INTRODUCTION

Wind waves belong to a high-frequency type of geophysical oscillations, and their characteristic periods are of the order of seconds. Long–term variations of wind wave field statistical parameters are produced by modulation of their generation conditions. The strongest manifestations are on the synoptic temporal scale, but significant scales of such variability also include annual, inter-annual and longer variations.

If wind wave generating conditions are constant, wind waves can be considered a quasi-stationary, small-scale geophysical process. For the deep sea case, the wave heights h obeys the Rayleigh distribution

$$F(h) = 1 - exp\left[-\frac{\pi}{4}\left(\frac{h}{\bar{h}}\right)^2\right] \qquad \text{(I.1)}$$

with $\bar{h}$ denoting mean wave height.

Forristall [Forristall, 1978] proposed another distribution, which is in frequent use now:

$$F(h) = 1 - exp\left[-2.26\left(\frac{h}{h_s}\right)^{2.126}\right] \qquad \text{(I.2)}$$

Here $h_s = 1.6\,\bar{h}$ is the so-called "significant" wave height. For a record consisting of n waves, it is equal to the average height of the one-third highest waves. After normalizing with respect to the zero-order moment of the spectrum m_0 the above distributions (1.1) and (1.2) read as follows:

$$F(h) = 1 - exp\left[-\frac{1}{8}\left(\frac{h}{\sqrt{m_0}}\right)^2\right] ,$$

$$F(h) = 1 - exp\left[-\frac{1}{8.42}\left(\frac{h}{\sqrt{m_0}}\right)^{2.126}\right]$$

Comparison of these distributions shows that they are close for small probabilities. At the same time, the relation (1.2) predicts somewhat smaller values of h_{max} for higher waves.

For example, the Forristall relation results in an estimate of the highest wave in a thousand waves, which is equal to 0.907 of the estimate obtained with the Raleigh distribution. Wave heights in a sequence are statistically connected, and their correlation function is as follows:

$$K_h(\tau) = D\, exp(-\alpha\,|\tau|) \qquad \text{(I.3)},$$

where D denotes the process variance, α is the decrement, and τ is the time lag.

The most fundamental starting point for derivation of equations governing the wave spectrum evolution is the equation for the conservation of the wave action density N (see e.g., [Komen et al., 1994; Lavrenov, 1998]):

$$\frac{\partial N}{\partial t} + \frac{\partial N}{\partial \varphi}\dot{\varphi} + \frac{\partial N}{\partial \theta}\dot{\theta} + \frac{\partial N}{\partial k}\dot{k} + \frac{\partial N}{\partial \beta}\dot{\beta} + \frac{\partial N}{\partial \omega}\dot{\omega} = G_s \qquad \text{(I.4)}$$

N is a function of latitude φ, longitude θ, wave number k, angle β between the direction of wave propagation and the parallel, angular frequency ω, and time t. In the deep sea case the source function G_S is represented as the sum of three terms:

$$G_S = G_{in} + G_{nl} + G_{ds}$$

G_{in} parameterizes spectral wave energy generation by the wind, G_{ds} is the wave energy dissipation, and G_{nl} represents the effect of weak nonlinear interactions on the wind wave spectrum change.

Present spectral wind wave models based on equation (1.4) are rather well developed. They incorporate a representation of all significant mechanisms affecting the wave spectrum evolution and are quite sophisticated numerically. Being forced by wind data (or atmospheric pressure), and data on boundary layer stability, the models compute the two dimensional (with respect of frequency and direction) spectrum $S(\omega, \beta)$ at nodes $\vec{r}_i$ of the numerical grid at times t_j.

For the statistical analysis of long term series, we will use in this study the results of hydrodynamic model simulations. The basic variable will be mean wave height $\bar{h} = \sqrt{2\pi m_0}$, where m_0 is the zero-order moment of the two-dimensional spectrum, i.e.

$$m_0 = \iint S(\omega,\beta)\,d\omega d\beta$$

at fixed locations r_i. The simulations were conducted at the Arctic and Antarctic Research Institute under the supervision of Dr. Igor V. Lavrenov. Another source of input data will be long-term synoptic wind wave observations $\bar{h}_t$ at automated buoys in several areas of the World Oceans [Buckley, 1988; Boukhanovsky et al., 2000] and estimates $h=\sqrt{h_{ws}^2+h_{sw}^2}$ from visual ship observations. Here h_{ws} and h_{sw} are wind sea and swell heights, respectively.

Time series of wind wave heights in mid-latitudes and subtropical areas of the World Oceans make alternating sequences of storms and weather windows. We define a storm of duration $\Im$ and intensity h^+ as a situation when the random function *h(t)* exceeds a predefined value *Z*. The period Θ during which the wave height is less than this threshold will be called a weather window of intensity h^-.

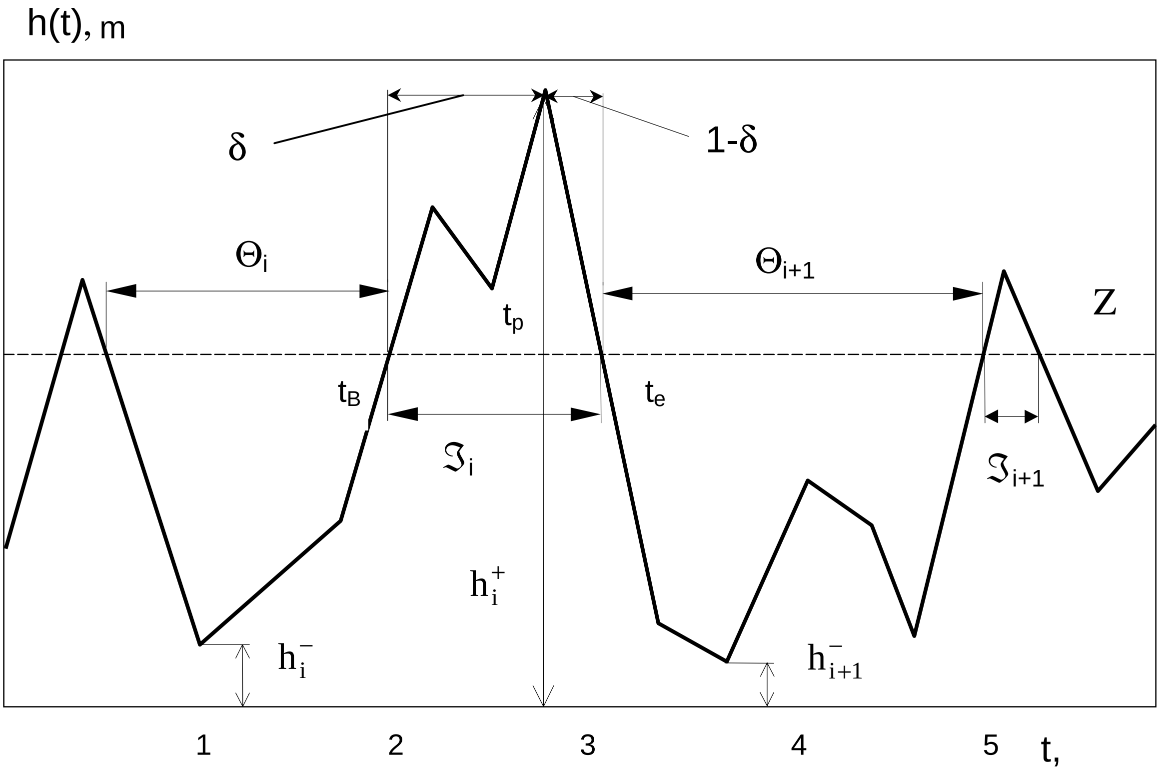

Figure. I.1. Parameters describing storms and weather windows

The parameter δ shows the asymmetry of the storm: $\delta=(t_p-t_b)/\Im$; t_b, t_p, t_e are times of storm start, maximum development, and end, respectively. Fig. 1 clarifies these definitions.

Wave observations or model simulation results can be represented in a more general way by the log-normal approximation of wave height distribution. The corresponding distribution density function reads as follows:

$$f(h)=\frac{s}{h\sqrt{2\pi}}exp\left[-\frac{s^2}{2}(ln\,h-ln\,h_{0.5})^2\right] \qquad (I.5)$$

where $h_{0.5}$ is the median, and s^{-1} is the r.m.s. deviation of the wave height logarithms. Fig. I.2 gives an example of wave height distribution plotted against probability (I.5) of non-exceedance.

If a wave height series *h(t)* at times of synoptic observations (i.e. with recording interval of 3 or 6 hours) is being considered as a sample of a stationary random function, then its auto-correla tion function for the synoptic variability range can

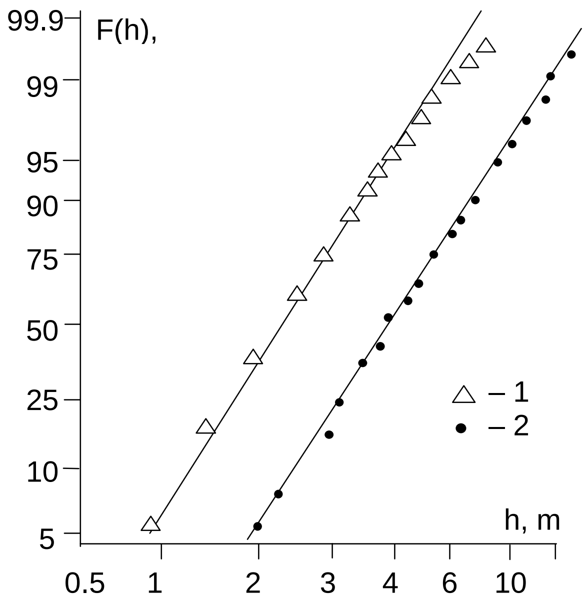

Figure I.2. Combined (wind sea and swell) wave height distribution for February (1) and August (2). Log-normal probability plot. Ocean Weather Station “Lima”: data of 1976-1980.

also be written as (I.3), but with other parameters. For example, from two to four consecutive individual waves within the quasi-stationary period are expected to correlate (the whole episode lasting 10-20 seconds). Wave observations at synoptic times are correlated, on average, for 1.5-3 days.

Wind waves also undergo an annual cycle. This results in a corresponding variation of monthly wave characteristics. For example, monthly mean wave heights $\bar{h}_t$ and parameters $h_{0.5}$ and s of distribution (I.5) vary in a cyclical mode from season to season and show stochastic fluctuations from year to year. Fig. I.3 shows the seasonal variation of parameters $h_{0.5}$ and s at Ocean Weather Station “M” located in the Norwegian Sea. Monthly parameters exhibit explicit seasonal variability, and some stochastic fluctuations are seen as variations of data in the same months of different years. The January median (shown as a horizontal line in boxes in Fig. 1.3.a) of $h_{0.5}$ estimates is approximately 3.2 m. During individual years it can vary from 2.2 to 4.0 m, making the inter-quartile range of (3.5-2.8) = 0.7 m. Such rhythmic variations can be expressed mathematically through a periodically correlated stochastic process (PCSP) with mean *m(t)* and variance *D(t),* which are periodic functions of time with period T = 1 year. Its covariance function $K(t_i,t_j) = K(t_i+T,t_j+T)$ depends on both arguments.

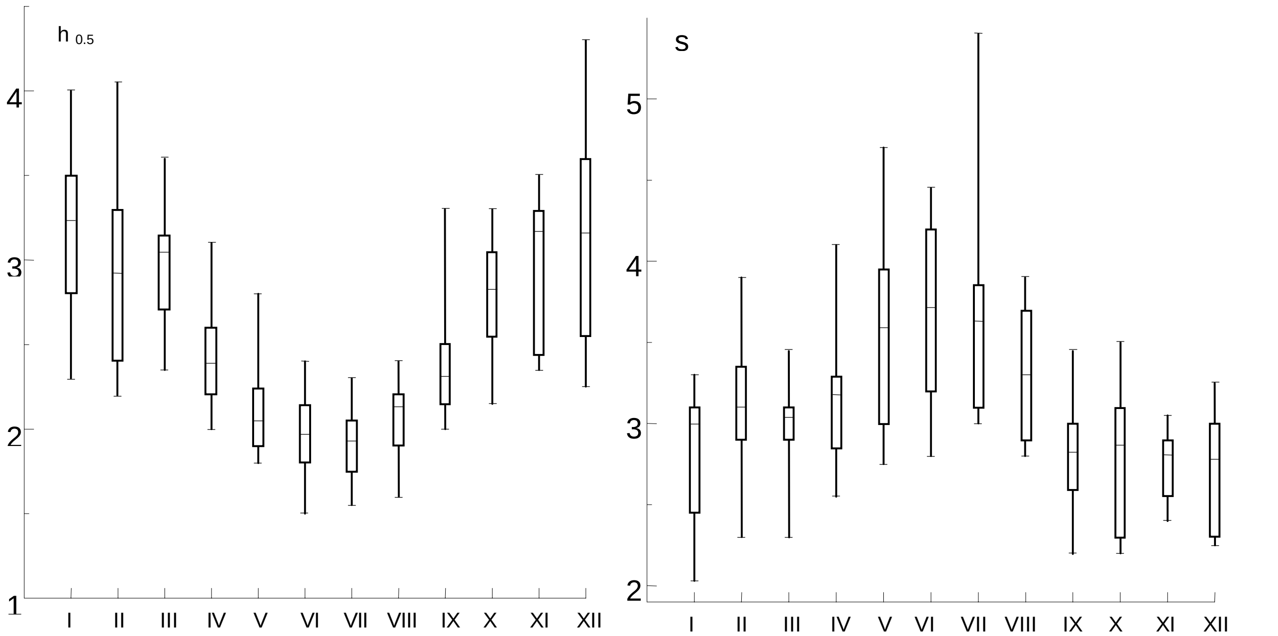

Figura I.3. Estimates of log-normal wave height distribution parameters $h_{0.5}$ and s at Weather Station “M”

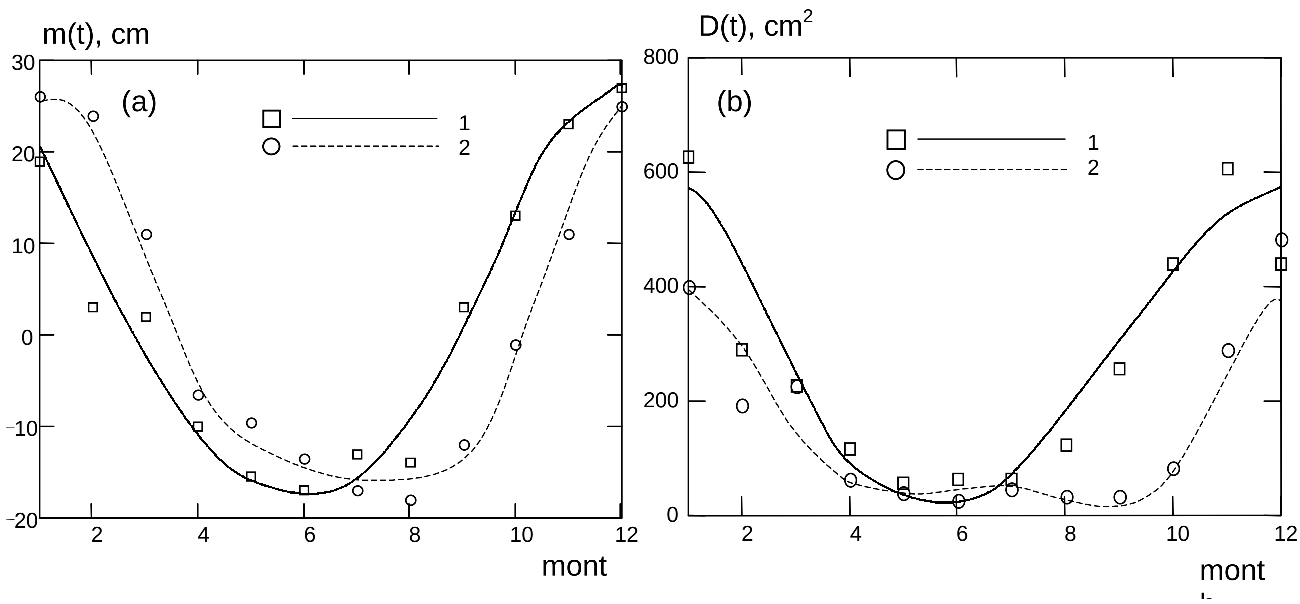

Figura I.4. Mathematicall expectation (a) and variance (b) of monthly mean wave height. All values are centered with respect to the mean annual wave height: (1)- The Baltic Sea, (2) The Black Sea.

PCSP samples, if they are taken at intervals equal to the correlation period *T*, produce stationary random series. Fig. I.4 shows functions *m(t), D(t)* obtained in experiments held in the Black Sea and Baltic Sea.

The following stochastic models can be used for the simulation of random series with a priori given properties.

Auto-regression model for the quasi-stationary and synoptic variability ranges

At the quasi-stationary and synoptic intervals of variability the wave process is best described by the stationary auto-regression model AR(p) of order p, namely

$$\xi_t = \sum_{k=1}^{p} \phi_k \xi_{t-k} + \varepsilon_t, \qquad \zeta_t = f(\xi_t) \qquad \text{(I.6)}$$

where ϕ_k are coefficients to be computed using the correlation function $K_\xi(\tau)$ as given by relation (I.2), ε_t is white noise with a given distribution function, which has to be compatible with the nonlinear functional transformation $f(\bullet)$ of function ξ_t into, respectively, the Rayleigh (I.1) or log-normal (I.5) distribution of ζ_t.

Stochastic model for sequence of storms and weather windows

A stationary pulse-like random process is a good model for sequence of storms and fair weather intervals. A sample can be generated as follows:

$$\xi(t) = \sum_{k=1}^{n} w_k \left(Z, t - \sum_{j=1}^{k-1} \left(\Im_j + \Theta_j \right) \right) \qquad \text{(I.7)}$$

where $\Im_j$ and Θ_j are, correspondingly, the duration of the storm and the weather window (with threshold value *Z*),

$$w(Z,t) = \begin{cases} Z + \left(h^+ - Z\right) u\left(t/\Im\right) & 0 \le t \le \Im, \\ Z - \left(h^- - Z\right) u\left((t - \Im)/\Theta\right) & \Im \le t \le \Im + \Theta \\ 0 & (t < 0) \cup (t > \Im + \Theta) \end{cases}$$

h^+, h^- are the highest wave height in storm and the minimum wave height during the weather window. Function *u(t)* prescribes the shape of the non-dimensional impulse. The triangular shape of this function

$$u(t) = \begin{cases} t/\delta & 0 \le t \le \delta, \\ 1/(1-\delta) - t/(1-\delta) & \delta \le t \le 1 \\ 0 & (t < 0) \cup (t > 1) \end{cases}$$

serves as a good first approximation. Parameter δ, as seen from fig. I.1, defines the asymmetry of function *u(t)*. If δ=0.5, the function is symmetric.

The actual generation of a series of random storms and weather windows is based on the Monte Carlo approach. First, the distribution function $F_\Xi(\cdot)$ and the matrix co-variation function $K_\Xi(\tau)$ are specified which fit the set of four random values $\Xi \sim (h^+, \Im, h^-, \Theta)$ or time series Ξ_t. Secondly, a non-dimensional storm shape function *u(t)* is chosen. Finally, an ensemble of storms and weather windows is generated numerically.

Stochastic model for extra-annual rhythms

This model is written as follows:

$$\xi(t) = m(t) + \sigma(t)\xi_t \qquad \text{(I.8)}$$

Here *m(t)* and *σ(t)* are periodic functions, and ξ_t is a non-stationary process AP(p) so that

$$\xi_t = \sum_{k=1}^{p} \phi_k(t) \xi_{t-k} + \varepsilon_t \qquad \text{(I.9)}$$

Coefficients $\phi_k(t) = \phi_k(t+T)$ are periodic functions of time.

A model that is capable of describing the year-to-year variability of monthly mean wave heights will therefore require twelve values of *m(t)* and 78 values of $K(t,\tau)$. It is possible to reduce the number of dimensions by considering the following representation of PCSP:

$$\zeta(t) = \sum_{k=-\infty}^{\infty} \eta_k(t) \exp\left(i\omega_k t\right) \qquad \text{(I.10)}$$

Here $\eta_k(t)$ are stationary random processes (components) with mathematical expectation m_k and co-variation function $K_k(\tau)$ that can be obtained by expressing functions *m(t)* and $K(t,\tau)$ as Fourier expansion series.

Relation (I.10) resembles a Fourier series expansion of $\zeta(t)$. However, both the coefficients and basis functions in it depend on the time variable, and hence (I.10) is not a Fourier expansion. A simpler model for PCSP can therefore be obtained by expanding the function $\xi(t)$ for *each* annual interval, as follows:

$$\zeta(t) = a_0 + \sum_{k=1}^{q} \left(a_k \cos \omega_k t + b_k \sin \omega_k t \right) \qquad \text{(I.11)}$$

where a_k and b_k are random values, and *q* is the order of the model.

For a stationary process it is possible to suppose that values a_k and b_k are independent, while for a non-stationary process they will be dependent. Table I.1 gives average values of means (m_{a_k}, m_{b_k}), variances (D_{a_k}, D_{b_k}), co-variation $K_{a_k b_k}$, and correlation ρ_{a_k, b_k} for coefficients of the model of annual rhythms. Hence, instead of model (I.8-I.9) with 90 parameters, a simpler model (I.11) with 20 parameters (see Table I.1) may be used. Monthly mean values of wave heights in the Black Sea were used for the computations. Corresponding values of $m(t)$ and $D(t)$ are shown in Fig. I.4. Coefficient a_0 in the table is equal to the annual average wave height. Coefficients a_1, a_2 correspond to the cosine component of the annual and semi-annual harmonics. Correspondingly, b_1 and b_2 correspond to the sine component. It becomes obvious from the table that a_k and b_k are strongly correlated. For example, the correlation coefficient between a_o and a_1 is 0.66.

All above models make it possible to describe wind waves as a multi-cyclic, multi-modulated random process. The multi-cyclic behavior of waves reflects the co-existence of sea and swell in the combined wave field. Multi-modulation is related to synoptic, seasonal, and extra-annual variability of the averaged wave parameters. Hydrodynamic properties of wind waves can be simulated by models based on equations for wave action density such as (I.4), and models (I.6) – (I.10) are available for the statistical description of the wave field.

At the same time, many practical computations of extreme wave height h_{max}, for example offshore and shelf engineering applications, employ the assumption that wave height series is a sequence of random values. The first approach of this kind is called the method of initial distribution. It is described in the following section.

Table I.1.

Statistical parameters of coefficients a_k, b_k of monthly mean wave height rhythms model (I.11). The Black Sea

Parameter	***m*,cm**	***D*, cm^2**	**K_{a_k,b_k} (cm^2) and ρ_{a_k, b_k}**				
			a_0	**a_1**	**b_1**	**a_2**	**b_2**
a_0	80	22	**1**	0.66	0.54	0.22	0.60
a_1	19	42	20	**1**	0.21	0.65	0.47
b_1	12	17	11	6	**1**	-0.26	0.52
a_2	2	39	6	26	-7	**1**	0.15
b_2	4	19	12	13	9	4	**1**

***Note**:* co-variation K_{a_k,b_k} is given below the diagonal and correlation coefficient ρ_{a_k, b_k} is given above the diagonal.

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