Weather Risks, Ratemaking, and Modeling the Tail Distribution: An Application of Extreme Value Theory[1]

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Abstract

Economic analysis of weather risk often depends on accurate assessment of the probability (P) of tail quantiles (Q) and extreme value theory can provide a promising estimation of the tail part risk. In this article, we apply statistical techniques to quantify weather tail risk and compare the results from standard statistical distributions with extreme value models for risk estimation and premium setting. We demonstrate that extreme value models can provide more statistically robust estimation in modeling weather tail risk and premium ratemaking of weather-based contingent claims. Rainfall data across selected regions in India during the 1871–2001period is used in the empirical analysis.

Key Words:

Weather risks, Ratemaking, Extreme value theory

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Weather Risks, Ratemaking, and Modeling the Tail Distribution: An Application of Extreme Value Theory

Economic analysis of weather risk often depends on an accurate estimation of the probability (P) or patterns depicting the stochastic nature of a random weather variable, especially the tail quantiles (Q). For example, accurate actuarial rates, which depend on the precise measurement of low tail risk, are essential elements of an actuarially sound insurance program. A few low-probability, high-consequence events often have dominant impacts on risk assessment and thus commercial investors often use the Value-at-Risk method to assess the risk in their portfolio that has a low probability at the tail part.

This article applies statistical techniques to quantify weather tail risk and compares the results from standard statistical distributions with an innovative approach — the use of extreme value theory for risk estimation and premium setting. The objective of this article is to provide evidence for the feasibility of applying extreme value models in modeling weather tail risk and to evaluate its effectiveness over alternative distribution models with regards to its economic impact on risk assessment and premium ratemaking.

This article is divided into five sections: The first section provides a strong motivation for why it is important to consider alternative statistical procedures for examining tail risk when pricing weather insurance products. The second section provides a description of tail distribution estimation for modeling and assessing weather risk. In the third section, the statistical model for modeling the tail distribution — extreme value theory — is introduced along with the statistical properties. The fourth section develops a research procedure that compares the estimation and actuarial performance of the standard distributions and the extreme value model using monthly rainfall data across different regions in India over the period from 1871 through 2001. The power and efficiency of the extreme value model are further demonstrated by modeling the tail risk. Finally, policy implications are developed in the fifth section.

Background and Motivation

Accurate ratemaking and efficient weather risk assessment depend on the precise forecasting of a future occurrence, especially for the tail-part risk. Until today, the most common method of forecasting is still to use historic records to derive the probability distribution of related variables (e.g., temperature, precipitation, crop yield, etc.) associated with various weather events, that is, the probabilistic method. Considerable disagreement exists about the most appropriate characterization of risk distributions. The approaches that have been used to represent risk distributions can be segmented into two primary methodologies: parametric and non-parametric.

When using the parametric approach, a specific family of distributions (e.g., normal, beta, gamma, etc.) is selected and parameters of this family are estimated based on observed data using either the maximum likelihood method or generalized method of moments. This approach works well when the underlying population distribution family is correctly assigned. In agriculture, parametric techniques have been extensively applied for estimating crop-yield distributions and premium ratemaking, such as the normal distribution (e.g., Day, 1965; Botts and Boles, 1958), beta distribution (e.g., Babcock and Hennessy, 1996; Nelson and Preckel, 1989), gamma distribution (e.g., Gallagher, 1986), lognormal distribution (e.g., Stokes, 2000), the S_u family (e.g., Ramirez, Misra, and Field, 2003), and a mixture of several parametric distributions (Goodwin and Ker, 2002). Different parametric distributions vary in terms of their flexibility and ability to capture the crop-yield process. Sherrick, et al. (2004) discusses the modeling of alternative distributional parameterization (i.e., the beta, the logistic, the lognormal, the normal, and the Weibull distribution) and their economic importance on crop insurance valuation. Parametric techniques are also commonly used in catastrophic risk modeling. For example, the Poisson distribution is often used to model rare and random events (e.g., earthquake occurrence, etc.); the Pareto distribution is used to estimate the flood frequency or fire loss; and the lognormal distribution is frequently used to track the earthquake motion, raindrop size, or tornado path (Woo, 1999). However, the prerequisites of functional form and distribution assumptions for the parametric approach may result in imprecise predictions and misleading inferences when the underlying distribution choice is incorrect.

Nonparametric methods have been developed for situations where no knowledge of a specific distribution family of the underlying population is assumed. The simplest nonparametric technique is the histogram and the most commonly used nonparametric methods are based on empirical distribution. Compared to the parametric approach, the nonparametric approach is free of functional forms and distribution assumptions and relatively insensitive to outliers. Therefore, this approach is invulnerable to specification errors and might result in more accurate and robust models (Featherstone and Kastens, 1998). In agriculture, in addition to empirical distribution methods and histograms, a variety of kernel functions have been used to estimate crop-yield distribution and rate crop insurance contracts, such as Goodwin and Ker (1998), Ker and Goodwin (2000), and Ker and Coble (2003). However, some nonparametric procedures (e.g., the kernel procedure) have relatively slow rates of convergence to the true density (Silverman, 1986) and results can be indeterminate when measuring rare events. Some efficiency might also be lost when some prior knowledge of the underlying distribution form is available or the sample is small.

When modeling weather risk and properly pricing weather risks whether it is in weather markets or in the development of stand-alone weather insurance products, the principle concern is not in estimating the whole distribution, but only the tail risk. The use of standard parametric or nonparametric methods might be misleading or biased when modeling the tail risk since traditional statistical methods mostly focus on the laws governing averages and are driven by data clustered in the center. This bias can further cause imprecise ratemaking when designing weather-based contingent claims. To overcome the disadvantage of applying standard methods in modeling tail risk, extreme value theory (EVT), which is grounded by the laws governing tail events, could provide a promising solution since it primarily quantifies the stochastic behavior of a process usually at the largest event, the smallest event, or at events over a threshold in a sample.

Extreme Value Theory — Let the Tails Speak for Themselves!

Basic statistical measures of risk, such as, mean, variance, and the third or fourth central moments, are all based on the center of the observed data. However, in weather risk estimation, a few low probability events will exert a high, or even dominant impact on risk assessment and the quantification of (P, Q) combinations needs to rely on the (asymptotic) form of tail distribution. Estimation and inference based on the whole distribution might be inaccurate since the data clustered in the center of the distribution will have too much influence over the estimators. Furthermore, alternative distributions that fit the observed data well might have different performances in a tail estimation, which can, in turn, bias the calculation of the insurance premiums and indemnity payments.

Recently, some researchers (e.g., Ker and Coble, 2003) have noticed this problem and suggest modeling the conditional risk distribution instead of the whole distribution in risk assessment. However, the risk estimation and economic analysis of alternative distribution specifications on modeling conditional weather risk have not been well documented. Specifically, the performance of alternative distributions on conditional tail-part risk valuation has not been addressed in most of the literature.

Extreme value theory (EVT) dates back to the late 1920s and early 1940s with the pioneering work of Fisher and Tippett (1928), and Gnedenko (1943). In 1958 Gumbel laid out the theoretical framework of the extreme value model in his classical book. Extreme value techniques have been extensively applied in many disciplines during the last several decades. Generally, there are two principal approaches to modeling extreme values, the block maxima model (BMM) and peak-over-threshold model (POT).

The first approach (BMM) models the largest or the smallest values for a series of identically distributed observations (i.e., annual maximum precipitation level, daily minimum temperature, the largest insurance claim, etc). In modeling weather risk and designing an efficient risk management system, it might be of particular interest when asking such a question as: “What is the probability that the maximum event for next year will exceed all previous levels?” In the actuarial industry, such information might be especially important in determining the buffer fund and probability of ruin that can jeopardize the position of the insurance or reinsurance company due to catastrophic loss.

Statistically, assume M_n to be the maximum of the process over *n* independent random variables with a common distribution function *F*, that is, $M_n = \max\{X_1, \cdots, X_n\}$.

In theory, the distribution of M_n can be derived by

(1) $$P(M_n \le z) = P(X_1 \le z, \cdots, X_n \le z\} = \{F(z)\}^n,$$

While $F^n(z) \to 0$ as $n \to \infty$ and the distribution of M_n degenerates to a point. Thus we need a location parameter, b_n, and a scale parameter, a_n, to normalize (M_n) into $W_n = \frac{M_n - b_n}{a_n}$. By Fisher-Tippet Theorem, the limiting distribution of W_n is given by a generalized extreme value (GEV) family such that

(2) $$\lim_{n\to\infty} P\left(\frac{M_n - b_n}{a_n} \le z\right) = G(z) = \exp\{-[1+\xi(\frac{z-\mu}{\sigma})]_+^{-1/\xi}\}$$

where μ and $\sigma (> 0)$ are location and scale parameters, and ξ is a shape parameter. We can further divide the GEV family into Gumbel ($\xi \to 0$), Frechet ($\xi > 0$), and Weibull ($\xi < 0$) distributions.

The GEV family can be easily transformed to model the smallest value by changing the sign. Furthermore, the GEV family can be extended to model the r^{th} largest or smallest order statistics and the parameters of the GEV family can be estimated in the presence of covariates, such as trends, cycles, or actual physical variables (e.g., the Southern Oscillation Index in the rainfall process). Maximum likelihood procedures can be employed to estimate the GEV parameters μ, σ, ξ. These estimators are unbiased, consistent, and asymptotically efficient. Although there is not always a straightforward analytical solution, the estimators can be found using standard numerical optimization algorithms.

Modeling only the extreme values can only be applied when the particular interest is in the largest or smallest event, and this method is also an inefficient approach if other data on the tail are available and of interest. POT can compensate for such shortcomings and be used to model all large (small) observations that exceed (fall below) a high (low) threshold, which are important in determining insurance or reinsurance premium rates, claims, and buffer funds. This approach might be more useful for practical applications since it is more efficient to use limited resources on extreme values instead of only the largest or smallest observation.

Assume *u* is the threshold, then the stochastic behavior of the events whose values are greater than *u* can be represented by the following conditional probability function:

(3) $$F_u(y) = P(X - u \le y \mid X > u) = \frac{F(u+y) - F(u)}{1 - F(u)}, y > 0.$$

Pickands (1975) and Balkema and de Haan (1974) have shown that if block maxima have an approximate GEV distribution, then threshold excesses have a corresponding approximate distribution within the Generalized Pareto Distribution family (GPD) and the parameters of GPD are uniquely determined by those of the associated GEV distribution of block maxima. For a large enough threshold u, the distribution function of $(X - u)$ conditional on $X > u$ can be approximated by

(4) $$H(y) = 1 - (1 + \frac{\xi y}{\sigma_u})^{-1/\xi} \text{ where } \sigma_u = \sigma + \xi(u - \mu).$$

If $\xi < 0$ (Weibull), the distribution of excesses has an upper bound; If $\xi > 0$ (Frechet), the distribution of excesses has no upper limit; If $\xi \to 0$ (Gumbel), the distribution can be simplified as an exponential distribution with parameter $1/\sigma_u$. Similar to the GEV distribution, maximum likelihood procedures can be utilized to estimate the GPD parameters given the threshold *u*.

The determination of the threshold *u* is crucial to perform the POT method. There always exists a tradeoff between variance and bias in determining the threshold. For example, too high a threshold will generate a few observations to estimate the parameters and may cause high variance while too low a threshold is likely to violate the asymptotic basis and may lead to a bias. Graphically, the mean residual life plot and Hill-plot (Coles, 2001) can be performed to determine the crucial threshold *u*. The goodness-of-fit test suggested by Gumbel (1958), and the Bootstrap methods suggested by Dekkers and de Haan (1989) can also be used to approach this problem.

Research Design

This study provides an empirical analysis of modeling weather risk using alternative parametric distributions and EVT. Premium rates of a hypothetical weather index with varying strikes are calculated and a statistical comparison is performed.

Data

Indian agriculture accounts for 24 percent of the GDP and provides work for almost 60 percent of the population. Monsoons in India can bring damaging cyclones and floods to the coastal plain. Heavy flooding in 2000 caused about 1,200 deaths in southern India and Bangladesh (Swiss Re, 2001). Officials in Andhra Pradesh report that by August 30, 2000 the floods had affected 3,080 villages and towns and submerged 177,987 hectares of farmland, causing damage officially estimated at 7.7 billion rupees. The real destruction may far exceed these figures (Peiris, 2000).

Parchure (2002) estimates that about 90 percent of the variation in the crop production of India is due either to inadequate rainfall or to excess rainfall. Generally, excess rain is concentrated in the months of June to September. However, the performance of the current crop insurance program in India can be considered disappointing (Skees and Hess, 2003; Parchure, 2002), and developing rainfall-based insurance can be considered an economically viable instrument (Veeramani, Maynard, and Skees, 2005).

In this study, historic monthly rainfall data from the months of June–September over the 1871–2001 period across 14 different subdivisions are used (Indian Institute of Tropical Meteorology, 2001). The time series data used to estimate an underlying distribution need to be identical and independent, thus a series of tests are necessary.

1) Deterministic trend or stochastic trend

The augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests were used to test for the existence of a stochastic trend on a region-by-region basis. The rainfall series for all 14 regions were found to be trend stationary and the unit root tests were rejected in all cases. The results suggest that a deterministic trend might be appropriate for the rainfall series.

2) Linear trend or higher order trend

The possible trend order was examined by regressing time series rainfall data against a possible time trend (e.g., linear, quadratic, cubic, or higher order) based on the significance of the F-test. Greene (2003) notes the conservative nature of this test in cases of non-normal errors. The results indicate that only two of the 14 regions were found to have significant linear trends (Region COAPR with a 10 percent significant level and Region SASSM with a 5 percent significant level). Region TELNG has a significant quadratic term and a fifth order term at the 10 percent level and a fourth term at the 5 percent level. Regions WMPRA and SHWBL have significant cubic terms at the 5 percent level and the 10 percent level, respectively. But none of them have significant lower order terms.

3) Autocorrelation and Normality

Durbin-Watson (DW) tests are used to indicate the incidence of a first order autocorrelation for lag one series (monthly autocorrelation) and lag four series (yearly autocorrelation). The results showed that the

DW test was only rejected in one region, SASSM, at a 5 percent significant level. A normality test (Kolmogorov-Smirnov test) failed to reject in only one region, NASSM, at a 5 percent significant level and in two regions, BHPLT and SASSM, at a 10 percent significant level. Since only two regions have a deterministic trend (CORPA and SASSM), a heteroscedasticity test is not performed in this study.

Given the sporadic violations of the i.i.d. assumptions, a linear trend was imposed for regions COAPR and SASSM and the time series rainfall data were detrended by a linear term to a base year of 2001. The raw rainfall data were used for the twelve other regions. The summary statistics of rainfall data are shown in Table 1.

Table 1. Summary Statistics of Rainfall in Selected Regions of India

	N	Mean	Median	Standard Deviation	Skewness	Kurtosis	Maximum	Minimum
BHPLN	524	2592	2457	1098	0.440	-0.193	5949	355
BHPLT	524	2750	2725	1051	0.340	0.273	7309	340
COAPR	524	1905	1827	818	0.678	0.395	4894	382
EMPRA	524	2983	2961	1325	0.139	-0.753	6780	177
EUPRA	524	2269	2298	1151	0.271	-0.414	5845	109
GNWBL	524	2887	2775	987	0.573	0.135	6158	700
NASSM	524	3628	3648	1038	0.212	0.040	7307	845
ORISS	524	2916	2842	1084	0.368	-0.218	6038	552
SASSM	524	3919	3749	1107	0.591	0.393	7892	1531
SHWBL	524	5014	4896	1655	0.444	-0.065	10129	1241
TELNG	524	1784	1707	779	0.743	0.783	5107	255
VDPBH	524	2357	2225	1068	0.388	-0.224	5969	170
WMPRA	524	2283	2277	1175	0.307	-0.496	5824	108
WUPPL	524	1915	1912	1142	0.244	-0.905	4949	4
Average		2800	2736	1106	0.410	-0.089	6439	484
Minimum		1784	1707	779	0.139	-0.905	4894	4
Maximum		5014	4896	1655	0.743	0.783	10129	1531

The mean of monthly cumulative rainfall during the period from June to September across the 14 regions averages 2,800mm, indicating that excess rainfall can be a significant risk. The sample means vary considerably ranging from a low of 1,784mm (TELNG) to a high of 5,014mm (SHWBL). Sample medians are slightly smaller than sample means in all regions except EUPRA and NASSM, ranging from 1,707mm (TELNG) to 4,896mm (SHWBL) with an average of 2,736mm. The variability of monthly rainfall is also different across the regions, with standard deviations ranging from 779 (TELNG) to 1,655 (SHWBL). The coefficients of skewness range from 0.139 (EMPRA) to 0.743 (TELNG), with an average of 0.41 across all regions. Positive skewness calls into question the use of symmetric distribution (e.g., normal distribution) to model rainfall. The coefficients of sample kurtosis range from -0.905 (WUPRL) to 0.783 (TELNG), with an average of -0.089. Both negative kurtosis (sub-Gaussian) and positive kurtosis (super-Gaussian) appear across the different regions, showing the possibility of both “less peaked” and “more peaked” density functions. Monthly cumulative rainfall levels vary significantly across regions. For example, the maximum rainfall ranges from as low as 4,894mm in COAPR to 10,129mm in SHWBL; the minimum rainfall fluctuates from 4mm in WUPRL to 1,531mm in SASSM. The summary statistics indicate that rainfall across regions displays significantly different distributions but predominantly positive skewness.

Research Procedure

The aim of this article is to provide evidence for the feasibility of applying EVT in modeling weather tail risk and to evaluate its efficiency over alternative distribution models with regards to its economic impact on risk assessment and premium ratemaking. In this study, four alternative distributions are selected as the parametric candidates and the GPD model is chosen as the extreme value candidate. The research procedure includes the following five steps:

Step 1. Fitting the Alternative Parametric Distributions

In selecting the parameterization of rainfall distributions, several considerations were given to 1) Stylized features of cumulative rainfall (i.e., non-negativity, skewness); 2) Flexible parameters to adequately characterize cumulative precipitation over time periods across different regions; 3) Previous studies and empirical evidence from climatological research (Ison, Feyerherm, and Bark, 1971; Woo, 1999). Four candidate distributions are considered in this study: the beta distribution[1], gamma distribution, lognormal distribution, and Weibull distribution.

Maximum likelihood methods were applied to solve for the parameters of the four distributions for each region sample. If any of the cumulative precipitation observations in the historical data serials are equal to zero, a censoring estimation suggested by Martin, Barnett, and Coble (2001), could be applied. The parameters for all four distributions were estimated separately across the 14 regions, and the fitted distributions differ meaningfully across regions.

Step 2. Rank alternative distributions

Each of the alternative distributions has two parameters to be estimated in this study and thus there are the same degrees of freedom when performing the maximum likelihood functions for the rainfall series.

Alternative distributions can be ranked for the goodness-of-fit according to some standard tests and visual Q -Q plot. SAS 8.2 provides several goodness-of-fit tests for the appropriateness of candidate distribution, such as the Kolmogorov-Smirnov test, Cramer-von Mises test, Anderson-Darling test, and Chi-Square test. A large p-value fails to reject the null hypothesis, suggesting that the candidate distribution might be appropriate to fit the sample data. However, these goodness-of-fit tests are not informative for comparing the tail behavior of the distributions and thus the Q-Q plot was also generated. The Q-Q plot provides the visual evidence of disparity in the tail part and it should be close to the unit diagonal if $\hat{F}$ is a reasonable model for the population.

Based on the standard goodness-of-fit tests and Q-Q plot, the appropriateness of the four distributions in fitting the rainfall series for each region can be ranked. The following example illustrates how to rank alternative distributions for the rainfall series in the WMPRA region. The plot of alternative distributions is shown in Figure 1. The statistics of standard goodness-of-fit tests are reported in Table 2. Q-Q plots of alternative distributions are provided in Figure 2.

Figure 1. Fitting rainfall at WUPPL by alternative distributions in WMPRA

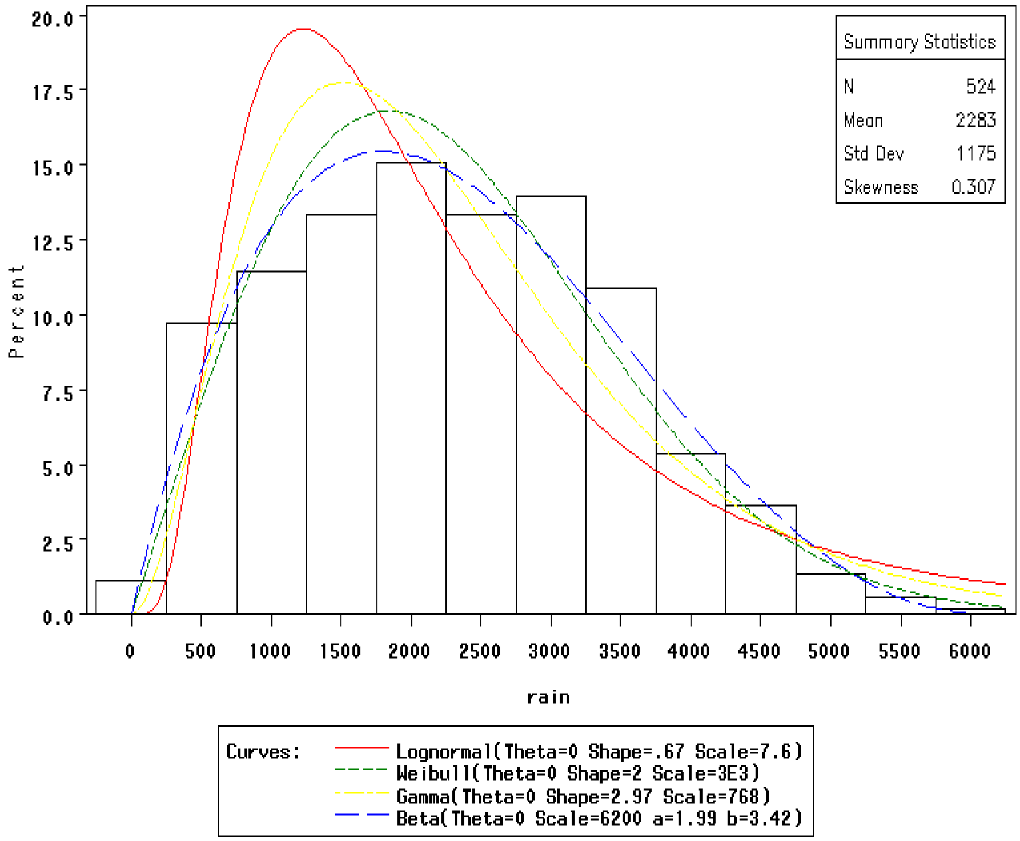
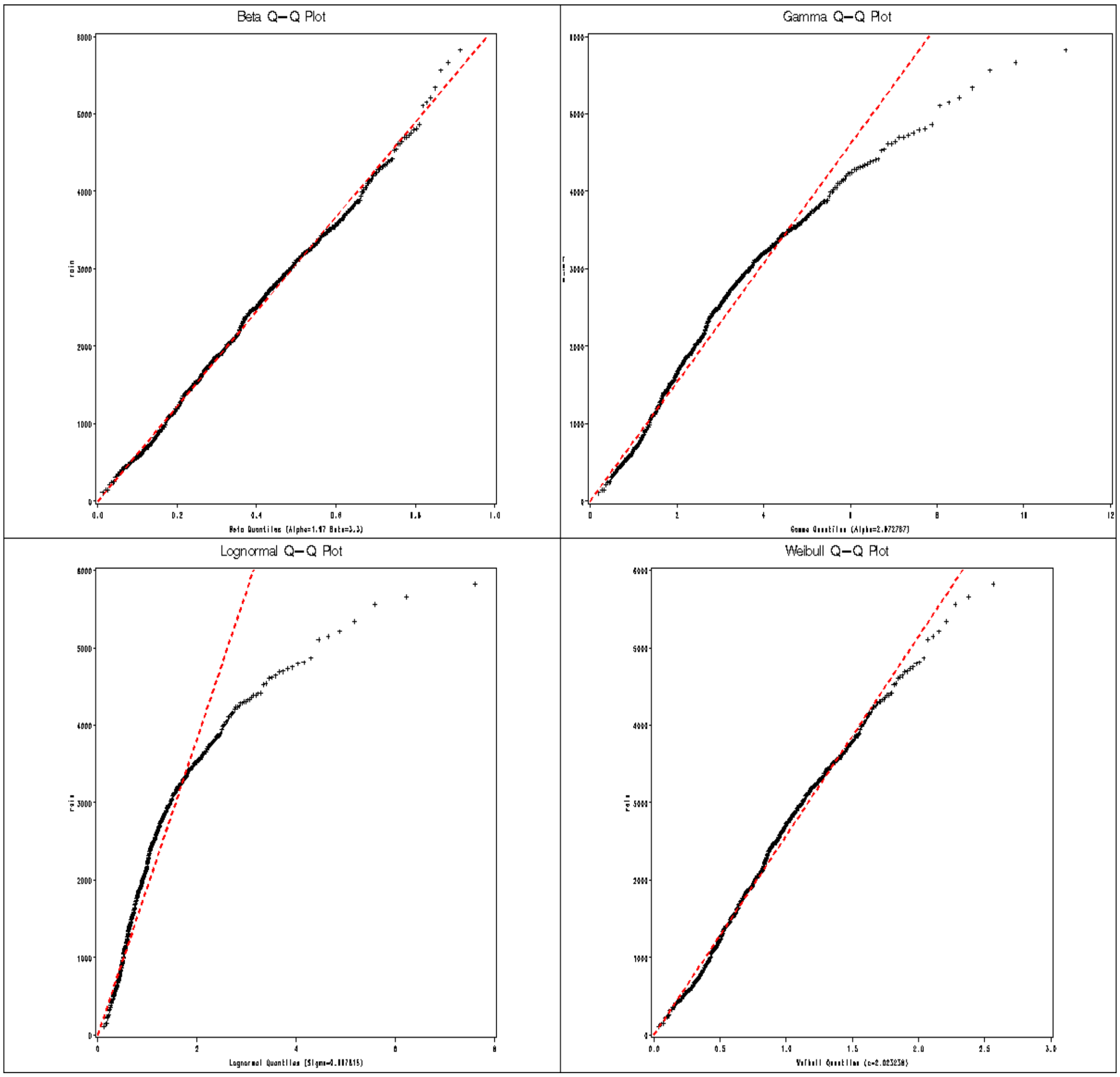


Table 2. Goodness-of-Fit Tests for Alternative Distributions in WMPRA

	Tests	Statistics		P-Value	
Beta	Kolmogorov-Smirnov	D	0.0314	Pr>D	>0.250
	Cramer-von Mises	W-Sq	0.0780	Pr>W-Sq	0.242
	Anderson-Darling	A-Sq	0.5204	Pr>A-Sq	0.2
	Chi-Square	Chi-Sq	12.5061	Pr>Chi-Sq	0.253
Gamma	Kolmogorov-Smirnov	D	0.0771	Pr>D	<0.001
	Cramer-von Mises	W-Sq	0.7494	Pr>W-Sq	<0.001
	Anderson-Darling	A-Sq	4.4289	Pr>A-Sq	<0.001
	Chi-Square	Chi-Sq	45.0614	Pr>Chi-Sq	<0.001
Lognormal	Kolmogorov-Smirnov	D	0.1012	Pr>D	<0.010
	Cramer-von Mises	W-Sq	1.8238	Pr>W-Sq	<0.005
	Anderson-Darling	A-Sq	10.8661	Pr>A-Sq	<0.005
	Chi-Square	Chi-Sq	133.2238	Pr>Chi-Sq	<0.001
Weibull	Cramer-von Mises	W-Sq	0.2435	Pr>W-Sq	<0.010
	Anderson-Darling	A-Sq	1.5594	Pr>A-Sq	<0.010
	Chi-Square	Chi-Sq	18.0760	Pr>Chi-Sq	0.054

Figure 2. QQ-plots of alternative distributions for the rainfall data in WMPRA



Both the Q-Q plot and the standard goodness-of-fit tests suggest that the beta distribution should be the most appropriate candidates since all four tests fail to reject the null hypothesis at a 10 percent significant level and the Q-Q plot is almost an ideal unit diagonal. The Weibull distribution should be considered second after the beta distribution. The lognormal and the gamma distribution both appear to be poor candidates for fitting the rainfall series by the goodness-of-fit tests. However, the tail behavior in the Q-Q plot suggests that the gamma distribution still provides a slightly better fit than the lognormal distribution.

After comparing the standard goodness-of-fit tests and Q-Q plots of these alternative distributions on a region-by-region basis, the number of times each candidate ranked first through fourth, along with the weighted average rank, and rank of average are summarized in Table 3. The results confirm that the appropriate distribution differs across the 14 regions and the Weibull distributions fit overall the best in the majority of the regions (5 regions in the first rank and 9 regions in the second rank, the weighted average of rank is 2.3). The fitting performance is nearly the same for the gamma and beta distribution and the gamma outperforms the beta distribution and takes the overall second position. The lognormal distribution is much inferior to the other three candidates and ranks only third in 2 regions and fourth in most regions with a weighted average of 5.4.

Table 3. Rankings of Alternative Distributions

	Alternative Distributions			
	Beta	Gamma	Lognormal	Weibull
1st	3	6	0	5
2nd	5	0	0	9
3rd	4	8	2	0
4th	2	0	12	0
Weighted Average	3.3	3	5.4	**2.3**
Rank of Average	3	2	4	**1**

The results are not surprising considering the microclimate pattern across regions. Sherrick, et al. (2004) also find similar results when using alternative distributions in modeling corn and soybeans in the United States. Their results suggest that the Weibull and beta distributions are overall ranked first and second in fitting corn yield and the logistic and Weibull distributions perform first and second in modeling soybean yield for selected farms at the University of Illinois.

Distributional choice has a tremendous impact on the risk assessment, and the selection of an appropriate underlying distribution can directly determine the economic effectiveness of risk hedging. Since the appropriate distribution differs across regions due to microclimate patterns, it might be best to find an appropriate candidate for each of the 14 regions based on the specification tests. However, such a method is time-consuming and costly for a large area. For example, crop-yield distributional modeling involves thousands of counties in the United States and rainfall series estimation includes hundreds of regions in most developing countries. Therefore, it is common to adopt the overall best distribution used in current crop insurance programs and weather index design. Unfortunately, even the overall best distribution can lead to misleading risk assessments and inaccurate premium ratemaking in some regions. For example, the Weibull distribution ranked best overall but only fit best in 5 out of the 14 Indian regions. Some efficiency in the other 9 regions may be lost when applying the Weibull distribution to model the rainfall series across the 14 regions.

Step 3. Estimate the rainfall series using EVT model

In this subsection, the POT model is used to model the excess rainfall risk and the GPD is chosen as the candidate distribution. First, the threshold (u) is decided, based on the mean residual plot on a region-by-region basis. As discussed earlier, an ideal mean excess plot should be approximately a straight line against the threshold. Next, the scale and shape parameters are estimated by the maximum likelihood method, based on the procedures provided above. Finally, a variety of statistical techniques, such as the

P-P plot, Q-Q plot, return level, and density function, are plotted to check the appropriateness of the GPD in modeling excess rainfall.

The estimated shape parameter is less than zero for all regions, suggesting that the excess monthly rainfall follows the Type III class of extreme value distribution, that is, the Weibull distribution. The various diagnostic plots for assessing the appropriateness of the GPD model fit to the rainfall data across the 14 regions. None of these plots call into question the validity of the fitted models.

Step 4. Compare the economic importance of estimations based on two methods

Weather-based contingent claims can provide an effective cross-hedging mechanism against the revenue uncertainty due to weather, such as heat-based insurance (Turvey, 2001), index products with rainfall application (Martin, Barnett, and Coble, 2001). A weather derivative is a contract between two parties that stipulates how payment will be exchanged between the parties depending on certain meteorological conditions during the contract period. Recently, Turvey, Weersink, and Chiang (2006) have developed a new method under situations where returns depend on not only the occurrence of the weather events, but also the timing, to price weather insurance for ice wine in the Niagara Peninsula of southern Ontario.

In this study, the design of the weather index follows the European precipitation options proposed by Zeuli and Skees (2005) but it is in the form of a call option, that is, indemnity payments are triggered when the actual monthly precipitation is above the pre-specified strike. The indemnity function is given by

(5) $$I(\tilde{w}) = \theta \times Max((\frac{X - x_c}{x_c}),0)$$

where x_c is the pre-determined trigger for obtaining the indemnity, and θ is the liability, that is, the maximum possible indemnity.

To formalize this study, the strike x_c is defined as a fraction of the proven precipitation level, $\bar{x}$, that is, $x_c = h * \bar{x}$. In this study, the mean of month rainfall during the 1871 – 2001 period is chosen as $\bar{x}$ and the available fractions vary from 1.2 to 1.5. The break-even premium rate is the standard basis for establishing insurance actuarial policy and can be calculated as the average of the percentage shortfalls above the strike following Ker and Coble (2003) and Skees, Black, and Barnett (1997):

(6) $$P_i = \int_{w_c}^{\infty} (\frac{X - x_c}{x_c}) dF_i(x) = \frac{P(X > x_c)(E(X \mid x > x_c) - x_c)}{x_c}$$

where the expectation operator and probability measure are taken with respect to the underlying distribution (i=1 means the beta distribution, i=2 means the gamma distribution, i=3 means the lognormal distribution, i=4 denotes the Weibull distribution, and i=5 denotes the GPD).

Given a risk distribution and strike level, the pure premium rates can be easily obtained and Table 4 presents the summary of actuarially fair premium rates estimated for the five rainfall distributions with varying strike levels. The paired t-tests for equality of means of alternative parametric distributions and GPD are also provided in this table.

Table 4. Pure Premium Rate of Weather Index under Alternative Distributions at Varying Strikes

h=1.2. Mean	0.0787	0.0861	0.0779	0.1184	0.0763
Std. Dev.	0.0293	0.0368	0.0288	0.0706	0.0292
Min	0.0341	0.0368	0.0367	0.0414	0.0355
Max	0.1323	0.1691	0.1368	0.3047	0.1409
h=1.3. Mean	0.0531	0.0609	0.0508	0.0906	0.0502
Std. Dev.	0.0234	0.0317	0.0239	0.0631	0.0250
Min	0.0170	0.0203	0.0180	0.0242	0.0172
Max	0.0973	0.1341	0.1010	0.2597	0.1079
h=1.4. Mean	0.0353	0.0432	0.0324	0.0703	0.0328
Std. Dev.	0.0187	0.0267	0.0191	0.0559	0.0205
Min	0.0087	0.0106	0.0078	0.0142	0.0075
Max	0.0751	0.1066	0.0744	0.2224	0.0822
h=1.5. Mean	0.0221	0.0309	0.0202	0.0553	0.0213
Std. Dev.	0.0141	0.0222	0.0145	0.0490	0.0163
Min	0.0039	0.0055	0.0029	0.0082	0.0029
Max	0.0541	0.0852	0.0534	0.1904	0.0623
Paired t-test					
h=1.2		2.8438**	-0.7391	3.3454***	-1.7576
h=1.3		2.9118**	-2.6339**	3.3207***	-2.3237**
h=1.4		3.2401***	-6.2201***	3.3816***	-3.1136***
h=1.5		3.7676***	-6.4180***	3.4638***	-1.0549

***: Significant at the 10% level; **: Significant at the 5% level; ***: Significant at the 1% level**

Among the four alternative distributions, the Weibull distribution, the overall best fitting candidate, tends to have lower pure premium rates, while the lognormal distribution, the overall worst fitting candidate, tends to have higher premium rates. Due to the diversified performance of the beta and gamma distribution, the pure premium rates obtained from these two candidates are generally between the lowest level obtained from the Weibull distribution and the highest level obtained from the lognormal distribution. The results suggest that some parametric distributions might underestimate the tail risk (e.g., the Weibull distribution) while other might overestimate it (e.g., the lognormal distribution). On the other hand, the pure premium rates obtained from the GPD lie in between those from the Weibull distribution and those from the beta and gamma distributions, suggesting that the GPD might be more appropriate in modeling tail-part risk. However, further statistical tests are needed.

The strike levels that trigger the indemnity payment vary when h equals 1.2, 1.3, 1.4, and 1.5, respectively. The premium rates tend to be lower with a higher strike level, and higher with a lower strike level. Furthermore, paired t-tests are performed where the GPD is chosen as the reference sample. The results show that the premium rates obtained from the Weibull distribution at h =1.5 and h=1.2, and the beta distribution at h=1.2 are not significantly different from those obtained from the GPD. The premium rates obtained from all the other distributions are all significantly different than those obtained from the GPD. The results demonstrate that alternative candidates have significantly different performances in economic implications.

Step 5. Perform sensitivity analysis of different strike levels

The last step compares the premium rates from the GPD and those from the first ranked candidate based on the goodness-of-fit test and the Q-Q plot. For each region, the pure premium rate based on the best candidate among the beta distribution, gamma distribution, or Weibull distribution, is chosen as the base case and compared with the performance of the GPD in modeling the tail risk. The results of the nonparametric sign test and Wilcoxon signed rank test applied to test the equality of means are shown in Table 5.

Table 5. The Actuarial Performance of the GPD and the Best Candidate

	h=1.2		h=1.3		h=1.4		h=1.5	
	Best	GPD	Best	GPD	Best	GPD	Best	GPD
BHPLN	0.0777	0.0824	0.0501	0.0566	0.0320	0.0362	0.0199	0.0212
BHPLT	0.0647	0.0659	0.0397	0.0429	0.0236	0.0260	0.0135	0.0164
COAPR	0.0825	0.0818	0.0573	0.0546	0.0393	0.0384	0.0268	0.0232
EMPRA	0.0829	0.0918	0.0553	0.0625	0.0359	0.0387	0.0230	0.0228
EUPRA	0.1089	0.1090	0.0770	0.0763	0.0533	0.0533	0.0360	0.0359
GNWBL	0.0554	0.0564	0.0342	0.0349	0.0209	0.0200	0.0128	0.0114
NASSM	0.0355	0.0341	0.0172	0.0170	0.0075	0.0087	0.0029	0.0039
ORISS	0.0684	0.0669	0.0452	0.0416	0.0292	0.0259	0.0190	0.0133
SASSM	0.0368	0.0380	0.0203	0.0221	0.0106	0.0113	0.0055	0.0054
SHWBL	0.0527	0.0505	0.0321	0.0303	0.0196	0.0189	0.0116	0.0084
TELNG	0.0853	0.0800	0.0595	0.0563	0.0412	0.0394	0.0284	0.0255
VDPBH	0.0870	0.0921	0.0586	0.0668	0.0388	0.0435	0.0254	0.0299
WMPRA	0.1114	0.1205	0.0786	0.0839	0.0548	0.0590	0.0370	0.0385
WUPPL	0.1368	0.1323	0.1010	0.0973	0.0744	0.0751	0.0534	0.0541
Mean	0.0776	0.0787	0.0519	0.0531	0.0344	0.0353	0.0225	0.0221
Std. Dev.	0.0286	0.0293	0.0232	0.0234	0.0182	0.0187	0.0136	0.0141
Min	0.0355	0.0341	0.0172	0.0170	0.0075	0.0087	0.0029	0.0039
Max	0.1368	0.1323	0.1010	0.0973	0.0744	0.0751	0.0534	0.0541
Paired t-Test	P-value	0.3560	P-value	0.2999	P-value	0.1683	P-value	0.6107
Sign Test		0.7905		0.7905		0.7905		0.4240
Wixcoxon Test		0.6257		0.4631		0.2412		0.6698

The means and variability of pure premium rates from the GPD are very close to those from the best candidate across different strike levels. Furthermore, all of these tests fail to reject the null hypothesis of the equality of pure premium rates based on the GPD and the best candidate with a high *p* value, demonstrating that the GPD performs as good as the best standard parametric method and is effective and robust in modeling and assessing tail risk and premium ratemaking

Policy Implications

Accurate estimation of tail events may be of particular interest to decision makers. The EVT can be considered the state-of-the-art procedure for estimating the downside risk of a distribution and provides promising potential for risk assessment and premium ratemaking of weather-based contingent claims.

The results also demonstrate that large differences in actuarially fair premium rates for a rainfall-based contingent claim can arise solely from the parameterization chosen to represent the underlying risk distributions, and misspecification in the risk distribution (e.g., the lognormal distribution) may lead to economically significant errors in weather index assessment of expected risks and premium ratemaking.

Furthermore, when modeling the tail risk, the GPD model is promising since it performs close to the best candidate chosen by different parametric distributions. What is evident from this study is that the distributional choice has a significant impact on rating and assessing weather-based contingent claims, and so the GPD model is effective and robust in modeling the tail risk.

However, this study addresses a limited set of parametric distributions and only one potential weather-based contingent claim (the rainfall index). Future work could consider a wide set of distributional choices, especially nonparametric techniques, and demonstrate the effectiveness of the GPD in a general case. In addition, the extreme value approach may involve a loss of information and the accuracy of estimation of a small sample size might be compromised in some realistic situations.

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