Traceability of GNSS measurements
version 0

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1 Preface

This document is (clearly) working document! At this time not all sections have been filled and new revisions will become available (hopefully) soon. Please consider this when reading this document. Finally I started writing what I thought was my task; I could have misunderstood this. In that case please let me know.

Comments and suggestions are appreciated (haandes@knmi.nl).

2 Introduction

3 Traceability

A measurement has metrological traceability if [5]:

- An unbroken and documented chain of calibrations links the instrument to an internationally agreed reference;
- Each chain link has documented uncertainty estimations;
- The final user has procedures to ensure that the instrument maintains its calibration with associated uncertainties over time;
- The influence factors affecting the measurement are quantified and a total measurement uncertainty is calculated, documented and reported with the result.

Establishing traceability routes and uncertainty analysis start both from a mathematical model $Y = f(x_1, x_2, ..., x_n)$ relating the output quantity Y to be measured to the n input quantities x_i. Traceability to reference standards is achieved if x_i are all traceable or constants.

When estimating the uncertainties in the calibration by comparison method, not only the ones corresponding to the standards calibration have to be considered but also other influence factors like their drifts between calibrations and the influence of the isothermal enclosures used. When other steps are required, additional uncertainties are added in each one in a similar way.

Examples of traceability can be found in [2, 5].

4 GNSS position traceability chain

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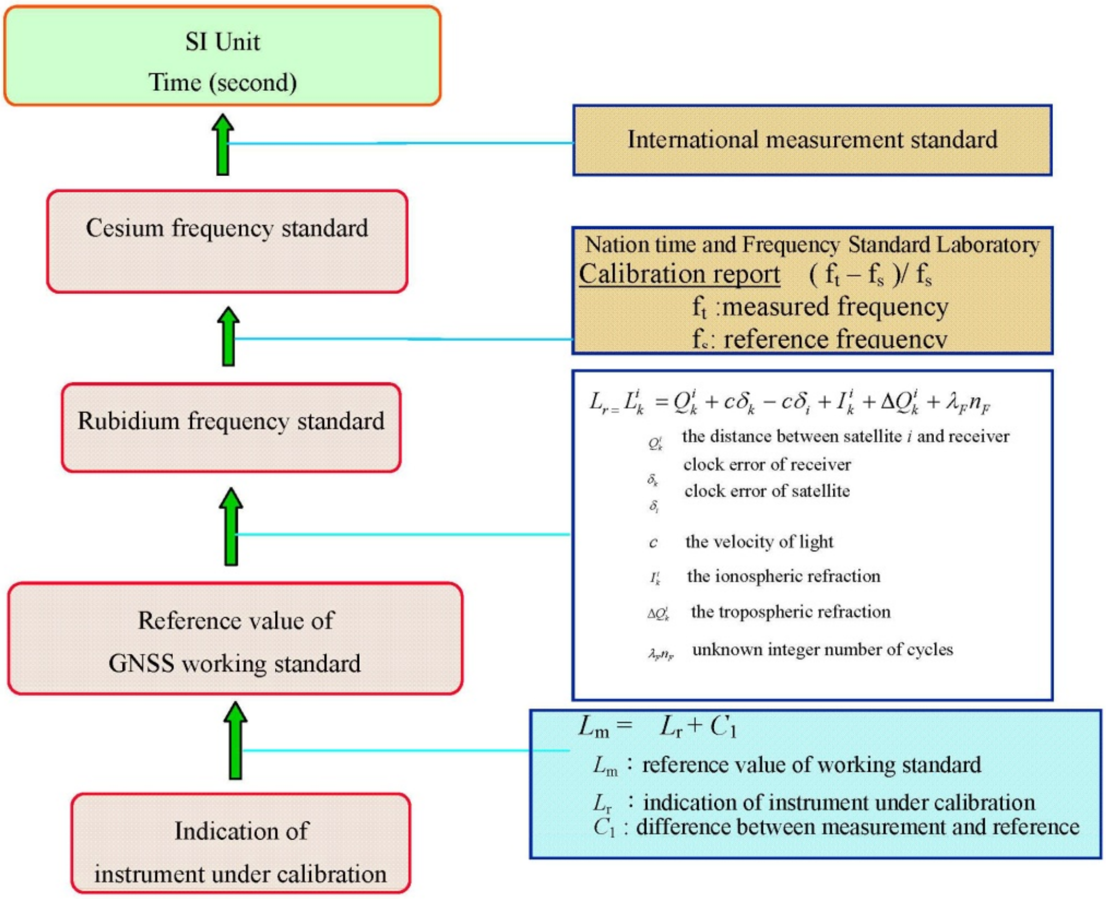

Figure 1: Traceability chain for GNSS positioning according to [11].

4.1 GNSS positions uncertainty

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Table 1: GNSS position uncertainties according to [11]

Standard uncertainty components	Variance estimates (a)	Divisor (b)	Standard uncertainties (c) = (a) ÷ (b)	Sensitivity coefficients (d)	uncertainty components (e) = (c) × (d)	Degrees of freedom
Measurement repeatability $u(x_2)$	3.1 mm	1	3.1 mm	1	3.1 mm	177
Centering and leveling $u(x_2)$	0.5 mm	$\sqrt{3}$	0.3 mm	1	0.3 mm	12.5
GNSS receiver frequency $u(x_3)$	6.4 mm	$\sqrt{3}$	3.6 mm	1	3.7 mm	50
Phase center offset $u(x_4)$	0.1 mm	$\sqrt{3}$	0.1 mm	1	0.1 mm	8
Satellite orbit $u(x_5)$	6.6 mm	$\sqrt{3}$	3.8 mm	1	3.8 mm	12.5
Atmospheric delay correction $u(x_6)$	4.4 mm	$\sqrt{3}$	2.5 mm	1	2.5 mm	12.5
IGS station coordinates $u(x_7)$	6.0 mm	$\sqrt{3}$	3.5 mm	1	3.5 mm	8

5 GNSS ZTD/IWV

5.1 Measurement of GNSS ZTD and IWV

The total zenith delay (ZTD) is determined by the integral of the refractive index in the zenith direction, given by

$$ZTD = \int_{H_a}^{r_s} 10^{-6}\mathcal{N}(h)dh \tag{1}$$

where the refractivity $\mathcal{N}$ is defined as $\mathcal{N} = 10^6(n-1)$ and according to [8, 10]

$$\mathcal{N} \quad = \quad k_1\rho R_d \; + \; \left(k_2 R_v - k_1 R_d + k_3 \frac{R_v}{T}\right)\rho_w = \mathcal{N}_h + \mathcal{N}_w, \tag{2}$$

for the neutral atmosphere. Here ρ is air density [kg m^{-3}], ρ_w water vapour density [kg m^{-3}], T is temperature [K] and $R_d = 287.05$ [J kg^{-1} K^{-1}] and $R_v = 461.51$ [J kg^{-1} K^{-1}] are the gas constants for dry air and water vapour. The empirical constants k_1, k_2, k_3 are given in Table 2. For slant delay observation, not discussed here,

Table 2: Empirical values of k_1, k_2, k_3

Reference	k_1 [K/hPa]	k_2 [K/hPa]	k_3 [10^5 K^2/hPa]
[3]	77.59 ± 0.08	72 ± 11	3.75 ± 0.03
[8]	77.61 ± 0.01	72 ± 9	3.75 ± 0.03
[9]	77.60 ± 0.01	64.79 ± 0.08	3.776 ± 0.004

the excess delay due to the curvature of the signal path needs to be taken into account. For zenith direction this excess path delay is zero.

The first term in Equation 2 is the hydrostatic refractivity[1], N_h, and the second term is called the wet refractivity, N_w. The integrals in the zenith direction of the hydrostatic and wet refractivity are called the Zenith Hydrostatic Delay (ZHD) and Zenith Wet Delay (ZWD) respectively, that is

$$\text{ZHD} = 10^{-6}\int_z N_h\,dz\ [\text{m}] \quad \text{and} \quad \text{ZWD} = 10^{-6}\int_z N_w\,dz\ [\text{m}]. \tag{3}$$

The integrated water vapour (IWV) or precipitable water vapour (PWV) is the integral of the specific humidity with height,

$$\text{IWV} = \int_z \rho_w\,dz. \tag{4}$$

Estimation of IWV from gb-GNSS requires additional information of surface pressure p_s and an estimate of the mean profile temperature T_m, then the ZHD can be approximated very accurately by the surface pressure and the latitude and height of the receiver, according to [7]

$$\text{ZHD}_s \quad = \quad 10^{-6}k_1 R_d \int_z \rho dz \approx \frac{2.2768 \cdot 10^{-5} p_a}{1 - 2.66 \cdot 10^{-3}\cos(2\phi_{gps}) - 2.8 \cdot 10^{-7} z_{gps}}, \tag{5}$$

where ϕ_{gps} is the latitude of the GPS receiver and z_{gps} the height of the receiver. The remaining atmospheric term can be rewritten as (following [4])

$$\text{ZWD} = 10^{-6}\left(k_2 R_v - k_1 R_d + k_3 R_v \frac{\int_z \rho_w/T dz}{\int_z \rho_w dz}\right)\int_z \rho_w dz. \tag{6}$$

and by defining the weighted mean temperature as

$$T_m = \frac{\int_z \rho_w dz}{\int_z \rho_w/T dz}, \tag{7}$$

the equation for ZWD becomes

$$\text{ZWD} \quad = \underbrace{10^{-6}(k_2 R_v - k_1 R_d + R_v k_3/T_m)}_{Q'(T_m)} \int_z \rho_w dz. \tag{8}$$

[1]The term N_h should not be confused with the dry refractivity, which is $N_d = k_1\rho_d R_d$, where $\rho_d = \rho - \rho_w$ the density of dry air.

The weighted mean temperature T_m can be approximated by a function of the surface temperature T_s [4, 1]; thus $Q'(T_m) \approx Q(T_s)$. The above equations show that IWV can be inferred from ZWD (or ZTD) observations in the following way

$$\mathrm{IWV} = \frac{1}{Q(T_s)}\mathrm{ZWD} = \frac{1}{Q(T_s)}(\mathrm{ZTD} - \mathrm{ZHD}_s). \tag{9}$$

The ratio $Q(T_s)$ between IWV and ZWD is around 6.5 [kg m^{-3}]. When only ZTD delay observations are available the ZWD is calculated by subtraction of ZTD by ZHD from the approximation derived by Saastamoinen.

5.2 GNSS ZTD/IWV traceability chain

The measurement of ZTD is a direct output of the GNSS processing in contrast to IWV, which requires additional calculations and consequently introduces extra uncertainties (e.g. in the values of k_1, k_2, k_3). Comparing GNSS ZTD with ZTD calculated using a reference measurement relies on the same empirical constants and thus almost the same uncertainties are present (except for the uncertainties related to Q). In Figure 2 the traceability chain

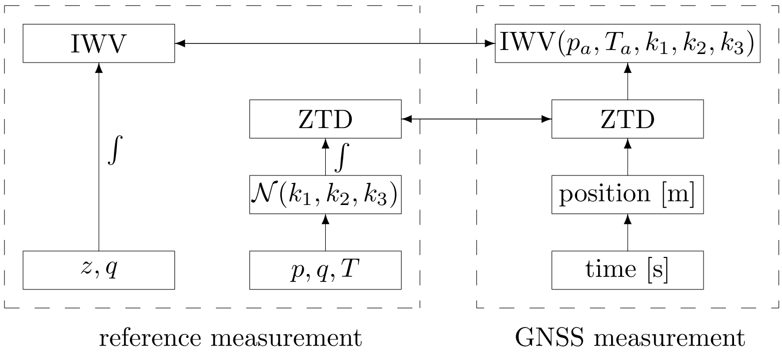

Figure 2: Traceability chain for GNSS ZTD and IWV.

for GNSS ZTD and IWV is presented, as well as the chain for the reference measurement of ZTD and/or IWV.

5.3 GNSS ZTD/IWV uncertainty

The uncertainties in constants k_1, k_2, k_3 are given in Table 2. In [6] a thorough evaluation of the uncertainties is presented. The final table (Table 4) is shown here in Table 3

Table 3: GNSS IWV uncertainties according to [6], Table 4.

Input variable	LDB0	LDRZ	NYA2	Uncertainty	Corresponding IWV uncertainty								
					LDB0			LDRZ			NYA2		
					[kg m^{-2}]	[%]	[%][e]	[kg m^{-2}]	[%]	[%][e]	[kg m^{-2}]	[%]	[%][e]
ZTD [mm]	2487	2376	2434	3.8, 3.7, 3.3[a]	0.59	1.8	79.9	0.58	2.2	82.2	0.49	2.1	77.0
Ground pressure P_0 [hPa]	1000.1	968.7	1005.6	0.2[b]	0.07	0.2	1.2	0.07	0.3	1.3	0.07	0.3	1.5
Constant[f]	2.2767	2.2767	2.2767	0.0015	0.23	0.7	12.2	0.22	0.9	11.9	0.23	1.0	17.0
Mean temperature T_m [K]	274.6	270.8	262.3	1.1[c]	0.13	0.4	3.8	0.1	0.4	2.5	0.09	0.4	2.6
k_2' [K hPa^{-1}]	22.1	22.1	22.1	2.2[d]	0.05	0.2	0.6	0.04	0.2	0.5	0.03	0.2	0.3
k_3 [$10^5 \times$ K^2 hPa^{-1}]	3.739	3.739	3.739	0.012[d]	0.10	0.3	2.3	0.08	0.3	1.6	0.07	0.3	1.6
IWV [kg m^{-2}]	33	26	23										
Conversion factor Q	6.4	6.5	6.7										
Total IWV uncertainty					0.66	2.0		0.64	2.4		0.56	2.4	

[a] The values are given by the mean ZTD uncertainty calculated from 1 year of data for LDB0, LDRZ, and NYA2, respectively. [b] For GRUAN sites equipped with surface barometers which are calibrated routinely. [c] Taken from Wang et al. (2005) based on the comparison between ECMWF reanalysis and radiosonde data. [d] Taken from Table 1 in Bevis et al. (1994). [e] Percentage of the total IWV uncertainty. [f] The constant given in Eq. (23).

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