{r}; d) = \sum_{l=1}^{N} c_l(\mathbf{r}) \frac{1}{\sqrt{2\pi\sigma_l(\mathbf{r})}} e^{-\left(\frac{d-D_l(\mathbf{r})}{\sigma_l(\mathbf{r})}\right)^2} \tag{1.5}$$

   Now, $3N$ fields $c_l(\mathbf{r})$, $D_l(\mathbf{r})$ and $\sigma_l(\mathbf{r})$ must be stored.

4. N-modal log-normal distribution for the number density

$$f(\mathbf{r}; d) = \sum_{l=1}^{N} \frac{n_l(\mathbf{r})}{\sqrt{2\pi} \log \sigma_l(\mathbf{r})} e^{-\frac{\log^2 \frac{d}{D_l(\mathbf{r})}}{2 \log^2 \sigma_l(\mathbf{r})}} \tag{1.6}$$

It is described by $3N$ fields $n_l(\mathbf{r})$, $D_l(\mathbf{r})$ and $\sigma_l(\mathbf{r})$.

5. N-modal log-normal distribution for the number density at fixed variance

$$f(\mathbf{r}; d) = \sum_{l=1}^{N} \frac{n_l(\mathbf{r})}{\sqrt{2\pi} \log \sigma_l} e^{-\frac{\log^2 \frac{d}{D_l(\mathbf{r})}}{2 \log^2 \sigma_l}} \tag{1.7}$$

It is described by $2N$ fields $n_l(\mathbf{r})$, $D_l(\mathbf{r})$ and N fixed numbers σ_l. (therefore, $p1 = \sigma_l$)

6. N-modal log-normal distribution for the number density at fixed variance and the prescription of number density and mass density. Again, equation (1.7) is used. However, it is not the field $D_l(\mathbf{r})$ that is stored, but it is expressed via

$$D_l = \left(\frac{m_l}{n_l \frac{\pi}{6} \rho_{p,l} e^{\frac{9}{2} \log^2 \sigma_l}} \right)^{1/3} \tag{1.8}$$

by the mass density $m_l(\mathbf{r})$.
It is described by $2N$ fields number density $n_l(\mathbf{r})$ and mass density $m_l(\mathbf{r})$, N values σ_l and N values for the particle densities $\rho_{p,l}$.
($p1 = \sigma_l$ and $p2 = \rho_{p,l}$)

(C. Hoose (2004) master thesis, Univ. Karlsruhe)

Application area: aerosol fields

7. N-modal exponential distribution function with prescribed specific mass $q(\mathbf{r})$:

$$f(\mathbf{r}; d) = \sum_{l=1}^{N} N_{0,l}\, e^{-\lambda_l(\mathbf{r})\, d} \tag{1.9}$$

with a fixed intercept-parameter $N_{0,l}$ for the mode l.

For the case of spherical particles and $N = 1$ (cloud droplets, rain droplets) the inverse length $\lambda(\mathbf{r})$ depends from the specific mass $q(\mathbf{r})$ and from the air density $\rho(\mathbf{r})$ by

$$\lambda_l(\mathbf{r}) = \sqrt[4]{\frac{\pi \rho_{w,l} N_{0,l}}{\rho(\mathbf{r})\, q(\mathbf{r})}}. \tag{1.10}$$

This formula also contains the density $\rho_{w,l}$ (e.g. density of liquid water, in general this value is the same for all modes l).
($p1 = N_{0,l}$, $p2 = \rho_{[w,l}$).

Application area: for $N = 1$ an exponential distribution is assumed for the most cloud physics particles (cloud ice, graupel, ...)

8. skew Gaussian function (e.g. for temperature distributions)

$$f(\mathbf{r}; T) = \begin{cases} c_r e^{-\frac{(T - T_0(\mathbf{r}))^2}{\sigma_r^2(\mathbf{r})}}, T > T_0(\mathbf{r}), \\ c_l e^{-\frac{(T - T_0(\mathbf{r}))^2}{\sigma_l^2(\mathbf{r})}}, T \le T_0(\mathbf{r}) \end{cases} \tag{1.11}$$

with 3 fields $T_0(\mathbf{r})$, $\sigma_r(\mathbf{r})$, $\sigma_l(\mathbf{r})$. The 'left-sided' and 'right-sided' variances $\sigma_{l,r}$ have the same physical dimension (temperature). To distinguish them, it is recommended to define two different GRIB-elements. c_l and c_r are approriate norms (not given here).

9. ...

Though the possible functional forms of distribution functions is extremely huge, in practice, only a few of them are used. However, the shown examples indicate, that even for the same underlying distribution function, there can exist differences about which parameter and fields are prescribed or derived by others or which variable is the independent one (in these examples this has been the diameter d, the particle mass m could be another one, ...). Consequently, this list can become quite large during the lifetime of GRIB2. In the end, this GRIB-template is a possible ansatz, to deliver a minimum of order together with *complete* information for the user of GRIB-data.