Appendix 1. *The Ekman wind spiral*

Assuming that any variable is the sum of mean term $\bar{A}$ and a turbulent fluctuation A', (Reynold's averaging): the continuity equation for A leads to terms of the form of $\overline{u'A'}$ which represent the flux of A in the direction of the wind component u, the x direction. In the case of horizontally homogeneous turbulence in the ABL, and assuming that in the ABL, a 3-way balance exists among the Coriolis force, the pressure gradient force and turbulent momentum flux divergence, the equations of motion become:

$$f(\bar{v}-\bar{v}_g) - \frac{\partial \overline{u'w'}}{\partial z} = 0$$

$$-f(\bar{u}-\bar{u}_g) - \frac{\partial \overline{v'w'}}{\partial z} = 0$$

where the pressure gradient force is expressed in term of the geostrophic wind, (u_g, v_g) and v and w the y and z wind components.

Generally the turbulent flux is assumed to be proportional to the mean gradient, where K_{mz} is a diffusity coefficient. The negative sign means that the flux is from high values of A to low values.

$$\overline{w'u'} = -K_{mz}\frac{\partial \bar{u}}{\partial x}$$

$$\overline{w'v'} = -K_{mz}\frac{\partial \bar{v}}{\partial x}$$

The solution of this set of equation gives for the northern hemisphere, with $v_g = 0$,

$$u = u_g(1 - e^{-\gamma z}\cos\gamma z)$$

$$v = u_g e^{-\gamma z}\sin\gamma z$$

where $\gamma = \sqrt{\dfrac{f}{2K_{mz}}}$

The figure besides shows the wind vector at different levels, along the Eckman spiral. Levels are in terms of the dimensionless height γz (.10π, .23π, .50π, 2.0π); the wind components are expressed in terms of $1/u_g$.

Eckman Wind Spiral
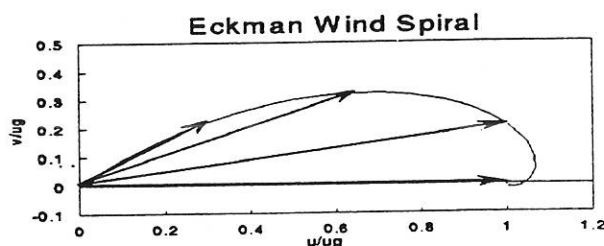
Reference: Holton, J.R., 1992. *An introduction to Dynamic Meteorology*, Academic Press, chapter 5.

Appendix 2. *The Surface Boundary Layer and the Similarity Theory.*

The Similarity theory provides models for wind and temperature profiles within the surface layer. In this layer, it is assumed that, under conditions of stationary homogeneous turbulence, the vertical fluxes of momentum and heat are constant. Molecular diffusion and Coriolis effects can be neglected.

The friction velocity u_* is defined as $\quad u_*^2 = -\overline{u'w'}$

Another scaling length is the Monin Obukhov length L, defined as $\quad L = -\dfrac{u_*^3}{\kappa \beta \overline{w'\theta'}}$

where κ is the von Karman 's constant (usually equal to 0.4), θ the potential temperature

$\beta = \dfrac{g}{\theta_0}$ the buoyancy parameter and, $\quad -\overline{w'\theta'}$ the vertical heat flux. A third scaling

parameter is defined : $\theta_* = -\dfrac{\overline{w'\theta'}}{u_*}$.

The application of these scaling parameters to the gradients of the mean profiles leads to

$$\frac{kz}{u_*}\frac{\partial \overline{u}}{\partial z} = \phi_m(z/L)$$

$$\frac{kz}{\theta_*}\frac{\partial \overline{\theta}}{\partial z} = \phi_h(z/L)$$

Functional relationships for ϕ_h and ϕ_m are obtained from experimental data (Businger at al., 1971).

These functions can be integrated to yield profiles for $\overline{u}$ or and $\overline{\theta}$.

For example in neutral conditions, the heat flux tends to zero, L tends towards $\pm \infty$, depending on whether neutrality is approached from stable or unstable conditions. In that case z/L tends to zero and ϕ_h , ϕ_m are close to unity for any value of z. This leads to the well known logarithmic profiles for $\overline{u}$ and $\overline{\theta}$.

Appendix 3. *The advection diffusion equation*

The advection-diffusion equation is the equation of continuity for the pollutant concentration C

.

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=-\frac{\partial}{\partial x}\overline{u'C'}-\frac{\partial}{\partial y}\overline{v'C'}-\frac{\partial}{\partial z}\overline{w'C'}+So+Si$$

This equation states that the time variation of the concentration of pollutant at a point depends on several different <u>physical processes</u>. These processes are:

a) advection by the mean wind $\quad u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}+w\frac{\partial c}{\partial z}$

b) turbulent diffusion , $\quad -\frac{\partial}{\partial x}\overline{u'C'}-\frac{\partial}{\partial y}\overline{v'C'}-\frac{\partial}{\partial z}\overline{w'C'}$

c) the source term So,

d) the depletion term Si. It generally takes into account the effects of clouds and precipitation (wet scavenging), radioactive decay, and deposition on the ground due to the various capturing properties of the surface (dry deposition).

<u>Wet scavenging</u> is the result of the interaction of the pollution plume with clouds and precipitation. When entering a cloud, some of the polluted particles are absorbed by the water droplets or ice crystals and eventually reach the surface in the precipitations, this is the rainout phenomenon. Washout occurs below the cloud, when the precipitation particles scavenge some of the pollutants. Both processes are major factors for ground contamination. They are usually modelled by assuming that the local depletion rate is proportional to the concentration, introducing a scavenging coefficient.

$$\frac{d}{dt}=-\Lambda C$$

The coefficient Λ is generally assumed to be proportional to the precipitation intensity. The wet deposition rate at the ground is then the cumulation of the depletion occurring at all levels of the atmospheric column:

$$D_w=\int_0^{H_w}\Lambda Cdz$$

where H_w is the height of the effective wet layer.

Dry deposition is the result of the interaction of the pollutant with the earths surface. The polluted particles are captured by the various elements of the surface. This phenomenon is usually considered by introducing a dry deposition velocity V_d. V_d usually depends on the type of surface, the type of pollutant and the state of the SBL. The dry deposition rate at the ground is then :

$$D_d = V_d C_s$$

where C_s is the concentration of pollutant near the ground.
In case of a radioactive pollutant, another factor of depletion is the radiological decay. The equation is:

$$\frac{dC}{dt} = -KC$$

where K is the radiological decay rate defined as $K = \frac{\ln 2}{T}$ (T is the half life of the isotope).