INVESTIGATING PARENT DISTRIBUTION OF TYPHOON-GENERATED ANNUAL MAXIMUM WAVE HEIGHT AND SAMPLE DISTRIBUTION OF RETURN WAVE HEIGHT ON THE EAST CHINA SEA

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1. INTRODUCTION

Identifying the parent probability distribution of storm-generated extreme waves may be useful to improve the reliability of the estimated return wave height. But the identification needs a sample of extreme waves with enormous size. From this point of view, Yamaguchi et al.(2004) obtained a sample of size 20,000 of typhoon-generated annual maximum(AM) wave height at each point of an 80 km grid set on the Northwestern Pacific Ocean by making use of their Monte-Carlo simulation model including a wave prediction model. First they estimated a parent distribution of typhoon-generated AM wave height based on an extreme value analysis of the huge sample. Then Yamaguchi et al. (2005) discussed the characteristics of probability distribution of an *r*-year return wave height data sample, in cases where the huge sample is divided into several hundred sets of a smaller size sample and *r*-year return wave height is estimated for the individual sample. Also, they conducted an investigation for low-generated AM wave height samples obtained by a Monte-Carlo simulation. A major conclusion is that identifying the parent distribution of AM wave height may lead to an improvement of reliability of the estimated return wave height.

In this study, a parent distribution of typhoon-generated AM wave height and sample distribution of *r*-year return wave height are investigated by analyzing a typhoon-generated AM wave height sample of size 10,000 on the East China Sea area with extended shallow water, in cases where a grid with a higher space resolution of 40 km and a shallow water wave prediction model are used. On the East China Sea, typhoon-generated wave heights are much greater than low-generated wave heights as Yamaguchi et al.(2006) made clear. This makes it possible to estimate extreme wave heights free from storm-type waves by analyzing only typhoon-generated extreme wave height data sample.

2. DESCRIPTION OF SAMPLE GENERATION METHOD AND DATA ANALYSIS

2.1 *Outline of Monte-Carlo Simulation Procedure*

The Monte-Carlo simulation consists of 4 steps. They are 1) a probabilistic generation for the parameters of a typhoon using a model developed by Hatada and Yamaguchi(1996) and extended by Nonaka et al.(2000), 2) a wind estimation, 3) a shallow water wave computation based on a model by Yamaguchi et al.(1984) and 4) an extreme value analysis using a model by Yamaguchi and Hatada(1997). In order to estimate extreme wave heights generated by actual typhoons, the step 1 can be replaced by the data set of parameters for influential typhoons of 315 cases over a period of 51 years from 1948 to 1998.

2.2 *Probabilistic Generation for Parameters of a Typhoon*

Figure 1 illustrates the definition sketch of the pressure pattern. An elliptical distribution is assumed for the pressure pattern in a typhoon such as

$$p = p_c + (1013 - p_c)\exp\left[-\left\{(x/a)^2 + (y/b)^2\right\}^{-1/2}\right] \quad (1)$$

where p is the pressure in a typhoon, p_c the central pressure of the typhoon, (x , y) a local Cartesian coordinate system with the origin located at the typhoon center and (a , b) the typhoon radii in x and y directions.

A typhoon is represented with 6 parameters, which are 1) position of the typhoon center(X_c , Y_c), 2) central pressure p_c , 3) inclination angle of an ellipse θ and 4) typhoon radii(a , b). The global coordinate system with the south-directed X axis and the east-directed Y axis is introduced in formulating the model. An elliptic distribution is used for modeling transformation of the pressure pattern from a circular distribution which reflects a decay of typhoon strength associated with its northward movement. A far field pressure is fixed as an empirical constant of 1013 hPa which is a good approximation and the mean typhoon radius R is defined as $(a + b)/2$.

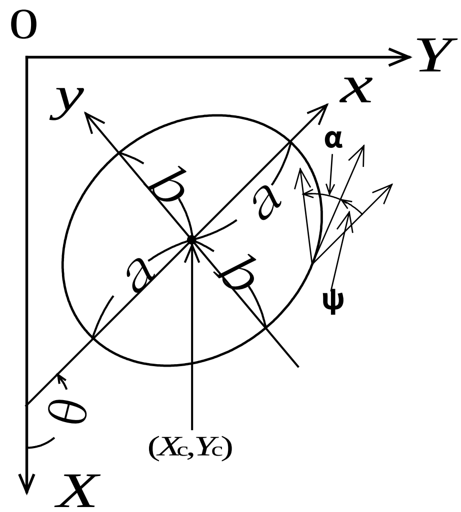

Figure 1. Definition sketch of pressure pattern in a typhoon.

The probabilistic generation model of a typhoon consists of 3 sub-models for 1) annual occurrence rate of strong typhoons, 2) generation of the parameters of a typhoon on the boundary and 3) change of the parameters with movement of a typhoon. The basic ideas are as follows: 1) the annual occurrence rate is calculated from the Poisson distribution, 2) each parameter may be expressed as a sum of its mean value calculated with use of a spline function on the boundary or regression equation on the inner region and the deviation from the mean value obtained with use of the cumulative distribution, 3) space-time correlation between each parameter at time step i and $i+1$ is taken into account with use of a linear regression equation under the assumption of a Markov process, 4) incremental change of each parameter is applied as a restraint to avoid excessive change of a parameter per one time step, and 5) regression equation and cumulative distribution are separately made on the sub-divided boundaries and sub-divided inner areas in order to take account of local effects. The 10 parameters to be generated at a time increment of 6 hours are X_c , Y_c , p_c , θ , R and b/a corresponding to a and b , ΔX_c . ΔY_c , Δp_c , $\Delta\theta$, where Δ means a parameter difference over one time step. A more detailed description can be found in Hatada and Yamaguchi(1996).

Figure 2 shows the modeling domain divided with a grid distance of 80 km, which also indicates sub-divided boundaries and inner regions. The East China Sea area where wave computation is carried out is enclosed by dotted lines in the figure.

Figure 3 illustrates a computation flow for sequential generation of typhoon parameters. The following explanations may be added to the above description according to the figure: 1) the model is made in 4 season-separated formulations for considering the season-dependent characteristics, 2) initial position(X_{c0}, Y_{c0}) of a typhoon center on the boundary is obtained from the cumulative distribution and 3) mean typhoon radius R and radius ratio b/a on the boundary are calculated with use of a regression equation and the cumulative distribution.

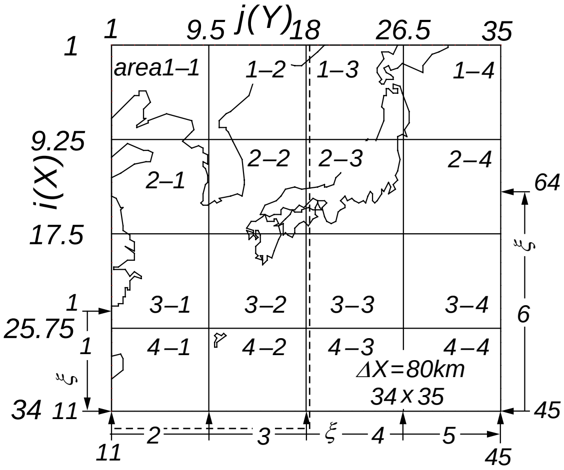

Figure 2. Modeling domain for typhoon.

Figure 4 exemplifies tracks of historical and simulated typhoons over 47 years. The spatial patterns of simulated typhoon seem to be similar to those of historical typhoons.

2.3 *Wind Estimation*

The wind estimation is due to the composition of gradient wind components and wind components related to the movement of a typhoon and the correction to 10 m winds. Gradient wind G is expressed as

$$G = V_g \Big/ \left[(1/2) + \left\{ (1/4) + V_g \big/ \left(f_c |R| \right) \right\}^{1/2} \right] \tag{2}$$

where V_g is the geostrophic wind and f_c the Coriolis coefficient. Also, wind related to a typhoon movement V is approximated with the following equation.

$$V = C \cdot G / G_{\max} \tag{3}$$

where C is the movement velocity of a typhoon and $G_{\max}$ the nearly maximum gradient wind velocity. Correction factor to a 10 m wind is taken as 0.6. The 10 m winds are calculated every half hour using linearly-interpolated values for every 6-hour typhoon parameters.

2.4 *Wave Computation*

A shallow water wave model by Yamaguchi et al.(1984) belonging to the second generation category is applied for every half hour computation of waves on the East China Sea grid with a space increment of 40 km, which is given in Figure 5. Shallow water areas are extensively distributed in the northwestern region. The 21 frequency data from 0.04 to 0.50 Hz and the 19 direction data with 20 degrees increment on the whole plain are used. A parametric inflow condition on the open boundary is imposed, in cases where the directional spectrum of each wave component is calculated from the product of JONSWAP-type frequency spectrum by Ross(1976) and $\cos^4\theta$ -based angular distribution function. Wave computations are conducted for each of about 40,000 influential typhoons simulated over 10,000 years and 315 historical typhoons over 51 years, and then a data sample of typhoon-generated AM wave height is selectively made at every sea grid point for each of the simulated and historical typhoon cases.

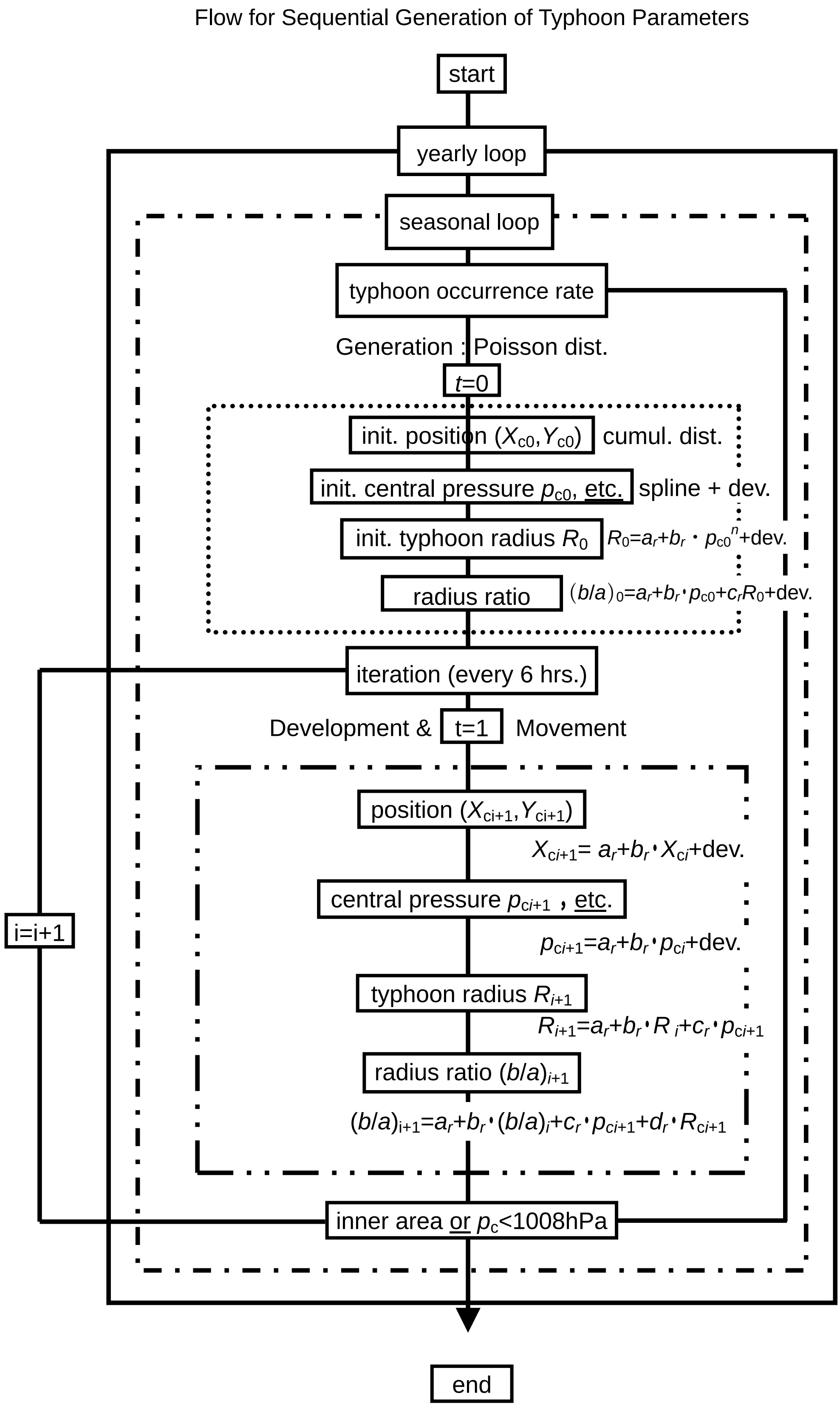

Figure 3. Computation flow for sequential generation of typhoon parameters.

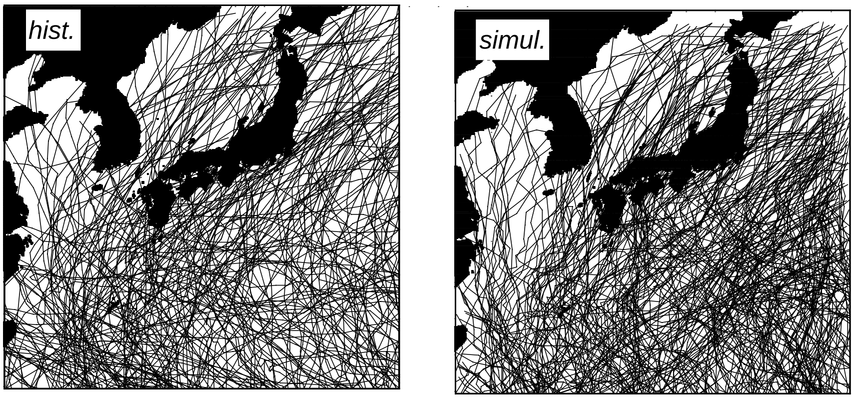

Figure 4. Tracks of historical and simulated typhoons over 47 years.

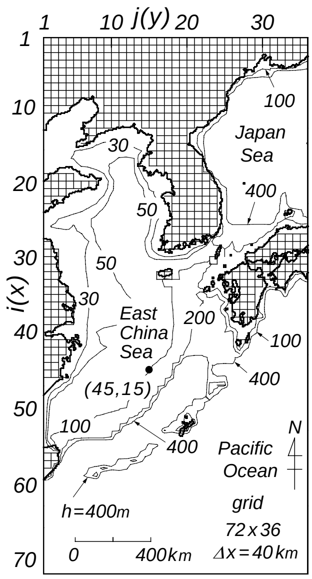

Figure 5. Wave computation grid with 40 km distance on the East China Sea.

2.5 *Extreme Value Analysis*

The extreme value analysis is due to a model by Yamaguchi and Hatada(1997). It uses the least square method(LSM) for the estimation of parameters (A, B) in the candidate probability distributions such as the Gumbel and shape parameter-fixed Weibull distribution, and a criterion of the largest correlation coefficient between calculated and input data for a selection of the optimum distribution. The Gumbel and Weibull distributions are respectively written as

$$F(H) = \exp[-\exp\{-(H-B)/A\}] \tag{4}$$

$$F(H) = 1 - \exp[-\{(H-B)/A\}^k] \tag{5}$$

where $F(H)$ is the non-exceedance probability distribution, H the stochastic variable, A the scale parameter, B the location parameter and k the shape parameter. The shape parameter k of the Weibull distribution is fixed as any of 27 kinds of the parameter ranging from 0.5 to 10. The two parameters(A, B) in the Gumbel and shape parameter-fixed Weibull distributions can be replaced with H_{50} and γ_{50} defined by H_{50}/H_{10}, where H_r is the *r*-year return value and γ_{50} the spread parameter proposed by Goda(2002).

The analyses are conducted for two purposes. One is to deduce the parent distribution of typhoon-generated AM wave height on the East China Sea by using one packet of sample with a size of 10,000. Also, *r*-year return wave height and its standard deviation on the East China Sea are obtained for historical typhoons. A jackknife method introduced by Yamaguchi and Hatada(1997) is applied for the estimation of standard deviation. The other is to investigate the characteristics of sample distribution of *r*-year return wave height, in cases where the 10,000-size sample of typhoon-generated AM wave height is regarded as 200 sets of a 50-size sample and the extreme value analysis using the LSM-based model is made separately for each sample.

The analysis for a small sample is made in two ways. The former is the known parent distribution case or the fixed shape parameter(FSP) case. The optimum distribution is fixed to either the Gumbel distribution or the Weibull distribution with the known shape parameter which is determined from the original sample with a size of 10,000. The latter is the unknown parent distribution case or the variable shape parameter(VSP) case. The optimum distribution is selected for every sample by a criterion of the largest correlation coefficient from the candidate distributions. Thus, the repeated calculation yields two sets of *r*-year return wave height sample with a size of 200. Statistics such as mean($\overline{H}_r$), standard deviation($H_{\sigma r}$), skewness(α_r) and kurtosis(β_r) are calculated for each set. Distribution of 200-size sample of *r*-year return wave height is approximated with the Gumbel or Weibull distribution by making use of the LSM-based extreme value analysis model.

3. ESTIMATION OF A PARENT DISTRIBUTION OF TYPHOON-GENERATED AM WAVE HEIGHT

Figure 6 shows spatial distributions of the 50-year return wave height H_{50}, the shape parameter k of the Weibull distribution or the Gumbel distribution (notation 'G') selected as the optimum distribution and the spread parameter γ_{50} defined by H_{50}/H_{10}. These parameters completely describe the characteristics of the Weibull distribution or the Gumbel distribution. The selected distribution may be regarded as a parent distribution of typhoon-generated AM wave height. The 50-year return wave height is more than 16 m on the southeastern area and reduces to 6 - 8 m towards the northwestern area on the East China Sea, reflecting the decay of typhoon strength associated with the northern movement. Even on the Japan Sea located in the northern region, the 10 m wave height area is widely distributed. The shape parameter decreases from 4 on the central and southeastern areas relatively close to Okinawa Islands toward about 1.4 in the northwestern area. On the Japan Sea, the parameter decreases from the Japan side toward the continent. More frequent passage of strong typhoons on the concerned area may give rise to a sharper distribution for typhoon-generated AM wave height data sample. Also, increase of the parameter toward the northwestern direction may be caused by scarceness of significant AM data associated with rare passage of intense typhoons. The spread parameter γ_{50} indicates almost the opposite spatial distribution to the case of shape parameter, in cases where the value ranges from 1.17 to 1.7.

Figure 7 indicates spatial distributions of the 50-year return wave height H_{50}, its standard deviation $H_{\sigma 50}$ and the spread parameter γ_{50} for historical typhoon case. The standard deviation ranging from 0.5 to 1.5 m is much larger than the maximum value of 0.07 m in the simulated typhoon case. Also, the return wave height and spread parameter yield a rather greater spatial variability than those in simulation cases because of a much smaller size sample of typhoon-generated AM wave height. But on the whole, the distributions of return wave height and spread parameter are respectively in qualitative and roughly quantitative agreement with those in the simulation case, although a significant difference is observed on the southwestern marginal area where the typhoon data are excluded in formulating the simulation model.

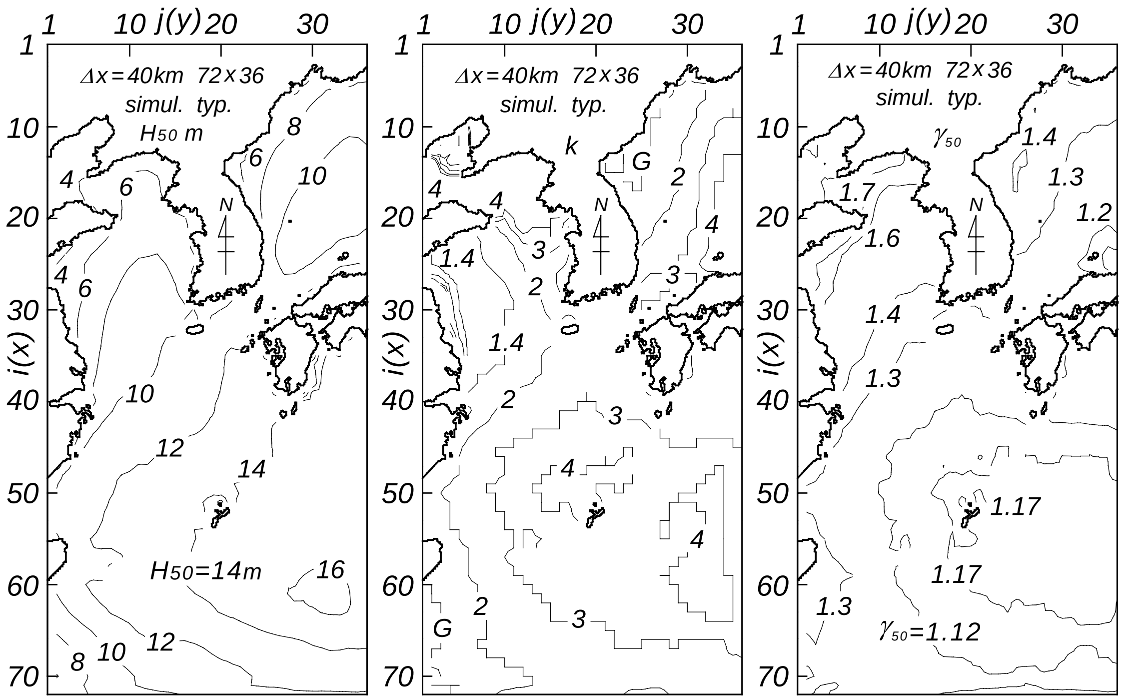

Figure 6. Return wave height, shape parameter and spread parameter(simulated typhoon case).

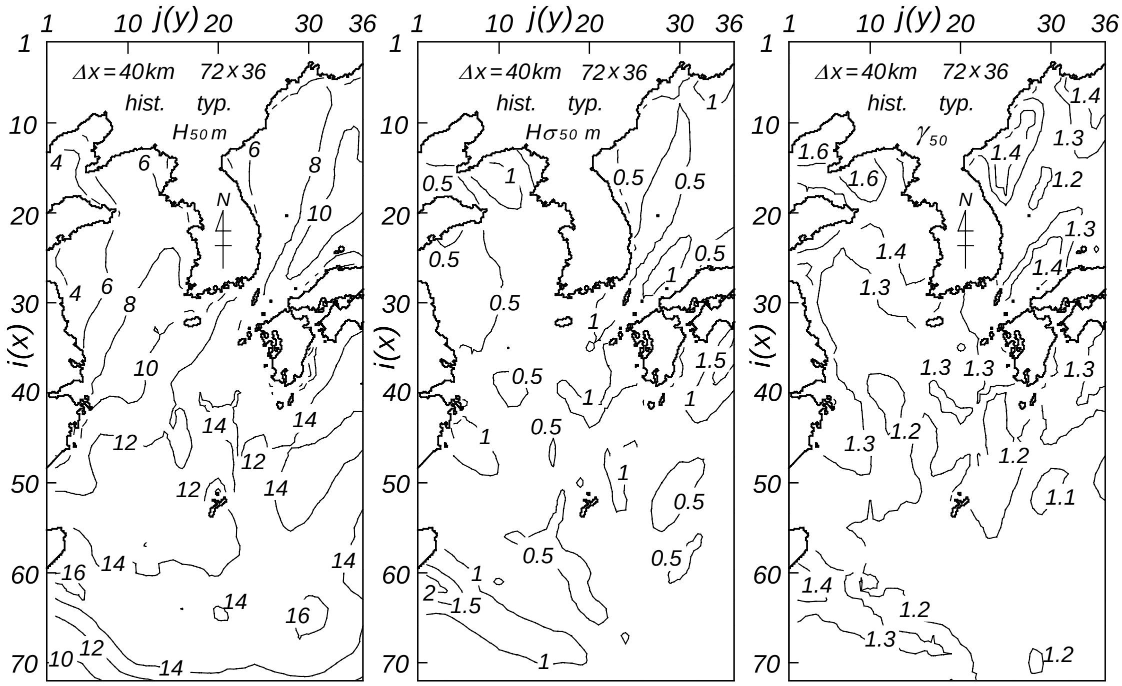

Figure 7. Return wave height, standard deviation and spread parameter(historical typhoon case).

4. VARIABILITY OF RETURN WAVE HEIGHT SAMPLE

Figure 8 shows variation of the statistics related to 50-year return wave height with sample size N at a representative point indicated by the black circle in Figure 5. They are sequentially evaluated from a 50-year return wave height sample with a size of 200 estimated in the FSP and VSP cases. Each of the wave statistics reduces the variability and takes almost constant value with increasing sample size N, although a small but discontinuous change is observed in the wave statistics calculated with use of higher moments such as kurtosis and skewness.

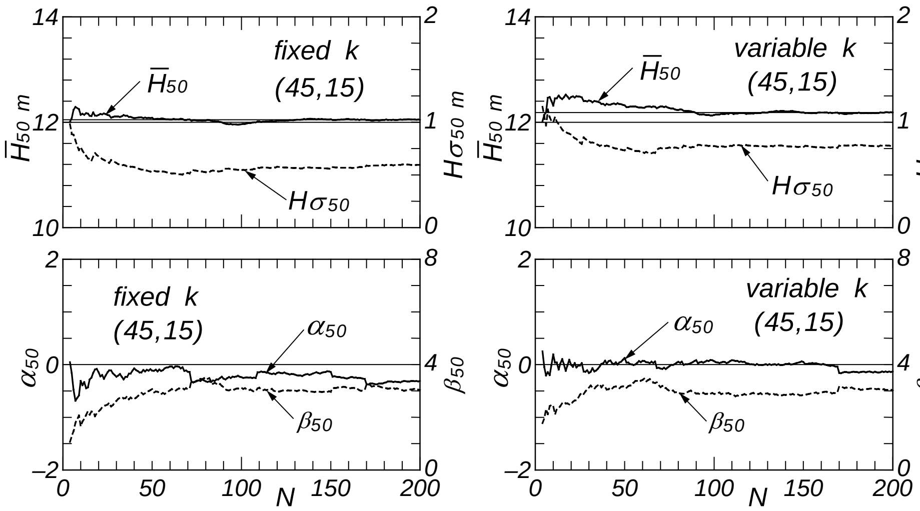

Figure 8. Variation of return wave height statistics with increase of sample size at a selected point.

Figure 9 gives spatial distributions of the sample mean $\overline{H}_{50}$ of 50-year return wave height H_{50}, the sample standard deviation $H_{\sigma 50}$ and the sample mean $\bar{\gamma}_{50}$ of spread parameter γ_{50} in each of the FSP and VSP cases. Comparison with Figure 6 reveals that in the FSP case, the mean value $\overline{H}_{50}$ nearly coincides with the 50-year return wave height H_{50} estimated using the original AM sample with a size of 10,000. It is also true for the mean value of spread parameter $\bar{\gamma}_{50}$. The following features can be read from the figures: 1) the FSP case yields a slightly smaller value for each of the mean values of return wave height and spread parameter than the VSP case. 2) the FSP case gives about 0.5 m smaller standard deviation than the VSP case. The latter means that identification of the parent distribution of typhoon-generated AM wave height reduces the variability of the estimated return wave height. As is exemplified in Figure 8, the skewness and kurtosis of 50-year return wave height sample are around zero and around 3 respectively. It may be said that the sample of return wave height is almost normally-distributed. Another important indication is that dimensionless quantities such as the ratio $H_{\sigma r}/A$ between standard deviation $H_{\sigma r}$ and scale parameter A of the parent distribution, the skewness and kurtosis of return wave height data sample are free from return period *r* in the FSP case.

Distribution of return wave height data sample with a size of 200 is approximated with the Gumbel or Weibull distribution, in cases where the LSM-based extreme value analysis model is utilized. The corresponding return periods are 50 and 100 years. Examples for each of the FSP and VSP cases are given in Figure 10 as a $Q-Q$ plot between individual sample data and calculated data, where ρ_{50} and k_{50} are the correlation coefficient and the shape parameter of the selected Weibull distribution in the case of 50-year return wave height data sample respectively. The subscript is changed from 50 to 100 in the 100-year return period case. As is indicated by the correlation coefficient of nearly 1, goodness of fit of the selected Weibull distribution to the sample data is excellent in any of the given cases. Also, it can be said that the shape parameter in the FSP case does not depend on the return period.

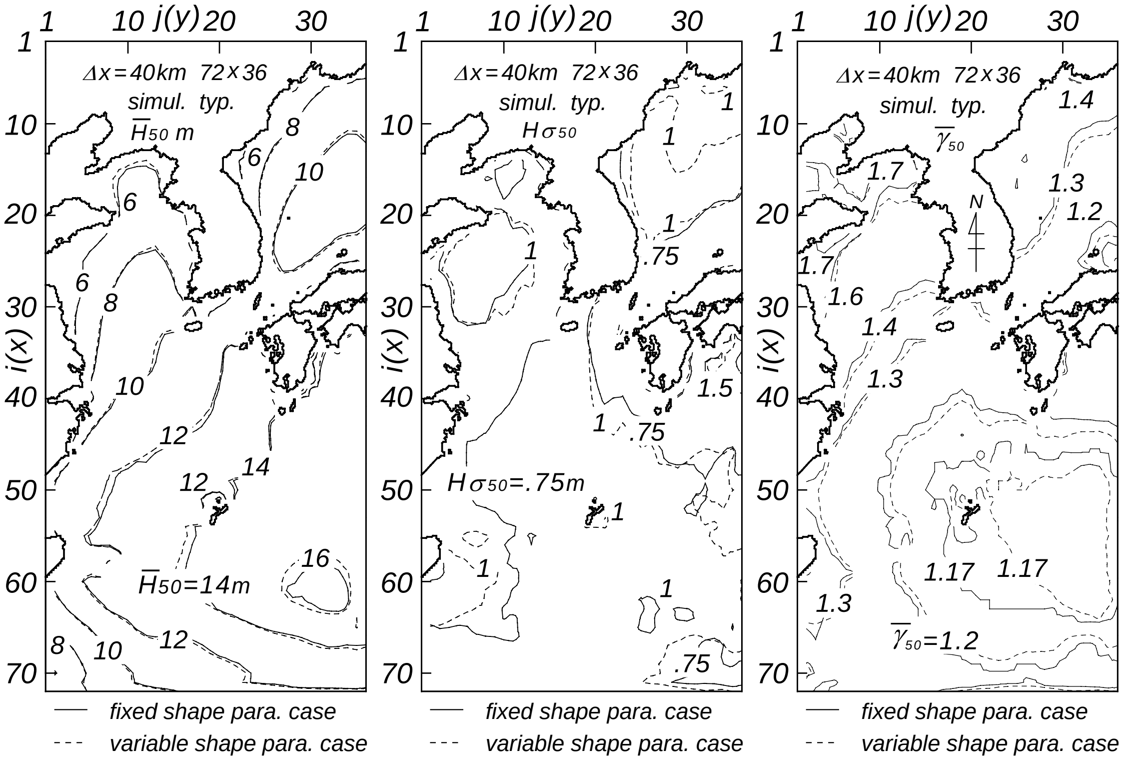

Figure 9. Return wave height statistics estimated by use of 200-size sample.

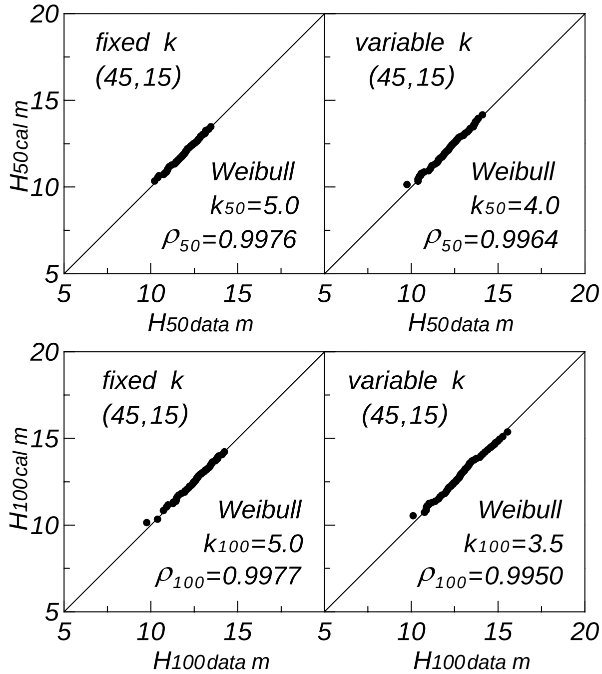

Figure 10. Q-Q plot to a sample of 50- or 100-year return wave height.

Figure 11 provides spatial distributions of the shape parameter k_{50} in the FSP and VSP cases and the shape parameter k_{100} in the VSP case, where 'G' indicates the Gumbel distribution. As was mentioned above, the shape parameter k_{50} coincides with the shape parameter k_{100} in the FSP case. In the FSP case, the shape parameter k_{50} takes a value of 3 to 5 on the East China Sea and a value of 3 on the Japan Sea. This suggests that the sample of 50-year return wave height is subject to a rather sharp-shaped distribution. In the VSP case, the shape parameter k_{50} gives a smaller value of 2 to 4 compared to the FSP case. Return wave height sample in the VSP case has a wider distribution than that in the FSP case. Moreover, increase of the return period from 50 to 100 year in the VSP case yields a further wider distribution for the return wave height sample, as an extension of area represented by the Gumbel distribution with a larger variability suggests. In the VSP case, a statistical variability of return wave height is augmented with an increasing return period.

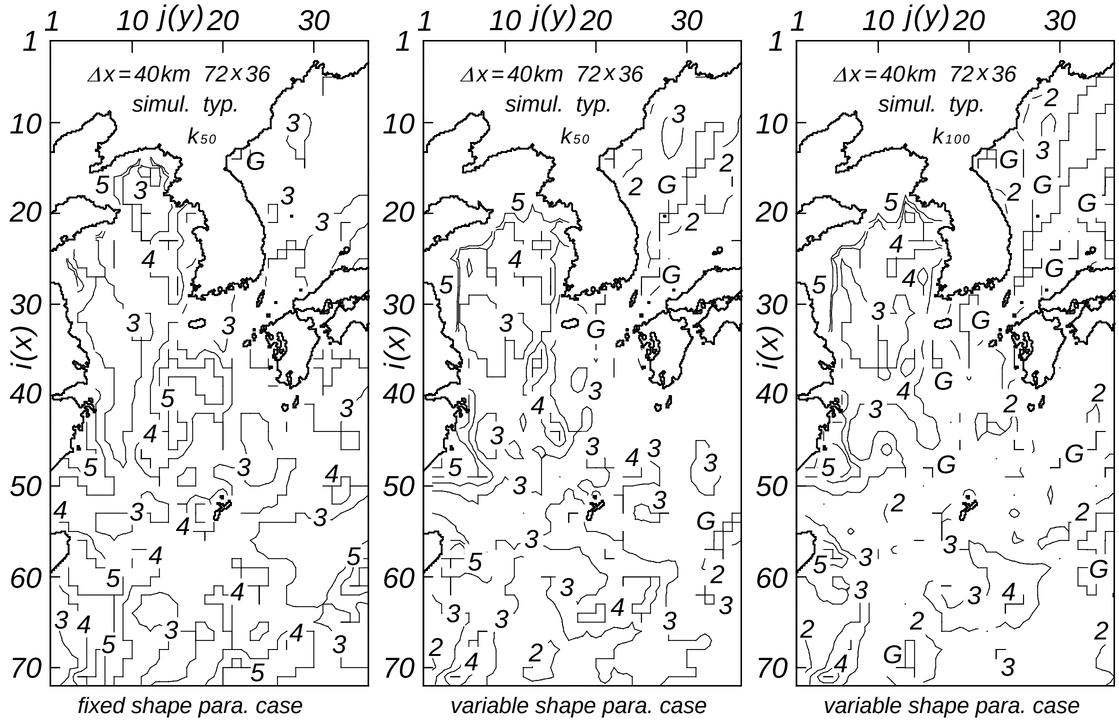

Figure 11. Shape parameter of the Weibull distribution fitted to a sample of 50- or 100-year return wave height.

Comparison between sample standard deviation of 50-year return wave height and that calculated using the fitted distribution is shown in Figure 12 for each of the FSP and VSP cases. The calculation gives a very close agreement with the sample value on most of the concerned region in any of the FSP and VSP cases. For the mean value of 50-year return wave height defined by a lower moment, the degree of agreement becomes higher.

Figure 13 demonstrates spatial distributions of a skewness difference $\alpha_{50\mathrm{cal}} - \alpha_{50\mathrm{data}}$ and a kurtosis ratio $\beta_{50\mathrm{cal}} / \beta_{50data}$ in the FSP and VSP cases. The subscripts 'data' and 'cal.' indicate sample value and calculated value respectively. A tendency that the agreement becomes lower with increasing order of the moment is inevitable. In spite of that, the discrepancy between sample value and calculated value may be insignificant. Thus, it can be said that the selected distribution approximates a sample distribution of return wave height with high accuracy.

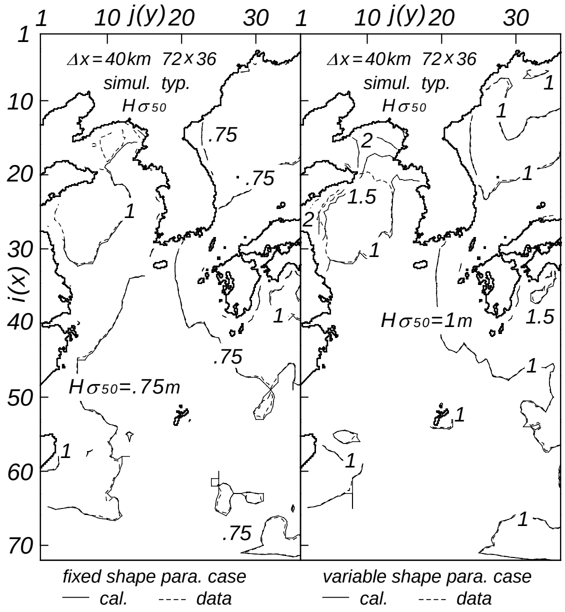

Figure 12. Comparison between standard deviation of 50-year return wave height sample and calculated standard deviation.

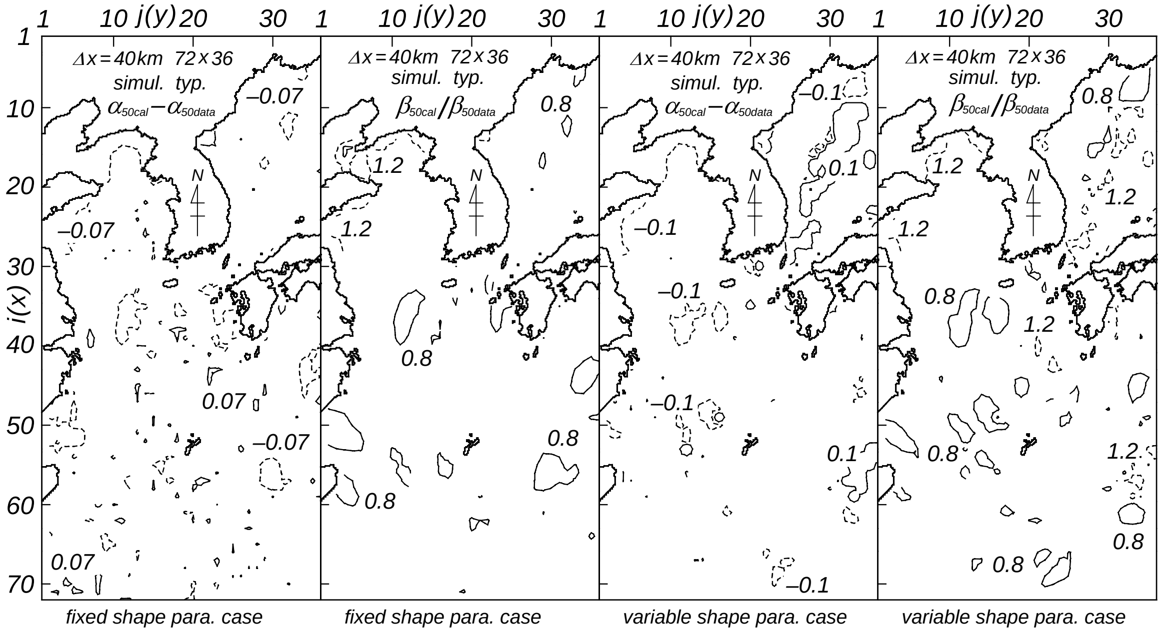

Figure 13. Comparison between higher moment statistics of 50-year return wave height data sample and calculated statistics.

5. CONCLUSIONS

The main results in this study are described as follows:

1) Monte-Carlo simulation of typhoon-generated waves over extremely long years makes it possible to directly estimate not only the parent distribution of typhoon-generated AM wave height but also a sample distribution of return wave height.
2) Space-dependent 50-year return wave height estimated using the simulated sample of typhoon-generated AM wave height is in reasonable agreement with that estimated using the sample for historical typhoon case. This is true for the spread parameter.
3) The parent distribution of typhoon-generated AM wave height is well-expressed by either the Weibull distribution or the Gumbel distribution.
4) Shape parameter and the other parameters of the parent distribution are significantly space-dependent. Frequent passage of strong typhoon may yield a sharper parent distribution associated with a greater shape parameter.
5) Identification of a shape parameter in the parent distribution may lead to an enhancement of statistical reliability of the estimated return wave height, that is to say, more efficient estimation of the return wave height.
6) Sample of return wave height may be subject to the normal distribution in a rough sense. In a detailed aspect, it is well-approximated by asymmetrical distribution such as the Weibull or Gumbel distribution.

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