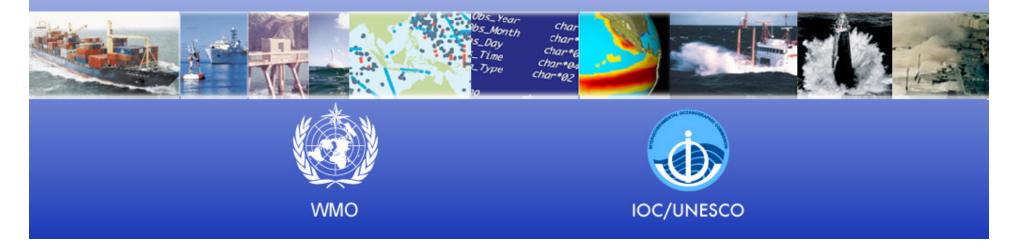


Joint WMO-IOC Technical Commission for Oceanography and Marine Meteorology

## **Wave Measurement Evaluation and Testing**

#### Val Swail<sup>1</sup> and Boram Lee<sup>2</sup>

#### <sup>1</sup>Chair, JCOMM Expert Team on Wind Waves and Storm Surges <sup>2</sup>World Meteorological Organization, Marine Meteorology and Oceanography Programme





#### **OBSERVATION REQUIREMENTS FOR WIND WAVES**

#### (developed by the JCOMM Expert Team on Wind Waves and Storm Surges)

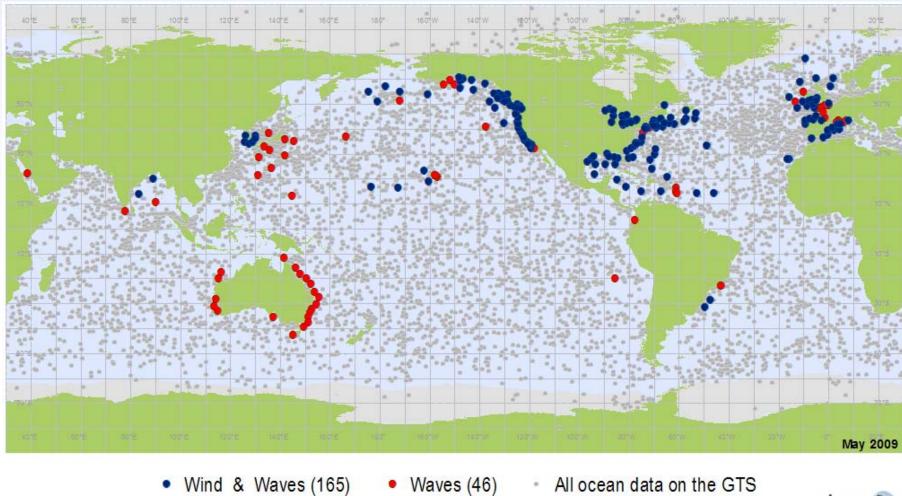
#### **Applications:**

- Assimilation into offshore wave forecast models
- Validation of wave forecast models (and hindcast and reanalysis)
- Calibration / validation of satellite wave sensors
- Ocean wave climate and variability
- Role of waves in coupling
- Coastal zone modelling erosion, sediment transport, inundation etc.
- Reference:
- OceanObs09 paper Swail et al.
- OceanObs99 paper Swail et al.
- DBCP-22 Meeting Report October 2006
- ETWS-II Meeting Report March 2007
- CBS/OPAG-IOS/ET-EGOC-3 Doc. 7.2.6





# Wave Data on the GTS









### OceanObs09

"Continuous testing and evaluation of operational and pre-operational measurement systems is an essential component of a global wave observing system, equal in importance to the deployment of new assets"

Swail et al., *Wave Measurements, Needs And Developments* For The Next Decade. OceanObs09 publication.



#### How to "ground truth" the "ground truth" ?



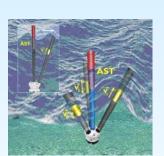


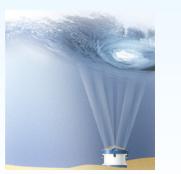
























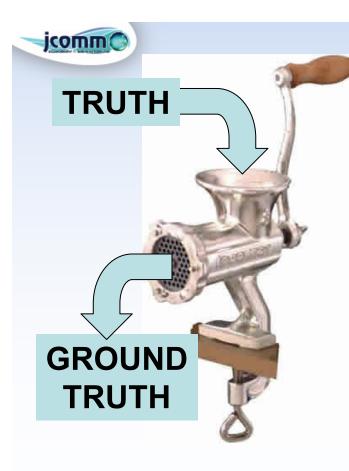


# How is your wave measurement?





Courtesy C-C Teng



## New System for obtaining "ground truth" for wave measurements

## Or

What about an independent group of assessors??



**Courtesy Don Resio** 



JCOMM Technical Workshop on Wave Measurements from Buoys New York, 2- 3 October 2008

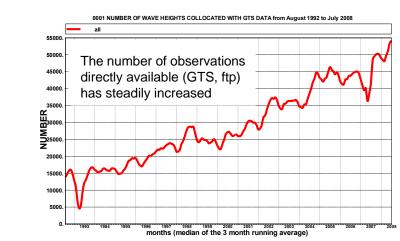
• <u>www.jcomm.info/Wavebuoys</u>



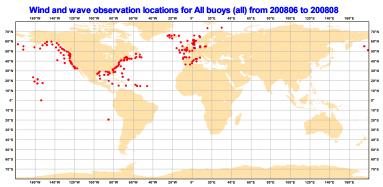


#### Introduction: wave in-situ data for in-house verification

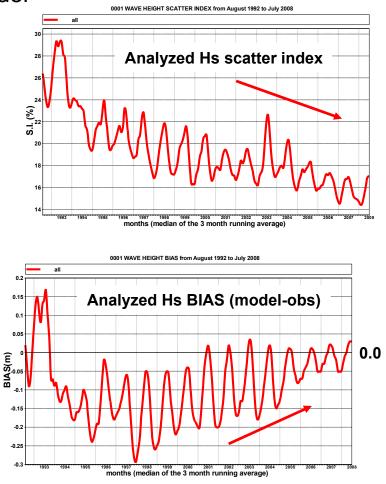
In situ wave observations have been used to assess the quality of the ECMWF wave model analyses and forecasts since 1992.



e.g.

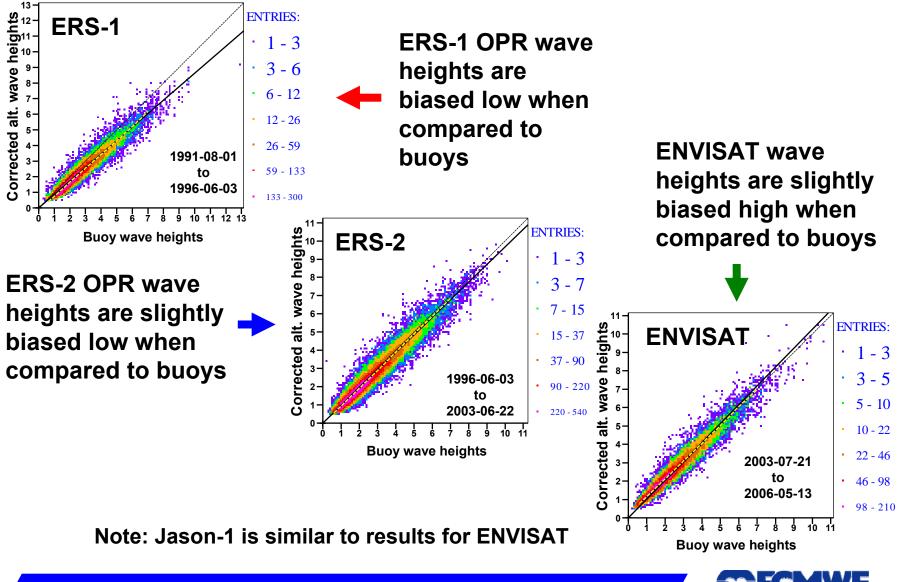


Locations of moored buoys, platforms and ships from which wind and wave observations are used in this verification.



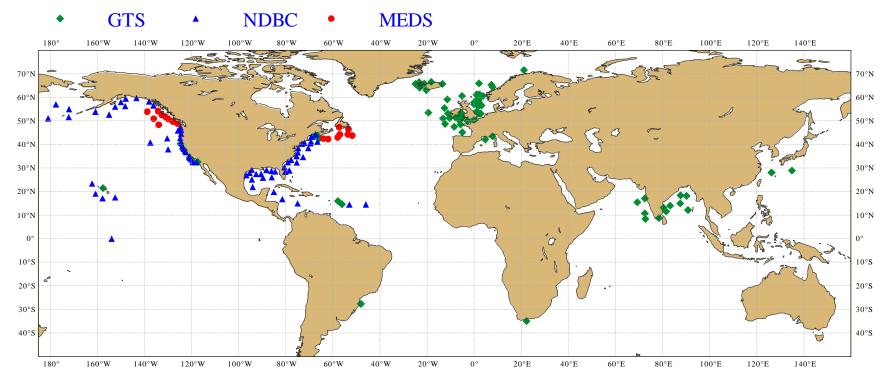


# Using all in-situ wave data for the interim reanalysis :





#### **Discrepancies in wave observations:** data used for the altimeter calibration



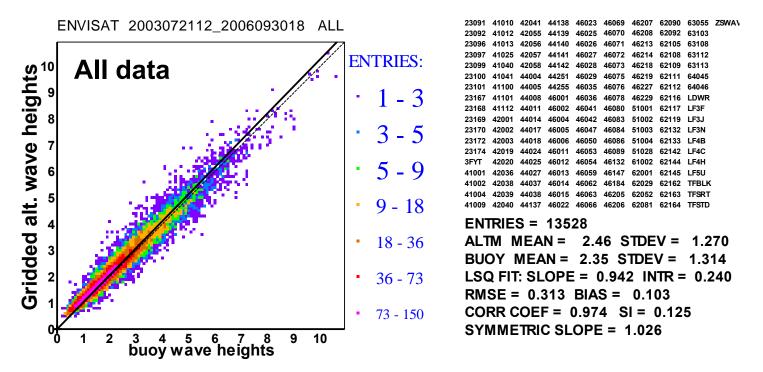
Data are from different sources:

NDBC (from NODC archive (ftp)), MEDS archive online.

GTS: data that were distributed by the Global Telecommunication System and archived at ECMWF. These are mainly from European buoys (UK, France, Ireland, Iceland), Japanese buoys, Indian buoys, Other American centres (Scripps, GoMoos,...), UK and Norwegian platforms and one South African platform (NDBC and MEDS are also on the GTS but slightly better data were obtained from the web).

#### **Discrepancies in wave observations:**

#### Collocation with ENVISAT



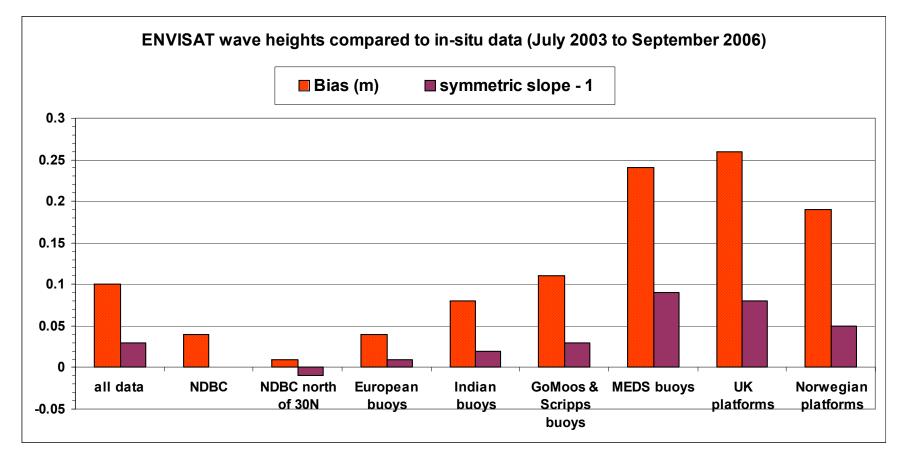
Comparison of gridded altimeter with buoy wave heights for 100. km, 5 % max RCE and 45. degrees max in mean wave dir

Triple collocations are used, in which a model hindcast is also used to determine whether or not altimeter and buoy should be collocated. RCE: Relative Collocation Error (abs(alt-buoy)/mean(alt,buoy)).

Model mean wave directions at both altimeter location and buoy should not be larger than 45°.



#### **Discrepancies in wave observations:**



Bias: altimeter Hs – in-situ Hs Symmetric slope: ratio of variance altimeter to variance in-situ





## Outcome of the meeting – Moored buoys

- Wave buoy data geographical coverage limited, especially directionality
- A thorough and comprehensive understanding of the performance of existing technologies under real-world conditions is currently lacking
- Continuity of established buoy networks, expansion of directional measurements priority for operations and climate assessment
- Expanding wave observing capabilities to other parts of the worlds oceans desirable from an operational point of view
- Guidelines of best practices for buoy wave measurements important in making buoy measurements consistent across networks and instrumentation types (and time).
- Agreed with the WIGOS Concept of Operations (CONOPS) recommendation that all wave observational data and metadata should adhere to WIGOS standards for instruments and methods of observation
- Agreed with development of best practices and standards documents related to waves and development of wave metadata within the Meta-T framework

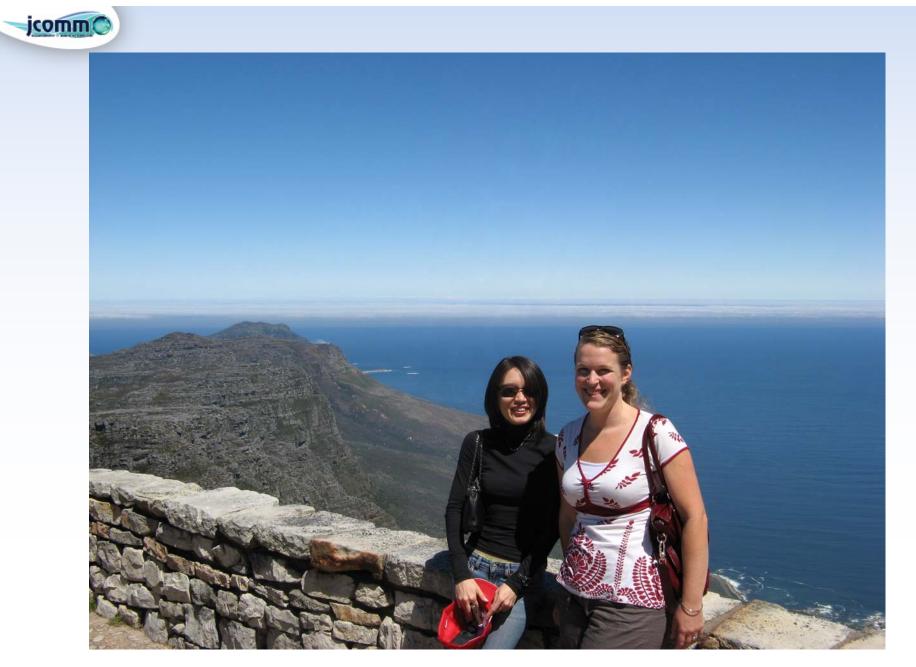
• No "perfect" wave measurement system against which to inter-compare other types of wave measurement. However, the Datawell sensors viewed as the best available and should form the basis for comparisons

## Outcome of the meeting – Moored buoys (2)

- Real need for independent performance testing to inter-compare various buoy networks, platforms, and instrumentation to establish consistency for the "first 5 standard" wave measurements.
  - Development of standardized procedures for buoy inter-comparison required.
  - Proper directional wave measuring device should reliably estimate "first 5"
  - multiple locations are required to appropriately evaluate the performance of wave measurement systems given the wide spectrum of wave regimes
  - Collocate different buoys with common reference (Datawell waverider) for at least a year at one or more reference sites;
  - Moving intercomparison technology was endorsed,
- For buoys not designed to follow wave slope/particle motion, may be better to do away with assumptions and transfer function correction measure buoy motion and then observe waves directly like from a fixed platform
- Raise awareness of sensor options, quality, prospects; transfer function problems.
- Develop a Pilot Project on Wave measurement Evaluation and Test for moored buoys for consideration at DBCP XXIV



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DBCP XXIV Cape Town 13-16 October 2008



## **PP-WET: Objectives**

- Develop the basis for an international framework for the continuous testing and evaluation of existing and planned wave buoy measurements
- Coordinate buoy inter-comparison activities.
- Develop technical documentation of differences due to hull, payload, mooring, sampling frequency and period, processing (e.g. frequency bands & cutoff), precision, transmission
- Develop training material to educate users about how to deploy and operate wave sensors appropriately.
- Contribute appropriate material to the JCOMM Standards and Best Practice Guide
- Establish confidence in the user community of the validity of wave measurements from the various moored buoy systems





## Why Do We Need to Test and Evaluate

- Measurements of surface gravity waves are estimates
  - From accelerations (double integrated)
  - From pressure response (invert to free surface)
  - From x,y velocities (invert to free surface)
- Only direct measurement of waves:
  - From capacitance or resistance gauges
  - From photo analysis
- Signal to noise:
  - Contamination of wave records
  - Agreement for *universal criteria* 
    - Reduces uncertainty in wave measurements
      - Provides consistency
      - Device to device
      - Underlying processes correctly evaluated

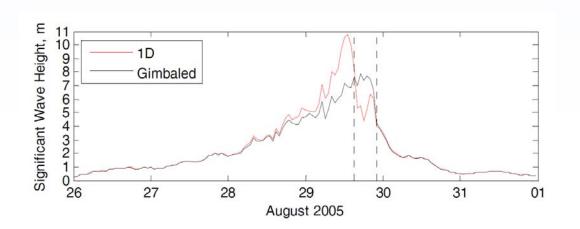






## **Deepwater: Wave Buoys**

- Impact is universal and dependent on buoy/device:
  - Non-directional buoys
    - 10% differences between US and Canadian buoys compared to altimeter records.
    - Heel in high wind and wave environments (Bender et al. 2009)
      - Mathematically gimbaled vs. strapped down accelerometers









## **Evaluation Procedure**

- Datawell Mark III: RELATIVE REFERENCE
  - This does not mean all directional wave measurements are required to be Datawell Mark III's
- Co-Located Procedure
  - Period of record consistent
    - Time consistency between platforms
    - Similar geographic/hydrographic
  - Analysis based on First-5
    - NOTE: S(f) is 1<sup>st</sup> of 5
  - Wave climate / environment dependent
    - Atlantic / Gulf of Mexico / Pacific / Great Lakes







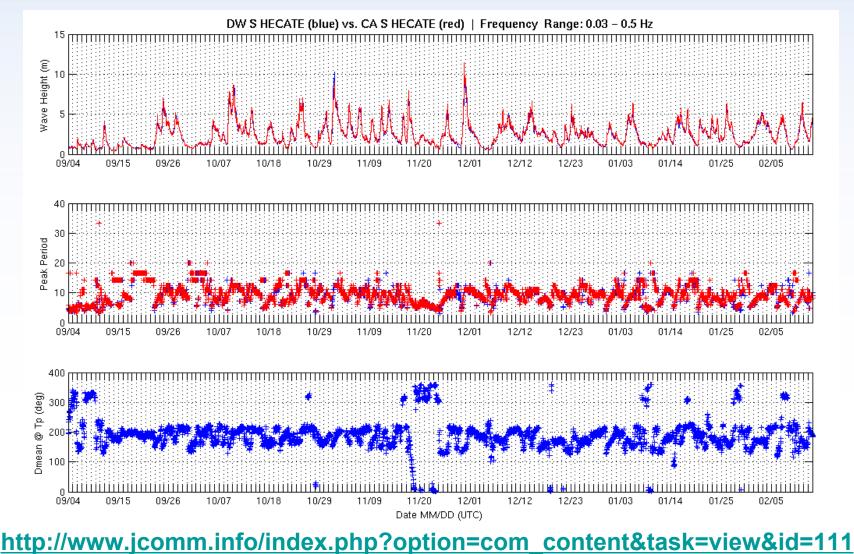


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### **Evaluation Procedure: Co-located**

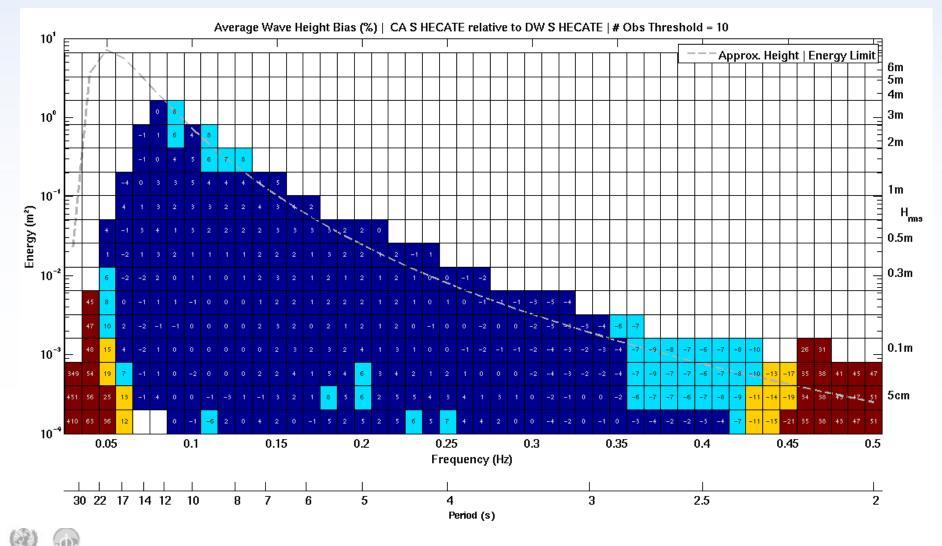
#### Analysis in the time domain by frequency criteria





### **Evaluation Procedure: Co-located**

#### Analysis in the frequency domain by moments

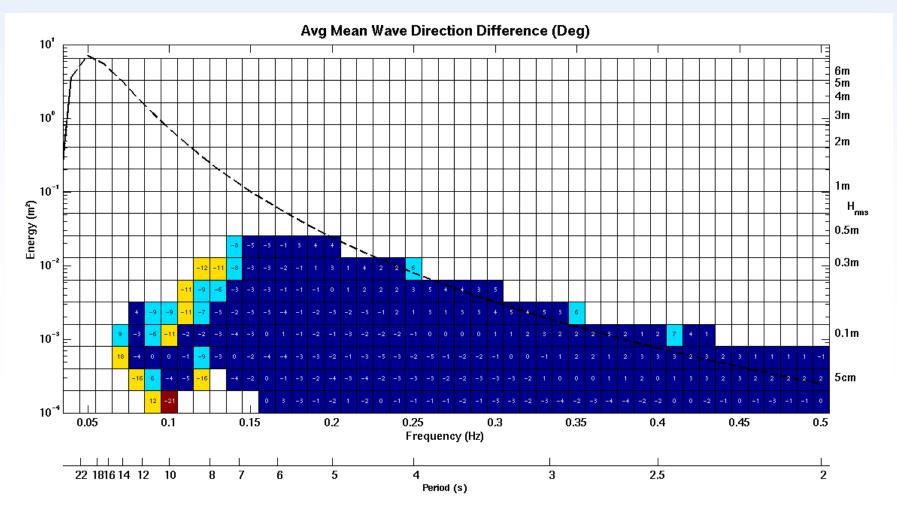


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## **Evaluation Procedure: Co-located**

Analysis in frequency domain for directional estimates when applicable

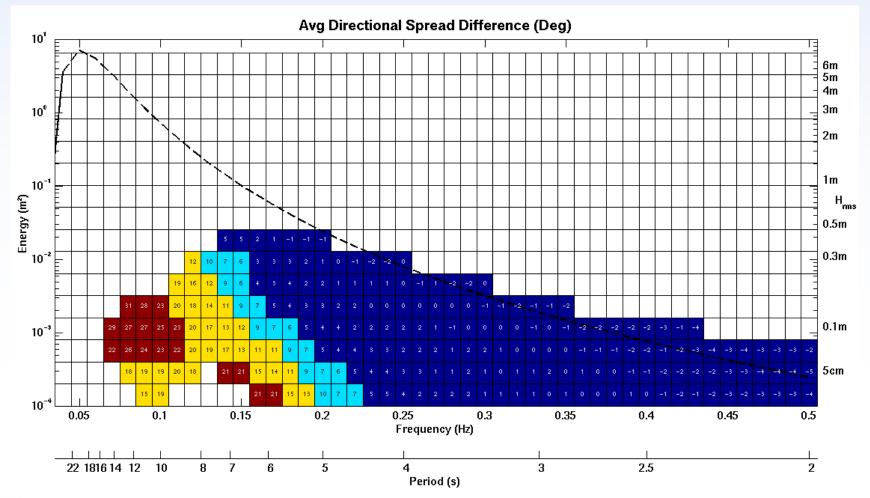






## **Evaluation Procedure: Co-located**

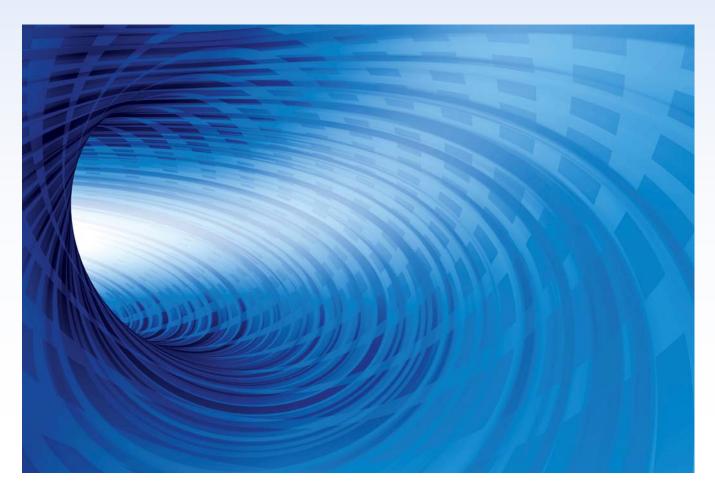
Analysis in frequency domain for directional estimates when applicable



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## A brief spin through some recent results







## **PP-WET Results (<u>www.jcomm.info/WET</u>)**

#### • Contract let to CDIP/SIO to develop

- Intercomparison web site
- Quality Assurance standards proposal
- Special metadata requirements for intercomparisons
- Provide intercomparison software to partners
- Advice on use of intercomparison methodology and web site (CB)
- Advice on intercomparison technical issues
- Conduct individual intercomparison analyses
- Intercomparison activities
  - Canada two co-location deployments see following slides
  - UK Comparison of heave sensor and TriAxys on K5 (not First-5)
  - Norway plan to submit Ekofsik platform wave data to CDIP for analysis laser, waverider, MIROS; investigation of possibility of deploying DWR
  - Korea multiple co-locations and analysis at leodo platform
  - OGP interest in providing co-located measurements to CDIP for analysis
  - Australia co-located MRU on Datawell buoy
  - India -co-locations underway, test data submitted to CDIP
- Special Session, discussion session, side meeting at 11<sup>th</sup> International Workshop on Wave Hindcasting and Forecasting – October 2009 – Halifax (www.waveworkshop.org)





## Intercomparison Metadata Form

SCRIPPS Institution of Con-	COASTAL DATA Monitoring and Predi				
Carlos Citations	ics ▼ Tools ▼ CDIP THE	EMES	Go 19:11 UTC		
Home Theme	Comparison Input Form				
Station Details					
-	Description	Input	Notes		
All Regions	ODAS ID		If given		
	WMO number				
Contact Us Publications	Station name		If given		
	Type of station		#NAME?		
Documents Latest News	Status		operational or ceased		
Personnel About CDIP List	Start date		dd/mm/yy		
	End date		dd/mm/yy if ceased		
Product list	Period of deployment		E.g. All year, seasonal		
	Country of ownership				
	Operating agency/institute				
	Degree of automation		#NAME?		
	Latitude (of deployment)		Up to 3 decimal places		
	Longitude (of deployment)		Up to 3 decimal places		
	Watch circle (m)				
	Hull type		As ODAS format descriptor		
	Hull manufacturer/model		If appropriate		
	Hull material		e.g. aluminium, plastic, foam etc.		
	Length (m)		If rectangular/boat shaped, to tenth of a m		
	Breadth/width (m)				
	Diameter (m)		If circular, to tenth of a m		
	Mooring type		As ODAS format descriptor		
	Operating environment		Open ocean, near-shore, coastal, lake etc		
	Water depth (m)				
	Elevation above sea level (m)		e.g. if on upland lake		
	Primary data collection system		Include Iridium		
	Primary (satellite) transmission time		e.g. specific time, on the hour etc.		
	Primary sat transmission ID		e a DCP no		





#### QARTOD CDIP FRF IOC NDBC NOBSKA NORTEK RDI SONTEK

#### IOC Quality Control Tests : Waves

The tests are from International Ocean Commission (IOC) Manual and Guides 26 prepared by the Commission of the European Community and the Committee for International Oceanographic Data and Information Exchange of the Intergovernmental Oceanographic Commission and published in 1993. The bulk of the wave tests are in SECTION 2.2, APPENDIX A WAVE DATA. Some of the tests have been edited from the Manual for clarity and to accommodate the QARTOD format. The table below will take you to the relevant tests. To view the Waves section, click here. Also included are

the relevant FORMALISED DESCRIPTION OF QUALITY CONTROL ALGORITHM.

#### DB - Directional Buoy

NB - Non-directional Buoy

This of the of the of the of the grant of digitized data)	TIME SERIES VALUES (Digital or digitized data)				
TEST: description (click name for more details)	DB	NB			
RAW DATA TIMING: verify number of collected values equals the number of expected values.		<			
GROSS ERROR LIMIT: test for values greater than 6 times standard deviation from the mean.	٠	<			
RATE OF CHANGE CHECK: test that the maximum allowable difference between adjacent samples.	٠	٠			
CONSECUTIVE EQUAL VALUES: test for occurrence of 10 or more consecutive points with equal value.		٠			
WANDERING MEAN CHECK: test for individual zero up-crossing period of > 25 seconds.		۷			
DATA STABILITY CHECK: test if the means or standard deviations of the segments (at least 8) differ from the mean or standard deviation of the entire sample.		<			
CHECK LIMITS TS: test for values greater than 4 times standard deviation from the mean.	٠	٠			
$\rm BUOY HEADING:$ Buoy heading directions should be checked to ensure that the values lie between 0 and 360°	٠				
RAW DATA INSPECTION AND EDITING: The routine inspection of the raw data should be one of the first checks carried out on receipt of the data from offshore.		٠			
STATIONARITY TS: check all channels for 10 or more consecutive points with equal value.	٠				
SPECTRAL VALUES					
TEST: description (click name for more details)	DB	NB			
ENERGY IN THE SPECTRUM: verify that energy in parts of the spectrum do not exceed					
expected values.	۲	<			
	<ul> <li>✓</li> </ul>				
expected values.		<u> </u>			
expected values. CHECK RATIO: the check ratio should theoretically be 1 at all frequencies. CHECK ON THE CROSS SPECTRA: Each of the cross-spectra has zero expectation at all frequencies. In reality, each should be at least an order of magnitude less than its		<i></i>			
expected values. CHECK RATIO: the check ratio should theoretically be 1 at all frequencies. CHECK ON THE CROSS SPECTRA: Each of the cross-spectra has zero expectation at all frequencies. In reality, each should be at least an order of magnitude less than its associated co- or quad-spectrum.		✓ NB			
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expected values. CHECK RATIO: the check ratio should theoretically be 1 at all frequencies. CHECK ON THE CROSS SPECTRA: Each of the cross-spectra has zero expectation at all frequencies. In reality, each should be at least an order of magnitude less than its associated co- or quad-spectrum. PARAMETER VALUES (Processed data) TEST: description (click name for more details) CHECK LIMITS PM: check that parameters do not exceed possible values. WAVE STEEPNESS: check that wave steepness <= 1/7. GAPS: Checks for gaps in the data should ensure that any defined periods of gaps are	J DB	۲ ۲			
expected values. CHECK RATIO: the check ratio should theoretically be 1 at all frequencies. CHECK RATIO: the check ratio should theoretically be 1 at all frequencies. CHECK ON THE CROSS SPECTRA: Each of the cross-spectra has zero expectation at all frequencies. In reality, each should be at least an order of magnitude less than its associated co- or quad-spectrum. PARAMETER VALUES (Processed data) TEST: description (Click name for more details) CHECK LIMITS PM: check that parameters do not exceed possible values. WAVE STEEPNESS: check that wave steepness <= 1 / 7. GAPS: Checks for gaps in the data should ensure that any defined periods of gaps are consistent with the number of data points nulled or absent. CHECKS ON INPUT DATA: Are direction data in degrees true or magnetic? Does	UB	۲ ۲			
expected values. CHECK RATIO: the check ratio should theoretically be 1 at all frequencies. CHECK ADTO: the check ratio should heoretically be 1 at all frequencies. CHECK ON THE CROSS SPECTRA: Each of the cross-spectra has zero expectation at all frequencies. In reality, each should be at least an order of magnitude less than its associated co- or quad-spectrum. PARAMETER VALUES (Processed data) TEST: description (Click name for more details) CHECK LIMITS PM: check that parameters do not exceed possible values. WAVE STEEPNESS: check that wave steepness <= 1/7. GAPS: Checks for gaps in the data should ensure that any defined periods of gaps are consistent with the number of data points nulled or absent. CHECKS ON INPUT DATA: Are direction data in degrees true or magnetic? Does magnetic correction applied ibe between 0 and 16° MEAN WAVE DIRECTION: Check that alvaues of mean wave direction (determined at	✓ ✓ DB ✓ ✓	۲ ۲			

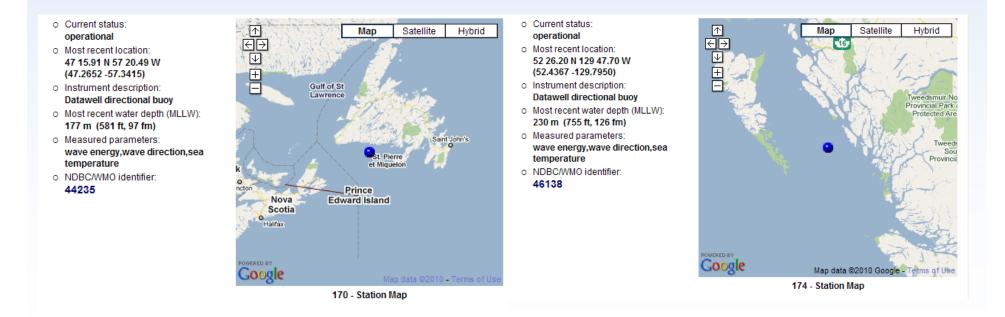


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170 co-located with operational 6m NOMAD 44255 plus TriAxys sensor 174 co-located with operational 3m discus 46185 plus TriAxys sensor



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DW\_S\_RAMEA\_v\_CA\_NE\_BURGEO 💌

#### Time Series Plots Wave Component Plots Metadata Documentation

Description	DW_S_RAMEA	DW_NE_BURGEO	Notes
ODAS ID	NULL	NULL	lf given
WMO number	44235	44255	
Station name	CDIP Station 170, Set p1 - SOUTH RAMEA ISLAND, CANADA BUOY	CDIP Station 655, Set p1 - NE BURGEO BANK BUOY	' If given
Type of station	Moored buoy	Moored buoy	#NAME?
Status	operational	operational	operational or ceased
Start date	05/10/10	01/01/10	dd/mm/yy
End date	NULL	NULL	dd/mm/yy if ceased
Period of deployment	seasonal	All year	E.g. All year, seasonal
Country of ownership	USA	CA	
Operating agency/institute	The Coastal Data Information Program (CDIP)	Environment Canada	
Degree of automation	Fully automated	Fully automated	#NAME?
Latitude (of deployment)	47.265 N	47.267 N	Up to 3 decimal places
Longitude (of deployment)	57.342 W	57.335 W	Up to 3 decimal places
Watch circle (m)	306	310	
Hull type	DR	NM	As ODAS format descriptor
Hull manufacturer/model	Datawell	Axys Technologies Inc	lf appropriate
Hull material	stainless steel	aluminium	e.g. aluminium, plastic, foam etc.
Length (m)	0.9	6.0	If rectangular/boat shaped, to tenth of a m
Breadth/width (m)	0.9	3.1	
Diameter (m)	0.9	NULL	If circular, to tenth of a m
Mooring type	ST	ST or AC ???	As ODAS format descriptor (ST=Semitaut,AC=All Chain,etc)
Operating environment	Open ocean	Open ocean	Open ocean, near-shore, coastal, lake etc
Water depth (m)	177	179	
Elevation above sea level (m)	0	0	e.g. if on upland lake
Primary data collection system	Iridium	Iridium	Include Iridium
Primary (satellite) transmission time	0025 on the half-hour	???	e.g. specific time, on the hour etc.
Primary sat transmission ID	NULL	777	e a DCP no

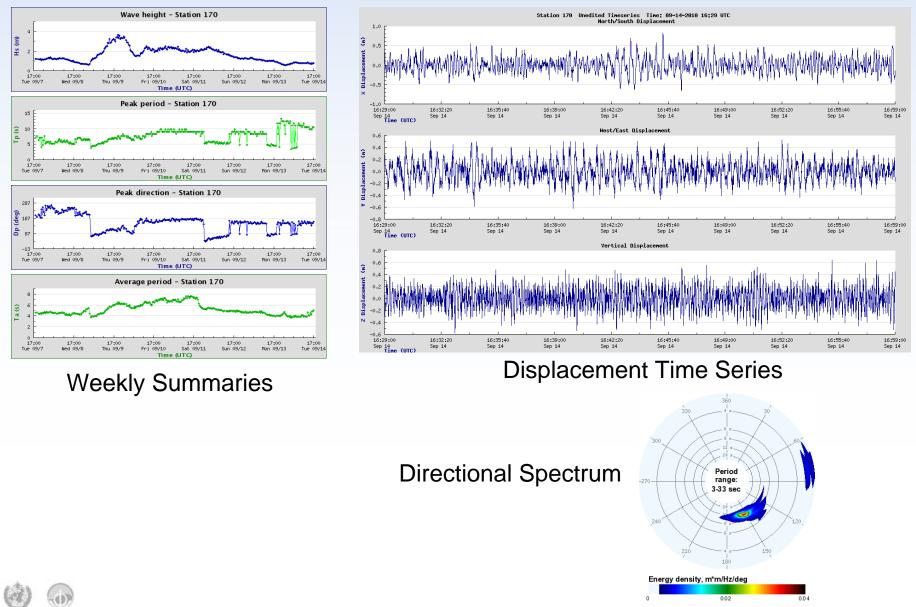
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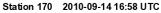


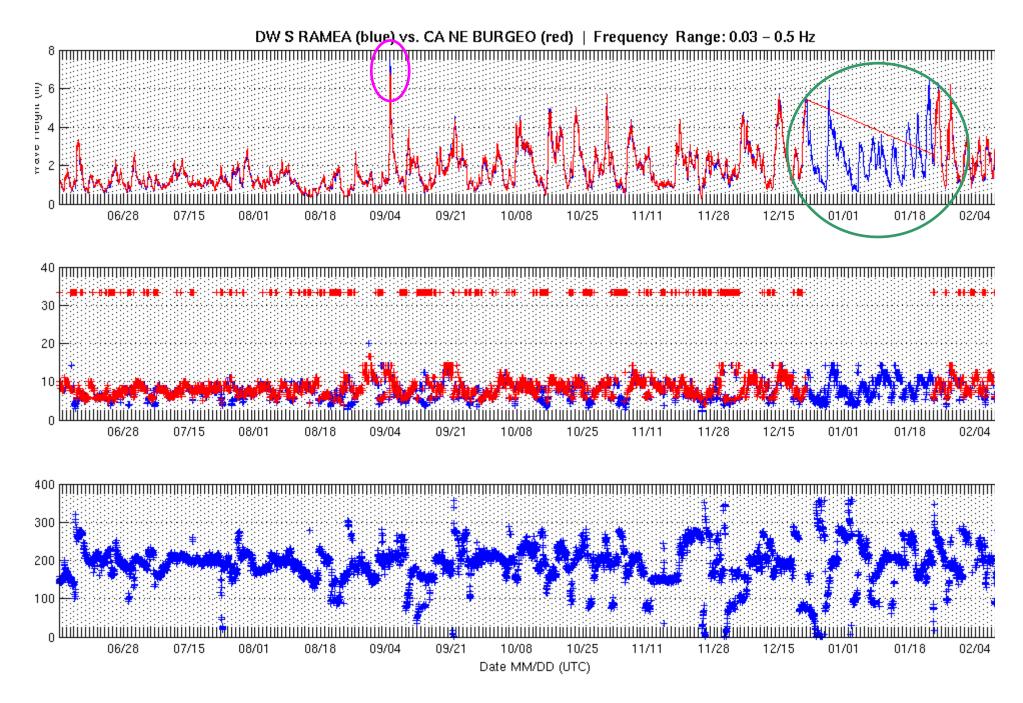


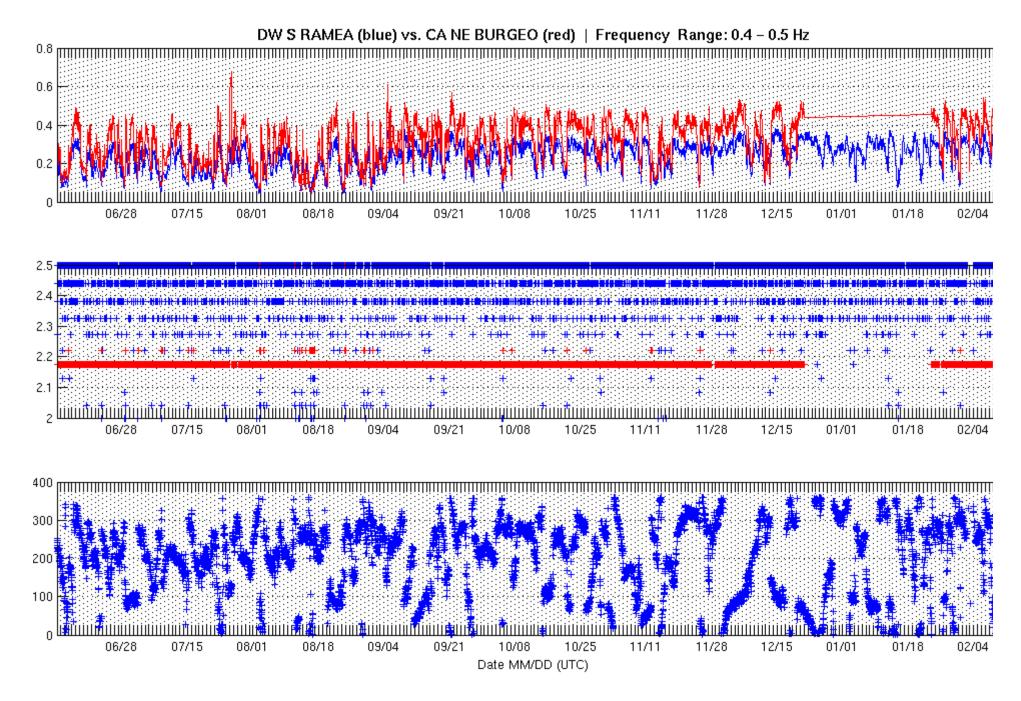
WMO

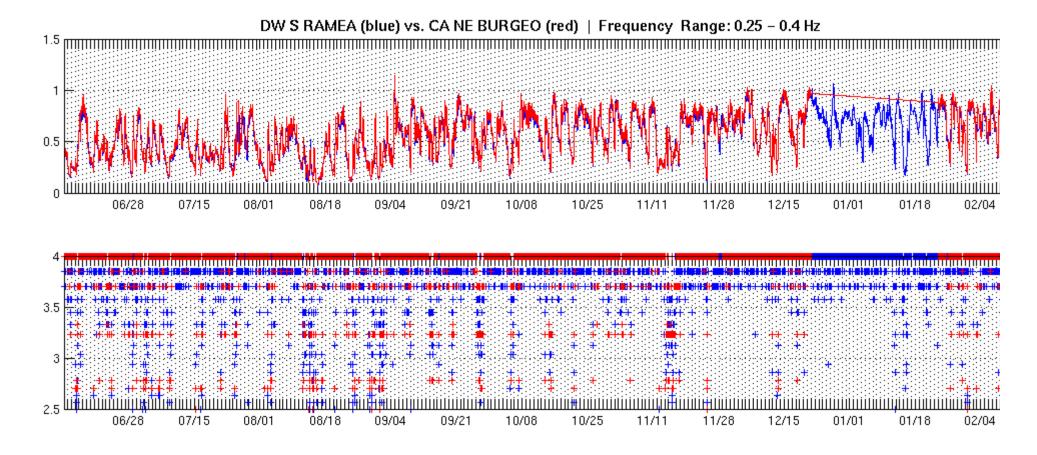
## **CDIP** Wave Summaries

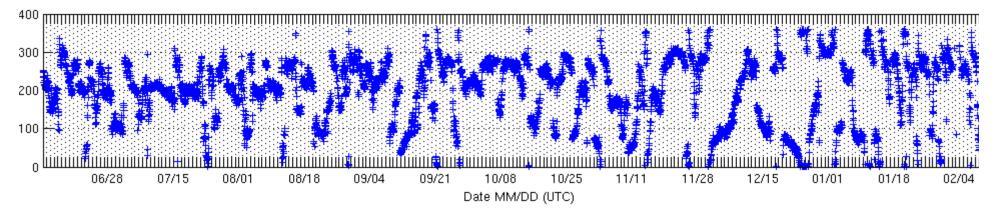




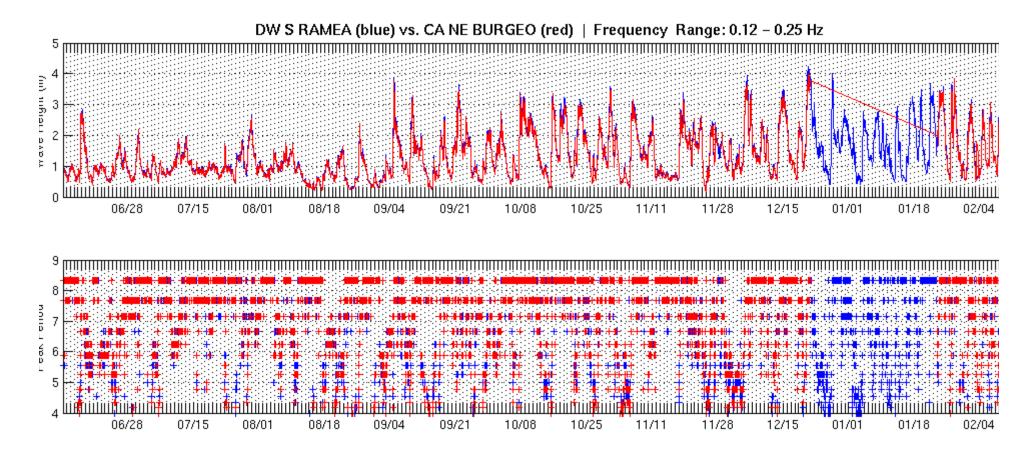


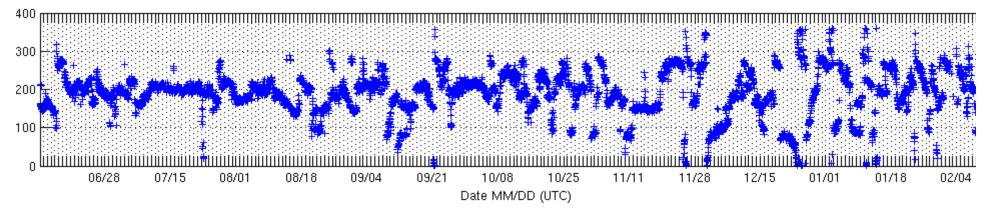




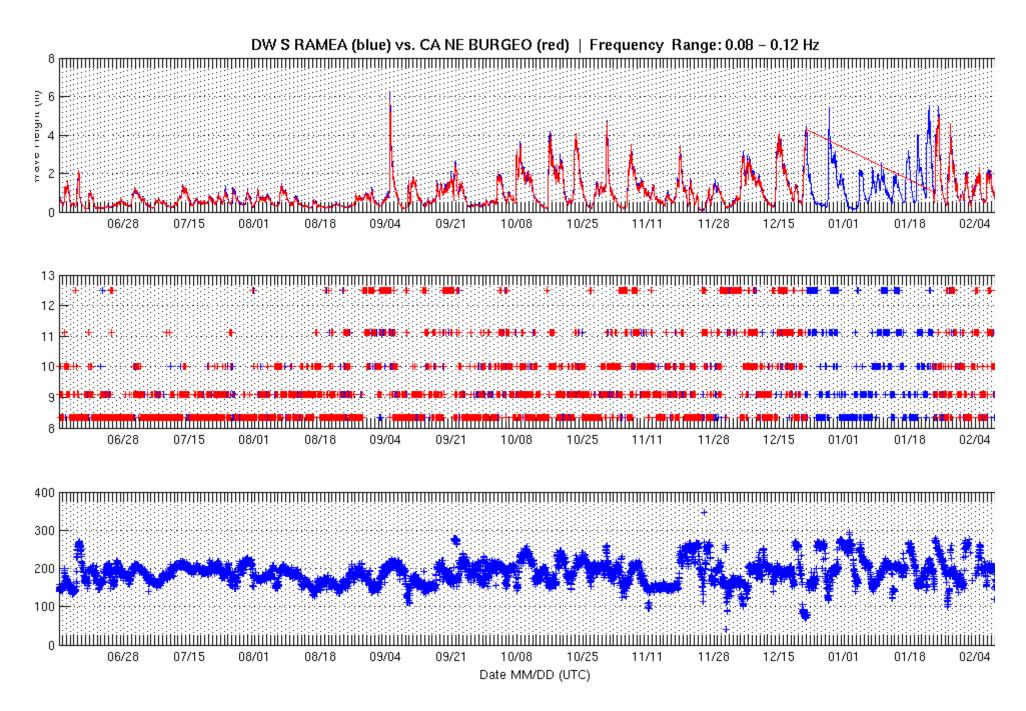


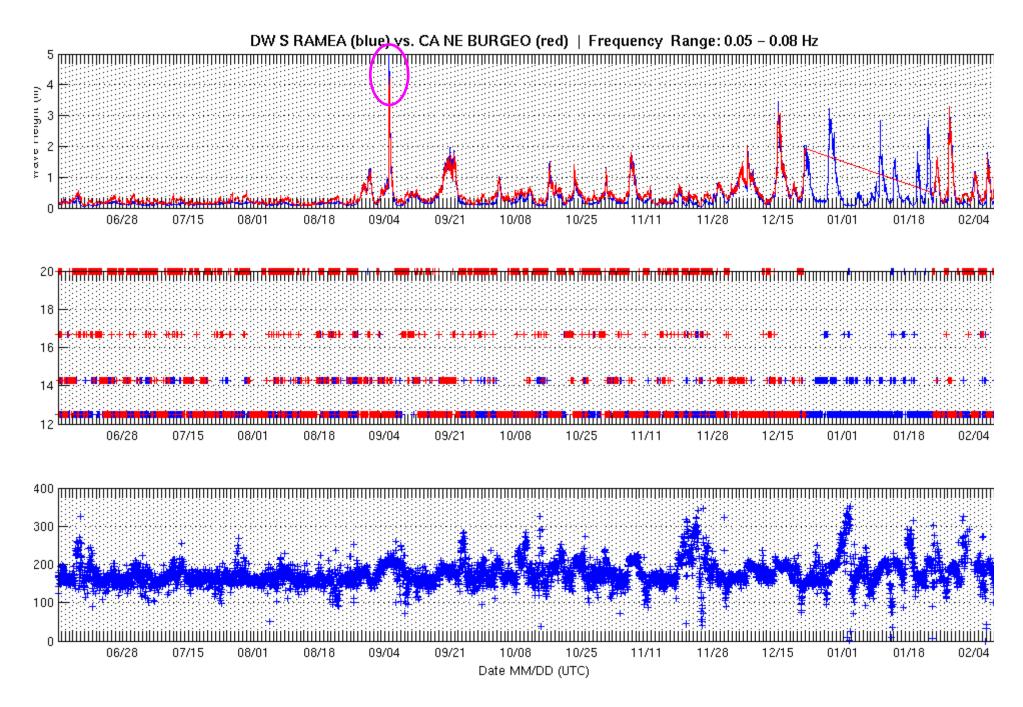
stra Comparison Tool, Version 1.0

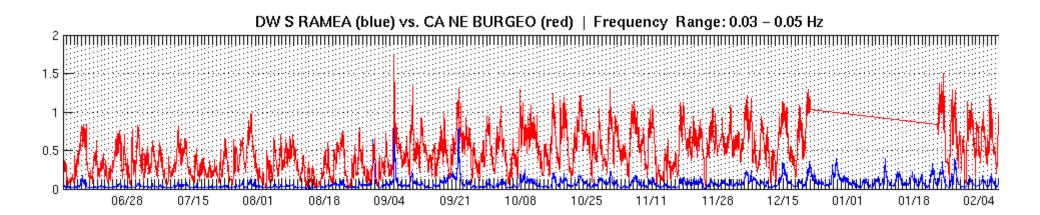


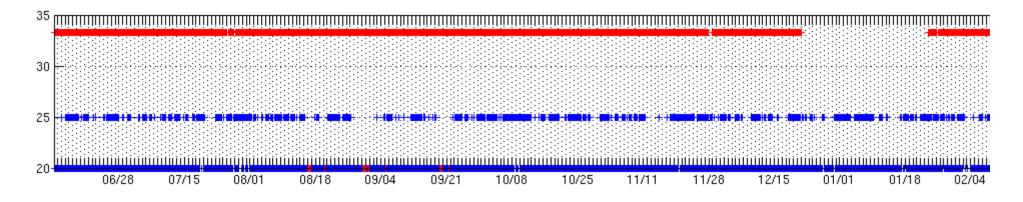


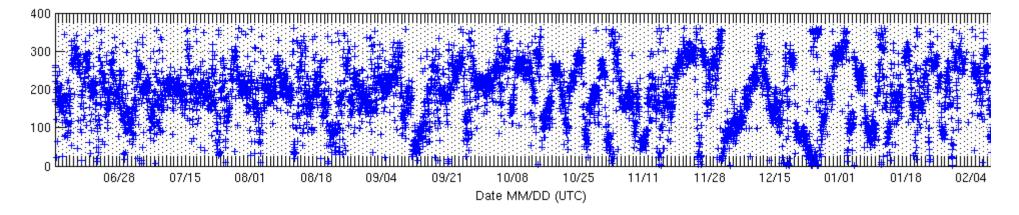
:tra Comparison Tool, Version 1.0



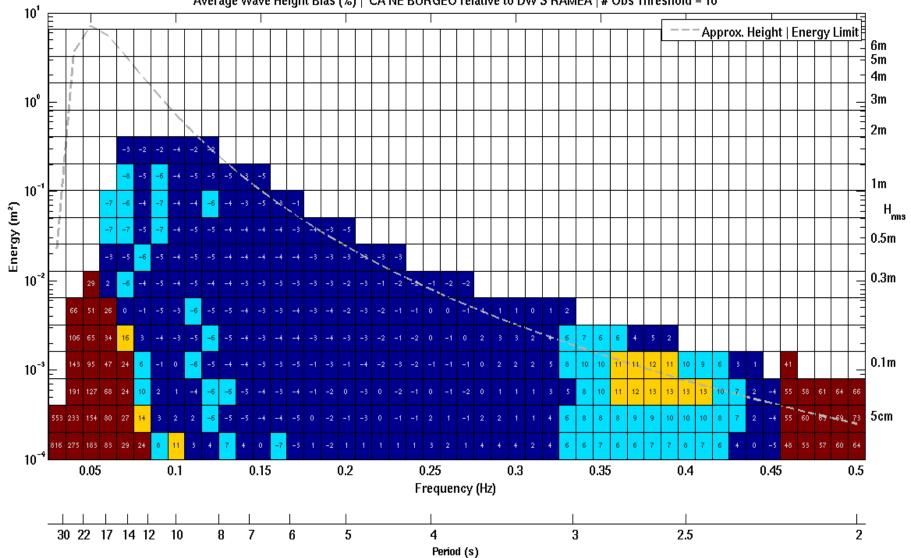








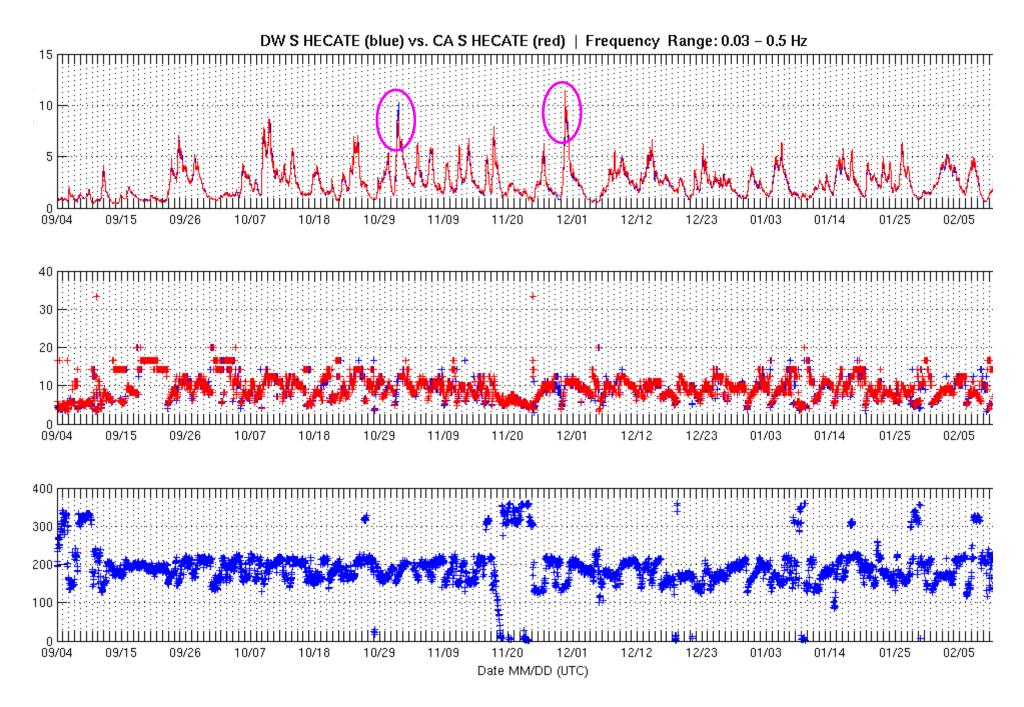


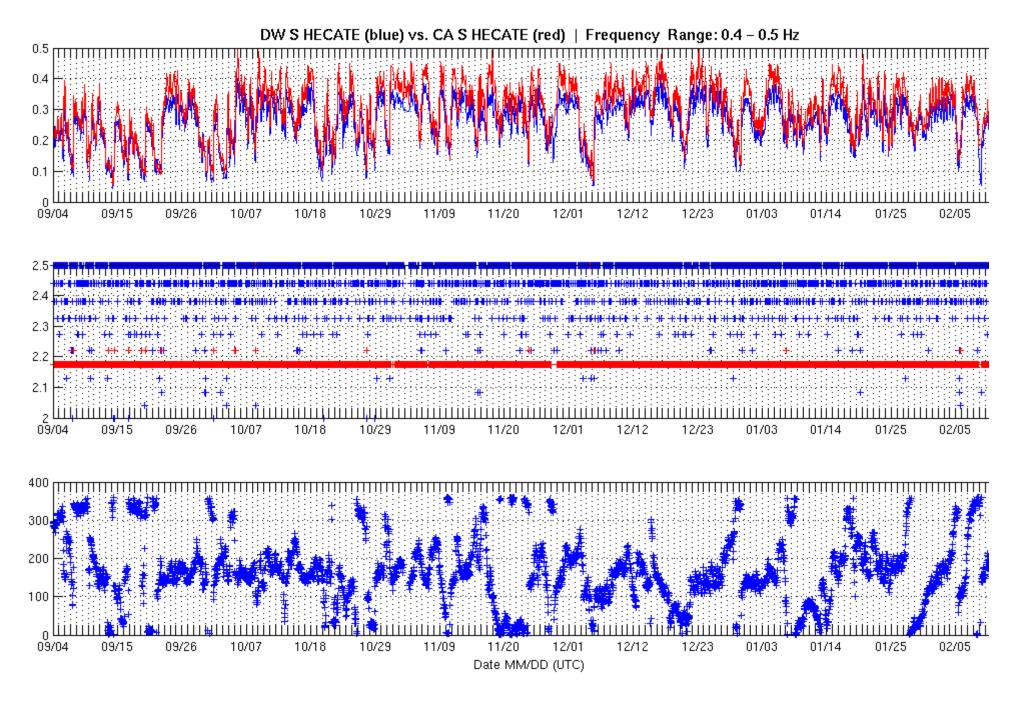


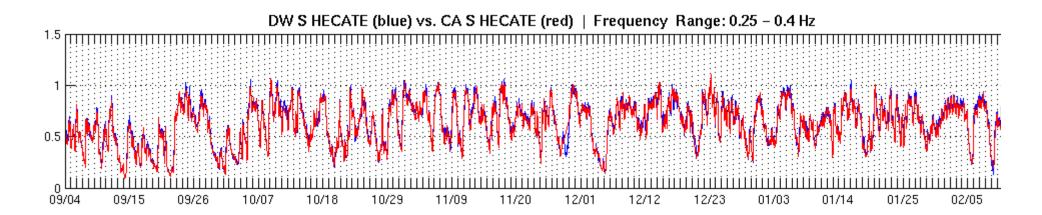
Average Wave Height Bias (%) | CA NE BURGEO relative to DW S RAMEA | # Obs Threshold = 10

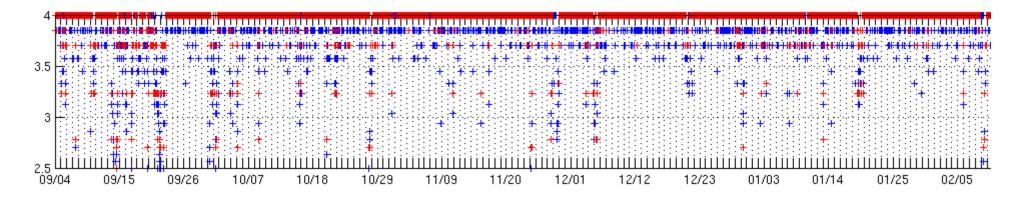
CDIP Wave Spectra Comparison Tool, Version 1.0

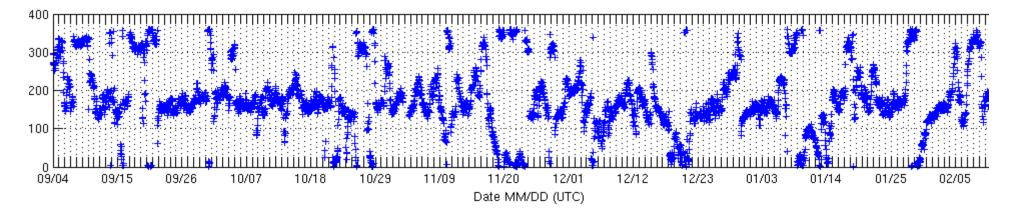
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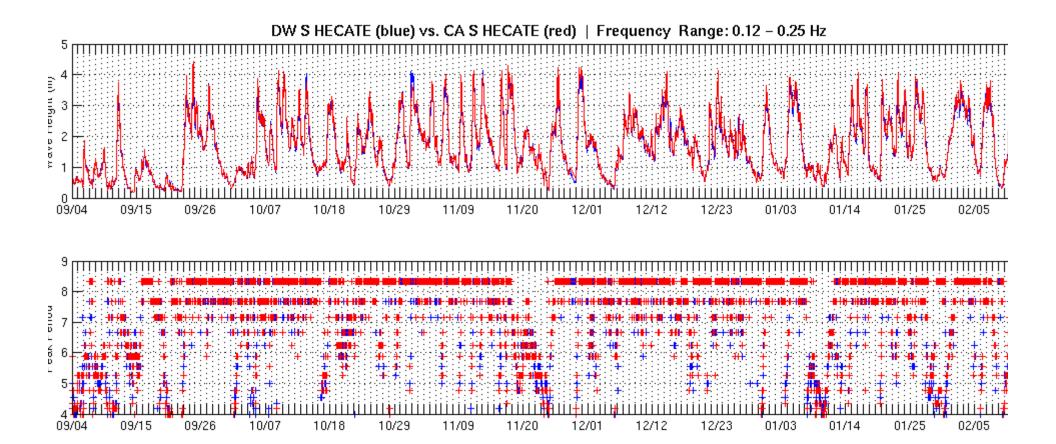


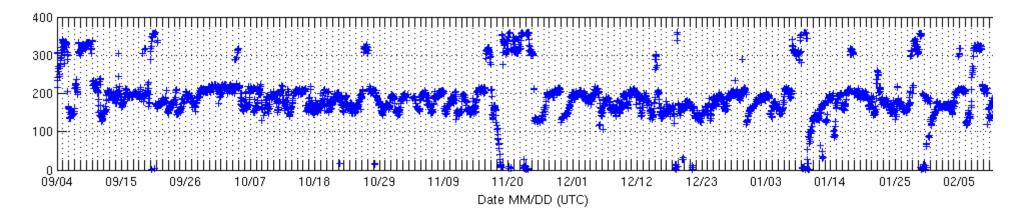


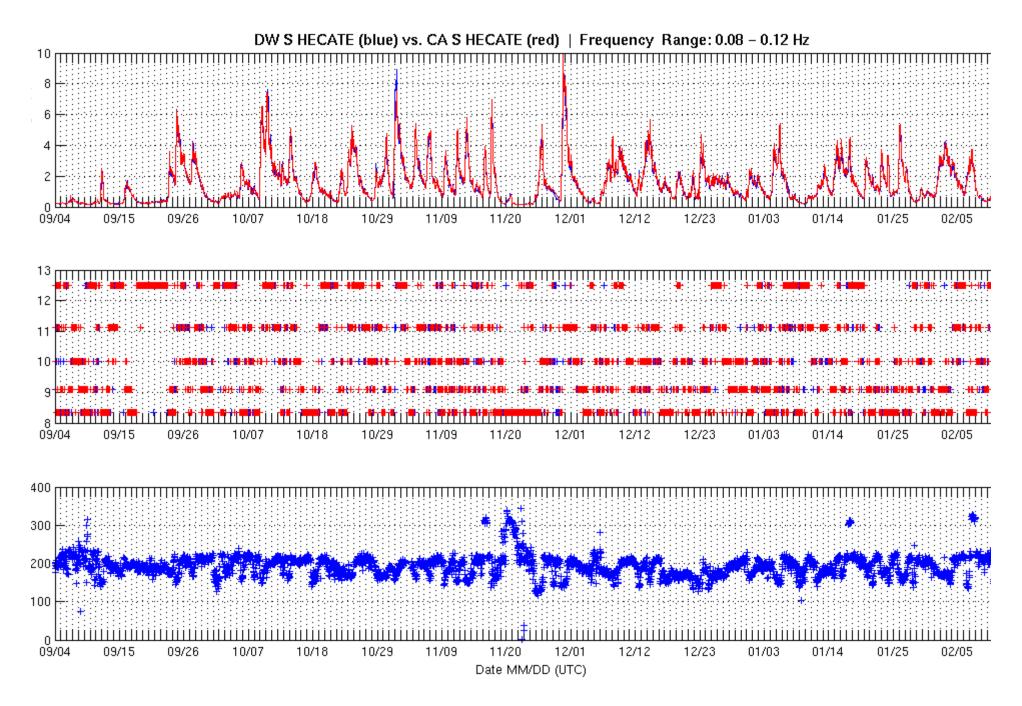


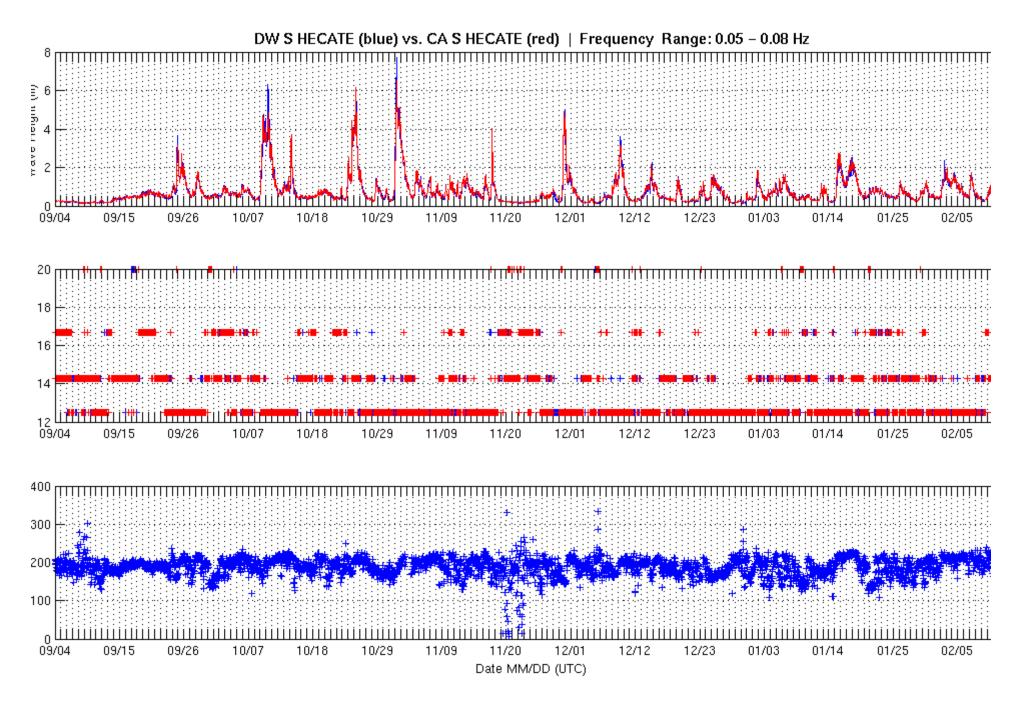


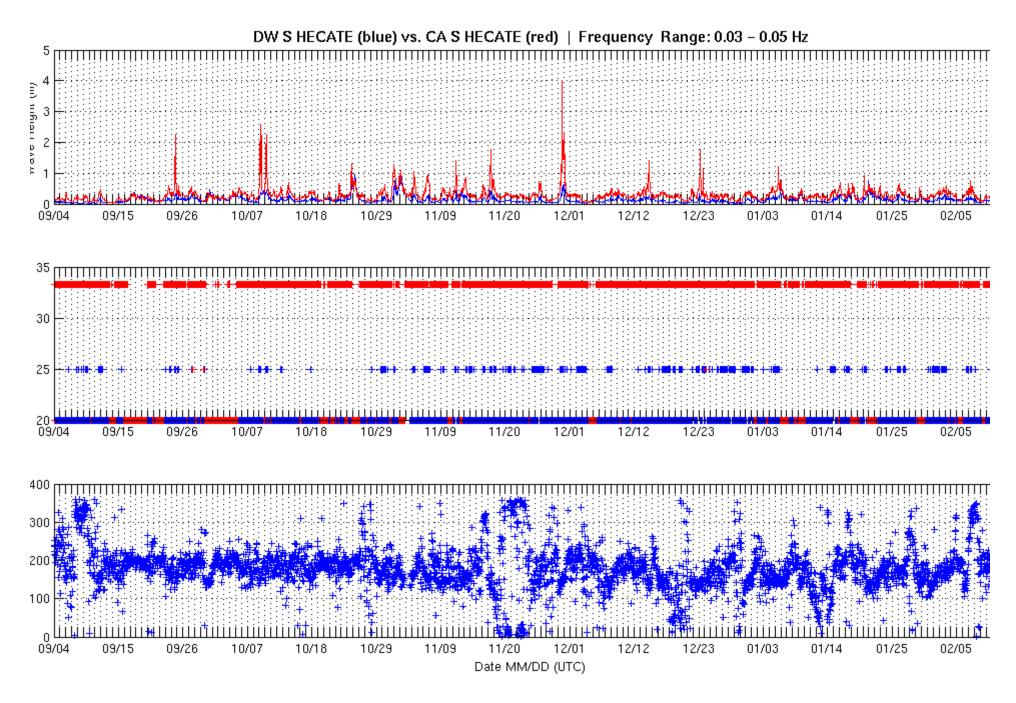




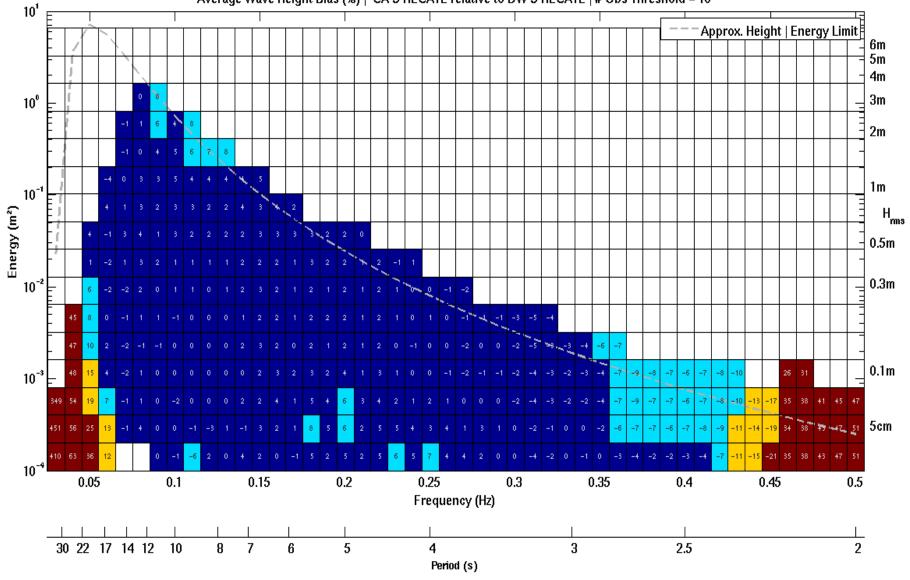








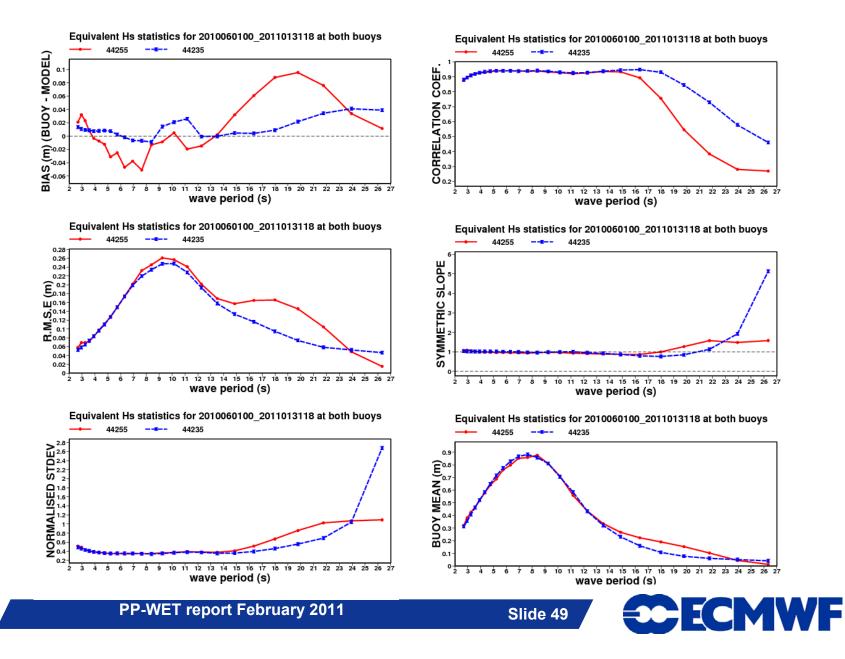




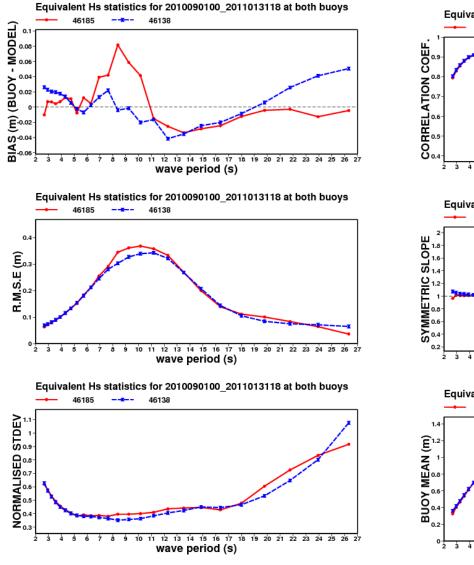
Average Wave Height Bias (%) | CA S HECATE relative to DW S HECATE | # Obs Threshold = 10

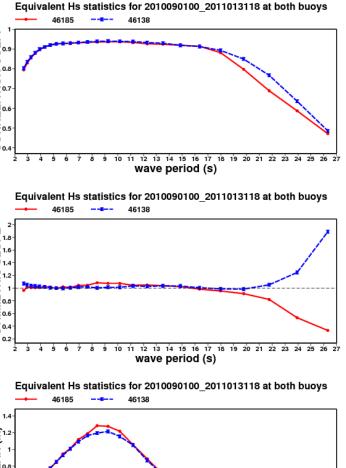
CDIP Wave Spectra Comparison Tool, Version 1.0

#### CA NE Burgeo (44255 red) vs DW Ramea (44235 blue)



### CA Hecate (46185 red) vs DW Hecate (46138 blue)





**PP-WET report February 2011** 

Slide 50





# Some Wave Related Activities which benefit from the results of this study

- MSC50 North Atlantic hindcast 1954-2010
- ERA-CLIM reanalysis; ERA-Interim reanalysis
- NCEP CFSR based reanalysis
- ICOADS Wave Climate summaries?
- JCOMM Extreme Waves Data Base
- JCOMM/DBCP Pilot Project on Wave Measurements from Drifters
- Coordinated Ocean Wave Climate Projections
- JCOMM Wave Forecast Verification Project
- Wave design criteria for offshore structures
- Trend and variability analysis for IPCC Assessments
- Thomas et al. wave climate homogeneity assessment (RHTest)
- Cardone et al. Very Extreme Storm Seas (VESS) altimeter analysis
- GlobWave altimeter and SAR data bases including direct validation of SAR 2-D spectra



# Thank you.





# **PP-WET Steering Team membership**

- Val Swail, Co-Chair (ETWS, EC)
- Bob Jensen, Co-Chair (USACE)
- David Meldrum (DBCP, SAMS)
- Jean Bidlot (ECMWF)
- Kwang-Chang Lim (KHOA)
- Chung-Chu Teng (NOAA/NDBC)
- Bill Burnett (NOAA/NDBC)
- Julie Thomas (UCSD)
- Hans Graber (U. Miami)
- Diana Greenslade (Australian Bureau of Meteorology
- Venkatesan (India)

- Bill O'Reilly (UCSD)
- Jon Turton (Met Office)
- Christian Meinig (NOAA/PMEL)
- Anne Karin Magnusson (Met.no)
- Kevin Ewans (Shell)
- George Forristall (ForOcean)
- Dong-Young Lee (KORDI)
- DBCP Technical Coordinator
- Secretariat support will be provided by WMO and IOC.
- Boram Lee (WMO)
- Etienne Charpentier (WMO)



# **First-5 Basics**

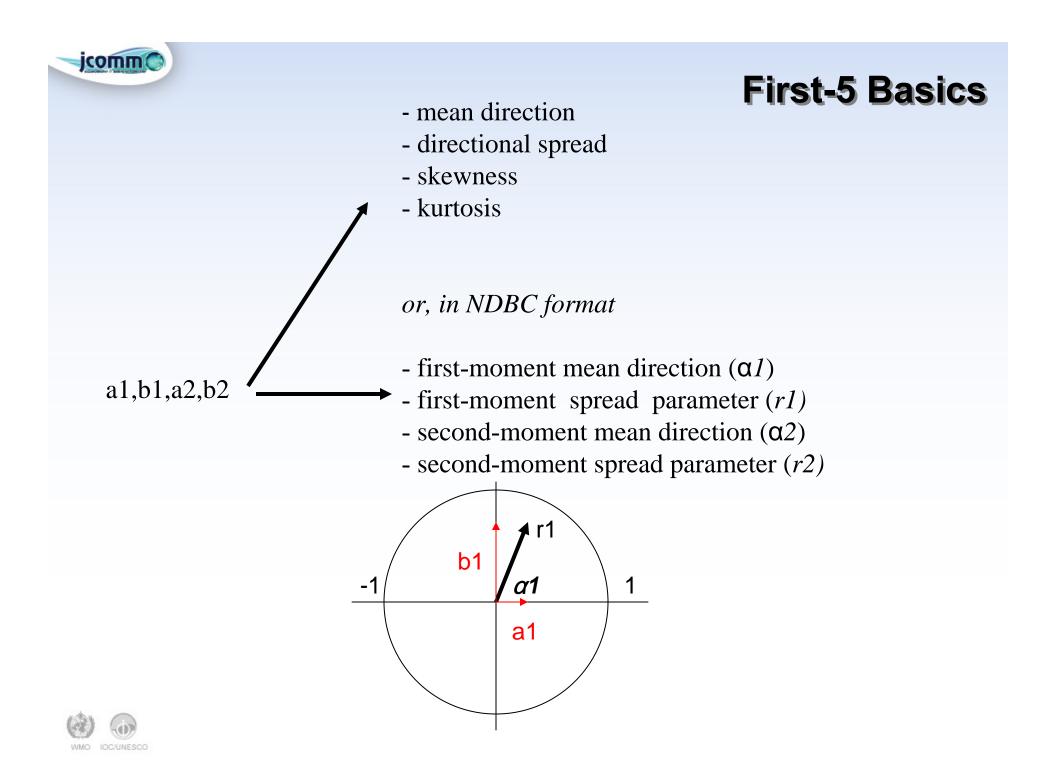
#### • First-5 Basics

B.Lee16

- Three components (x,y,z or derivatives)
- Time series analysis
- Results in S(f), a1(f), b1(f), a2(f), b2(f)

freq Hz	Band width	energy m*m/Hz	Dmean deg	a1	b1	a2	b2
0.0250	0.0050	0.0028	321	0.1920	-0.1567	-0.3925	-0.6345
0.0300	0.0050	0.0035	115	-0.1076	0.2259	-0.5132	-0.5796
0.0350	0.0050	0.0046	173	-0.2883	0.0348	-0.2973	-0.5084
0.0400	0.0050	0.0062	303	0.2602	-0.4085	-0.1606	-0.6449
0.0450	0.0050	0.0106	241	-0.0693	-0.1232	0.1890	-0.4245
0.0500	0.0050	0.0664	295	0.2434	-0.5111	-0.0182	-0.3324
0.0550	0.0050	0.4436	272	0.0230	-0.8426	-0.5614	-0.1069
0.0600	0.0050	2.4041	287	9.2594	0.0467	0.0400	-9.3178
0.0650	0.0050	4.6515	295	0.3985	0.8367	-0.5535	-0.6727
0.0700	0.0050	5.2140	298	0.1100	-0.0304	-0.1750	-0.7209
0.0750	0.0050	1.9294	310	0.5513	-0.6680	-0.2944	-0.7309
0.0800	0.0050	1.4582	349	0.7292	-0.1430	0.2632	0.0403
0.0850	0.0050	2.5656	328	0.7689	-0.4840	0.2847	-0.6974
0.0900	0.0050	0.6455	352	0.7463	-0.1086	0.4258	-0.0207
0.0950	0.0050	0.6295	329	0.7213	-0.4297	0.2088	-0.6399
0.1013	0.0075	0.7499	0	0.6994	0.0019	0.2030	0.0206
0.1100	0.0100	0.5782	27	0.6616	0.3353	0.1029	0.4937
0.1200	0.0100	0.3596	23	0.7253	0.3028	0.2794	0.4324
0.1300	0.0100	0.1433	10	0.5246	0.0925	0.1332	-0.0804
0.1400	0.0100	0.0918	11	0.5567	0.1123	0.2326	0.1826
0.1500	0.0100	0.1041	17	0.6158	0.1886	0.2376	0.2832
0.1600	0.0100	0.0779	6	0.5846	0.0592	0.0527	0.2101
0.1700	0.0100	0.0458	11	0.4591	0.0926	-0.0412	0.1988

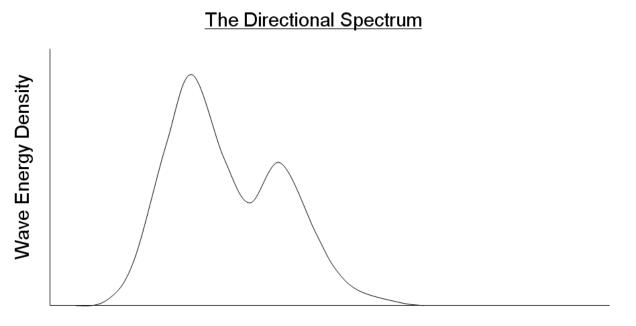






## **First-5 Basics**

#### The Outcome and Minimum Requirements for Directional Observations



Wave Direction θ

 $\frac{S(f,\theta)=S(f)[a1 \cdot \cos(\theta)+b1 \cdot \sin(\theta) + a2 \cdot \cos(2\theta) + b2 \cdot \sin(2\theta) + a3 \cdot \cos(3\theta)+b3 \cdot \sin(3\theta)+a4 \cdot \cos(4\theta)+b4 \cdot \sin(4\theta)+\dots \text{ infinity and beyond}]$ 





## **First-5 Basics**

