

Energy-Flux Balances and Source Term Parameterizations

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Objectives

- 1. nonlinear 4-wave interactions
- 2. a look at energy – flux balances
- 3. relation to source term parameterizations
- 4. challenges for improvements ...

1. introduction: energy – flux balances

Review

$$dE(f) / dt = S_{nl} + S_{in} + S_{ds}$$

Quadruplet wave interactions determined by locus

$$\omega_1 + \omega_2 = \omega_3 + \omega_4 \quad k_1 + k_2 = k_3 + k_4$$

replaced by DIA as

$$\omega_1 = \omega_2 = \omega \quad \omega_3 = \omega(1 + \hat{\lambda}) \quad \omega_4 = \omega(1 - \hat{\lambda})$$

where $\hat{\lambda} = 0.25$ and $q_3 = 11.5^\circ$ and $q_4 = -33.6^\circ$

⇒ Many fewer interactions are selected

Review

Snl = boltzmann integral

- $S_{nl} = 2 \int dk_3 T(k_1, k_3)$

where $T(k_1, k_3) = \int ds J C^2 \theta D(n_1, n_2, n_3, n_4)$

Tracy and Resio, 1982

Locus of interactions

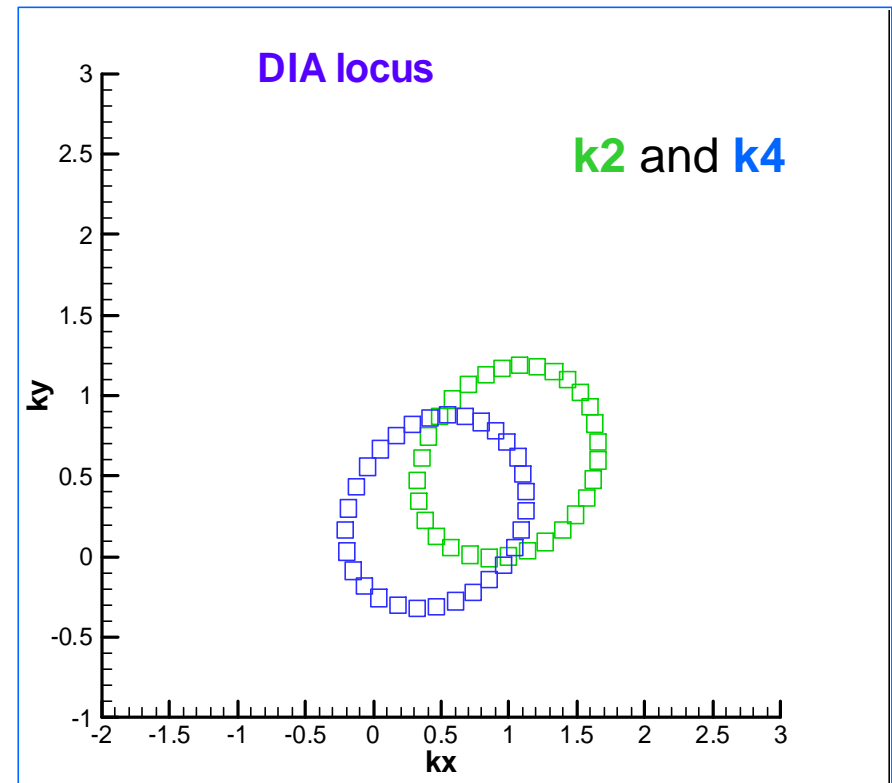
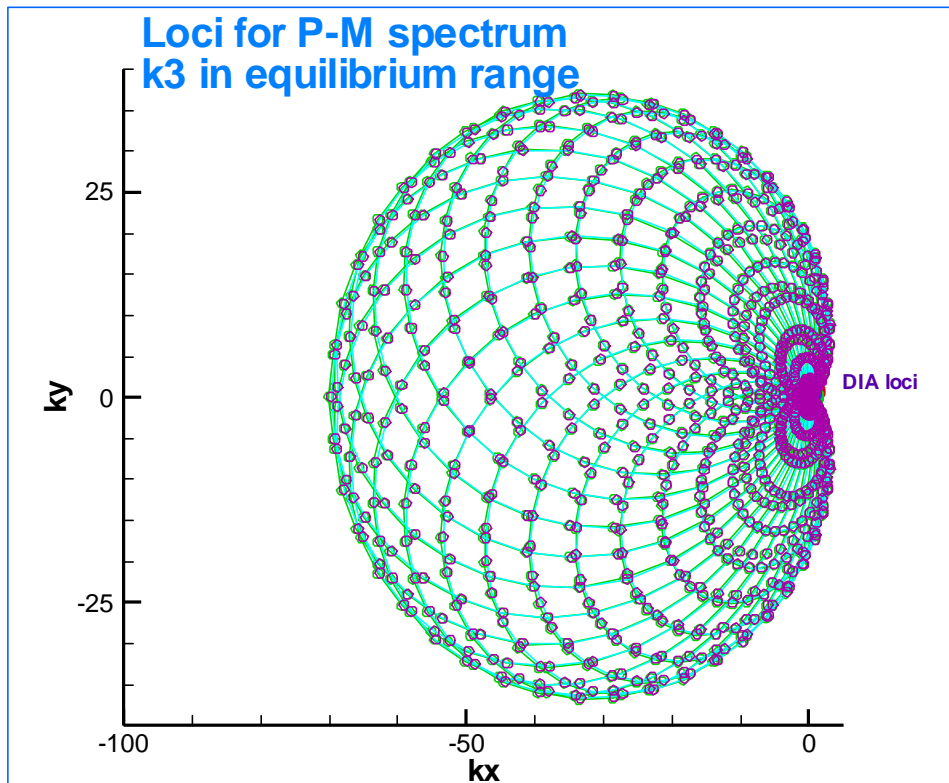
Jacobian

Coupling coefficient

action densities

$$\theta = \begin{cases} 1 & \text{when } |k_1 - k_3| = |k_1 - k_4| \\ 0 & \text{when } |k_1 - k_3| > |k_1 - k_4| \end{cases}$$

Comparison of P-M loci with DIA



Tracy and Resio, 1982

$$f_1/f_p = 2.0$$

$$k_{1x} = 1$$

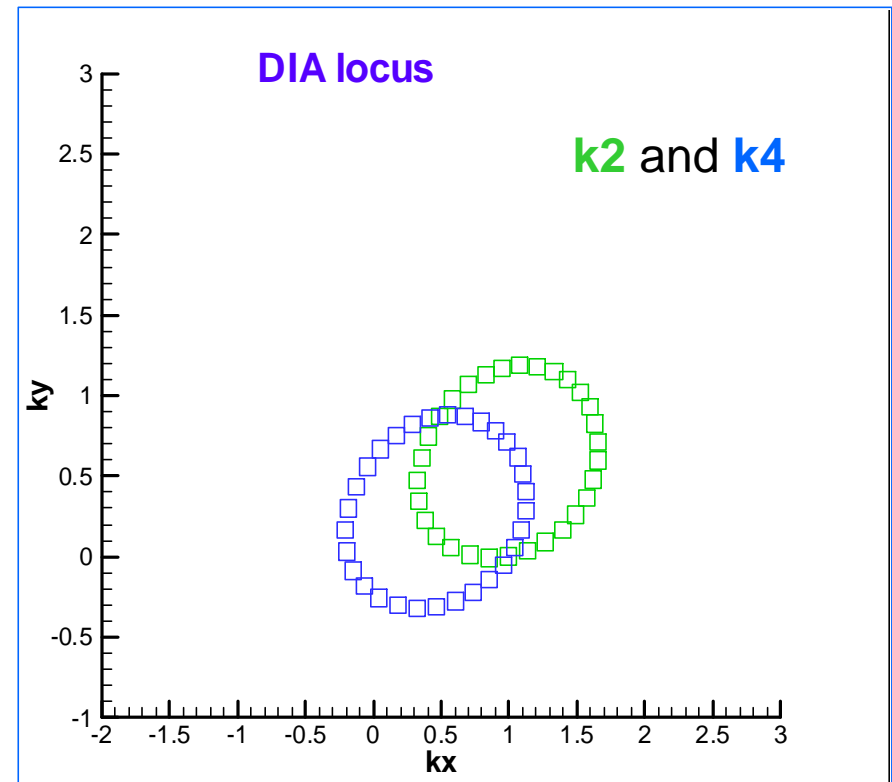
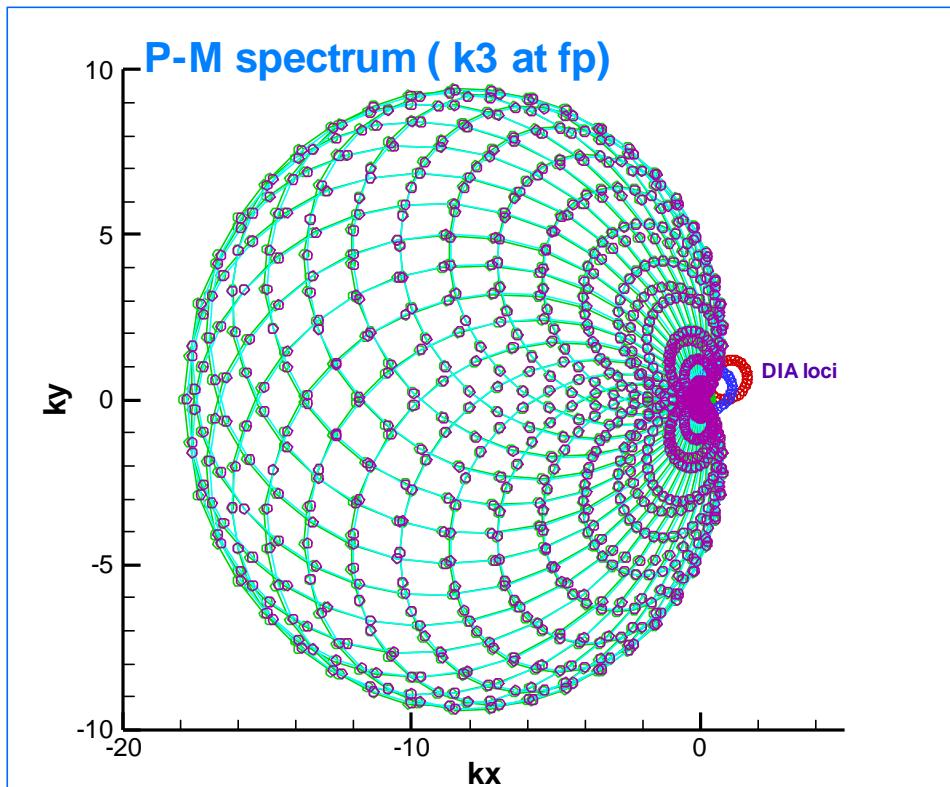
$$k_{1y} = 0$$

$$q_3 = 11.5^\circ$$

$$q_4 = -33.6^\circ$$

$$\lambda = 0.25$$

Comparison of P-M loci with DIA



$$k_{1x}=1$$

$$k_{1y}=0$$

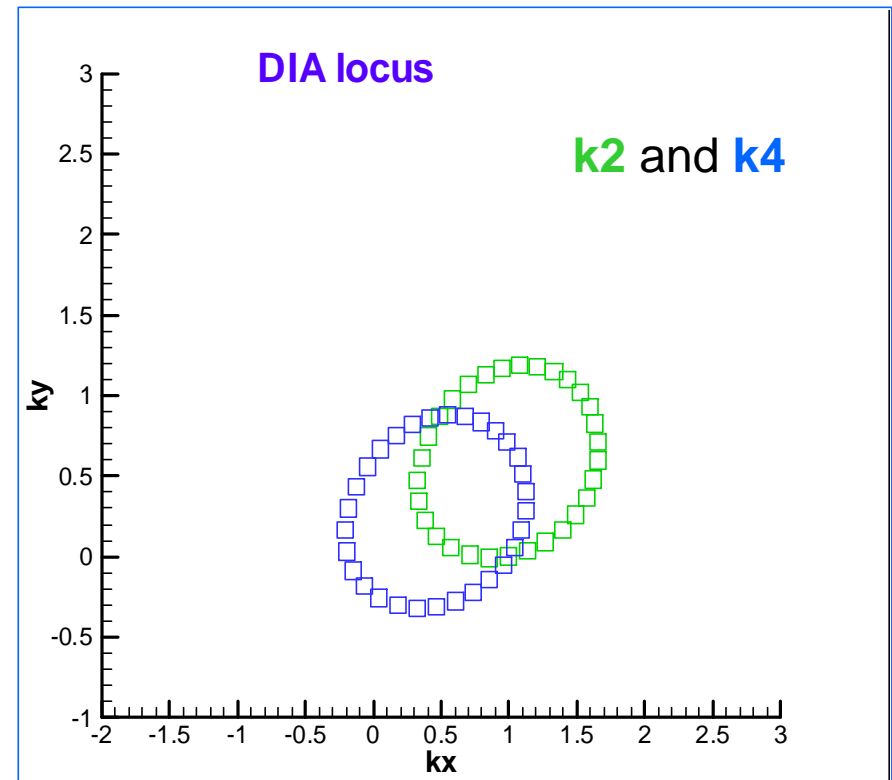
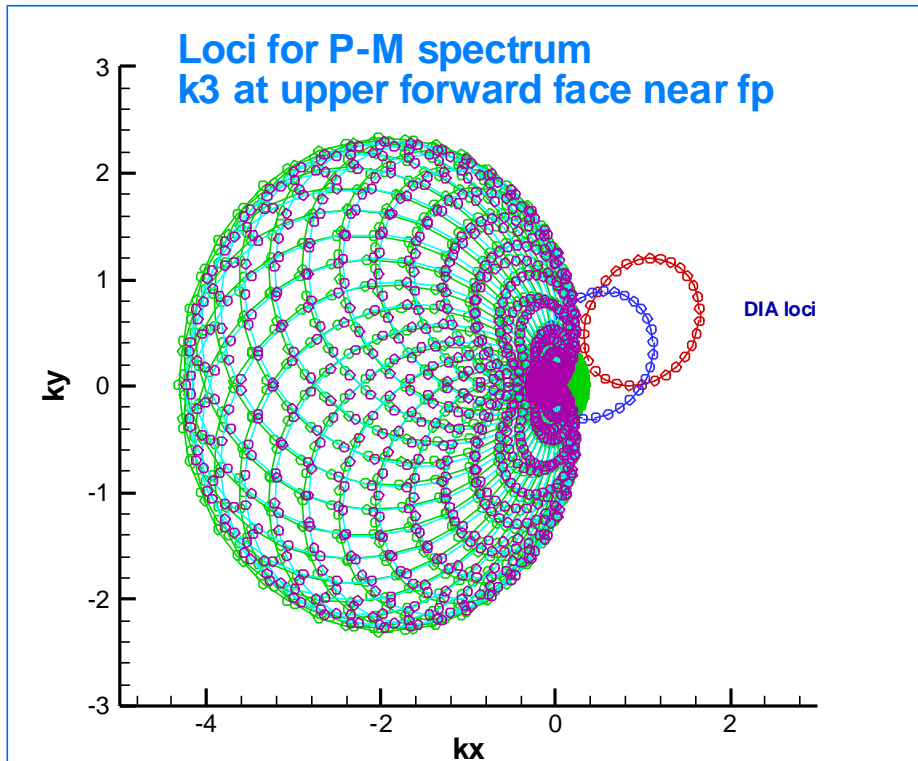
$$q_3 = 11.5^\circ$$

$$q_4 = -33.6^\circ$$

$$\lambda = 0.25$$

$$f_1/f_p = 1.0$$

Comparison of P-M loci with DIA



$$k1_x=1$$

$$k1_y=0$$

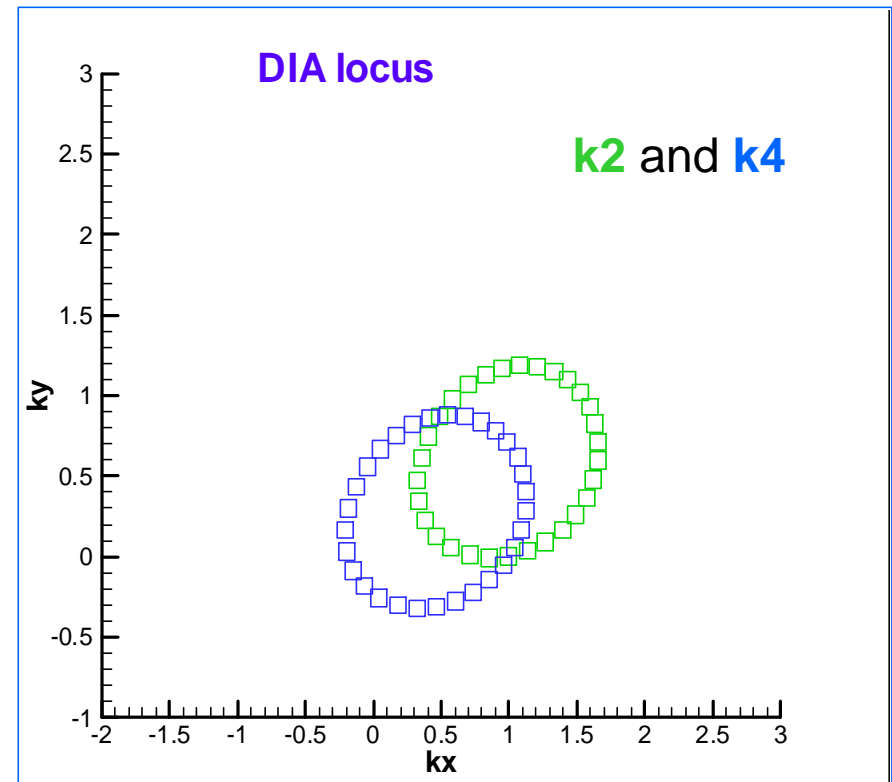
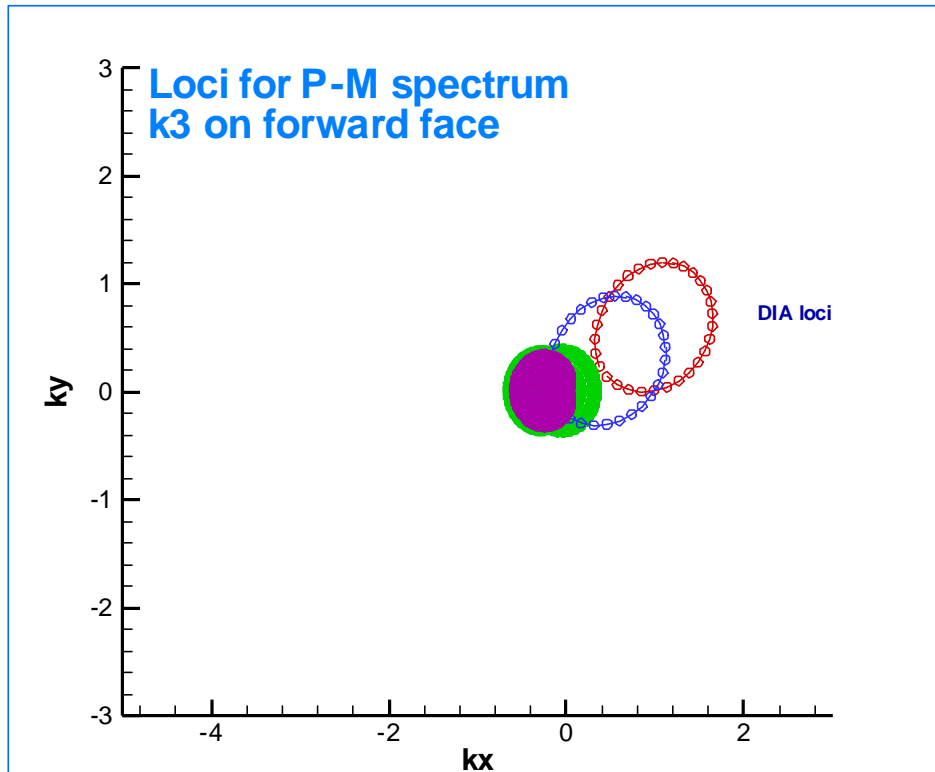
$$q_3 = 11.5^\circ$$

$$q_4 = -33.6^\circ$$

$$\lambda = 0.25$$

$$f_1/f_p = .95$$

Comparison of P-M loci with DIA



$$k_{1x}=1$$

$$k_{1y}=0$$

$$q_3 = 11.5^\circ$$

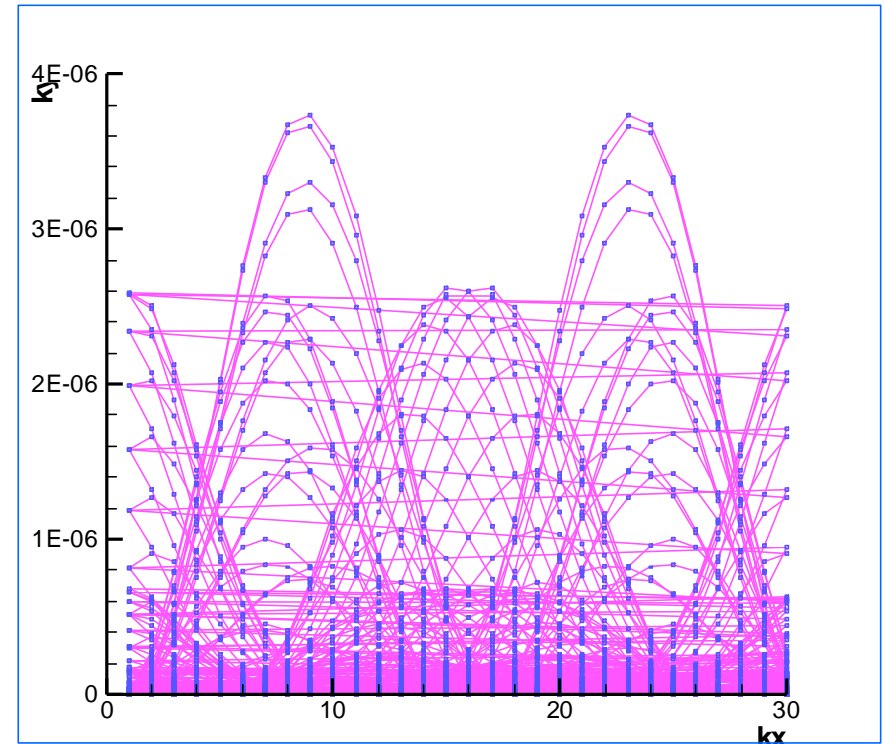
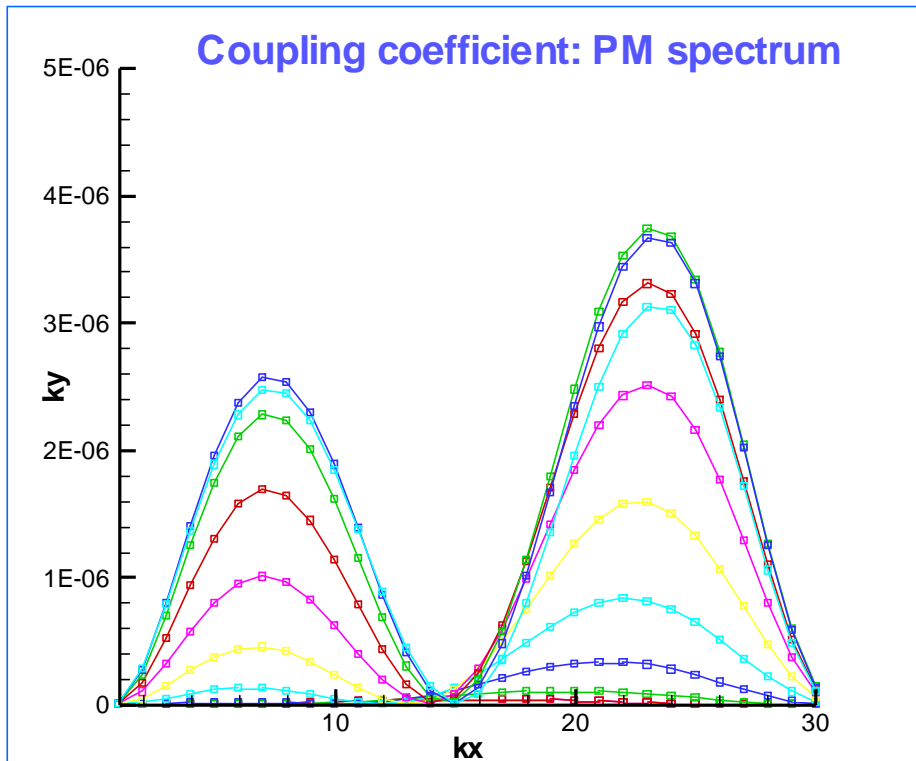
$$q_4 = -33.6^\circ$$

$$\lambda = 0.25$$

$$f_1/f_p = .82$$

Review

Coupling coefficients



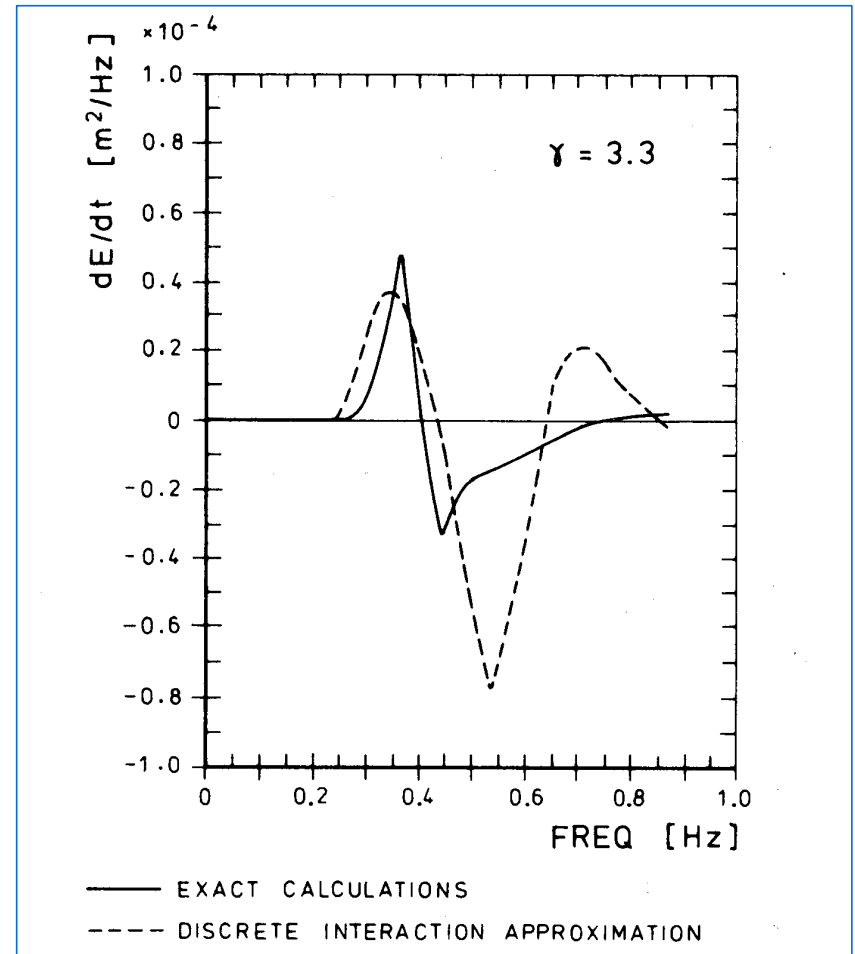
Selected C values....

$f_1 = .95$ and $f_3 / f_p = n \times 7 \dots$

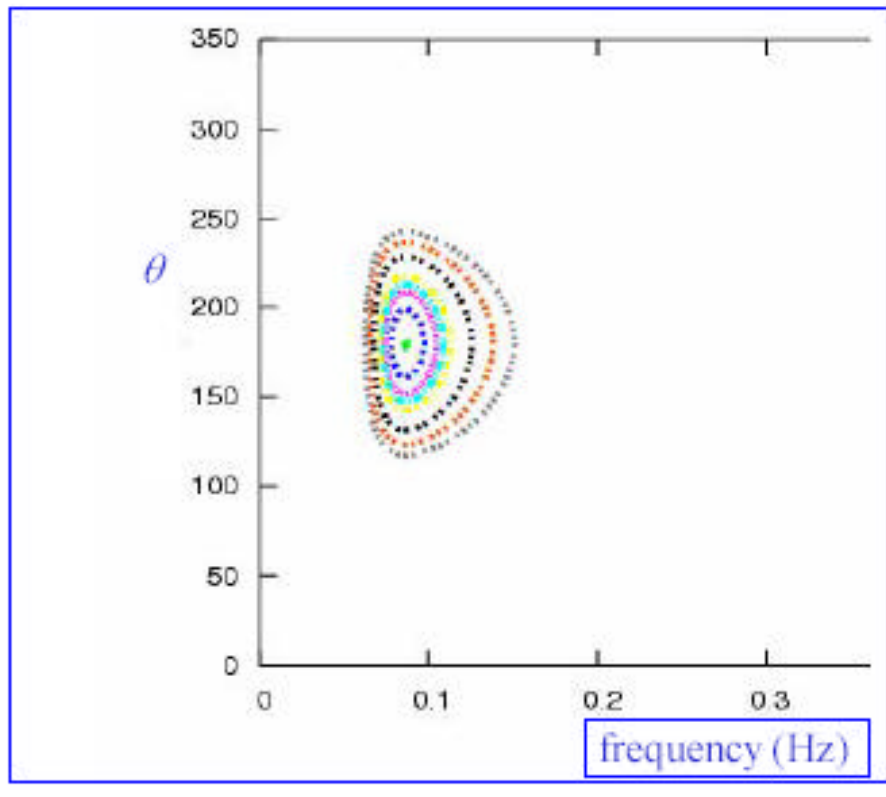
Impacts on Snl computations

For simple SWAMP-2 type waves DIA is reasonable approximation....

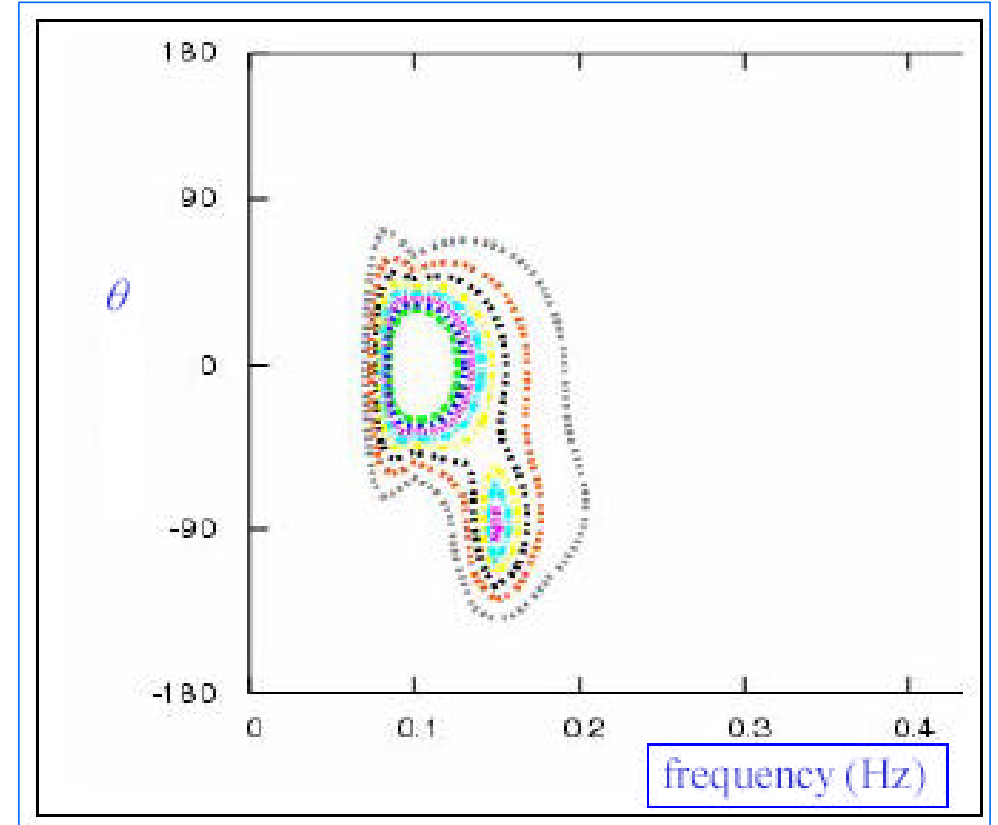
DIA for standard JONSWAP spectrum
(from Hasselmann et al. 1985)



Other examples ..

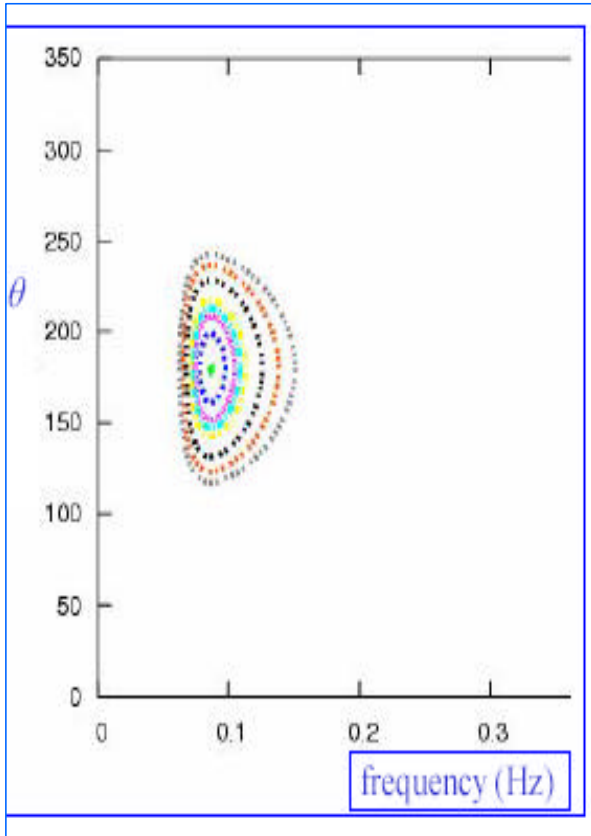


Simple peak spectrum

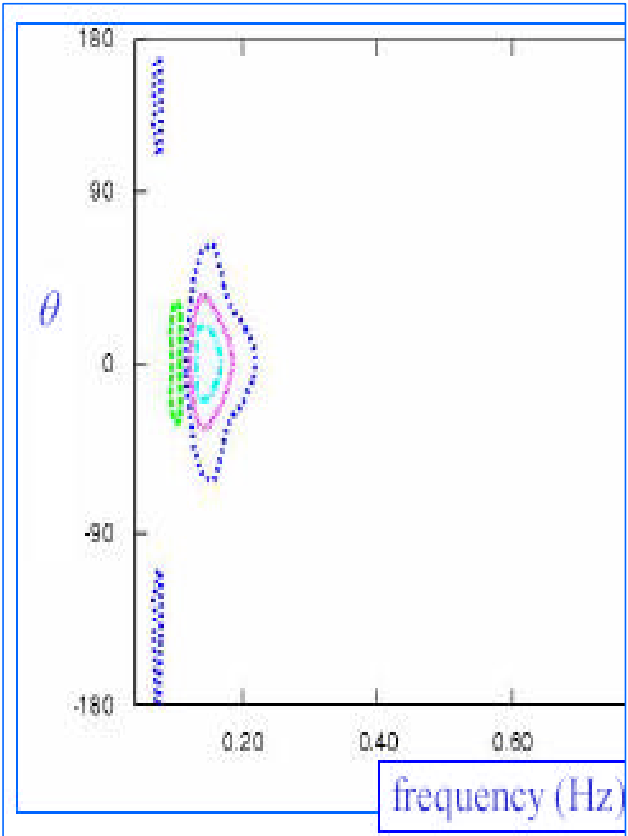


Double-peaked sheared spectrum

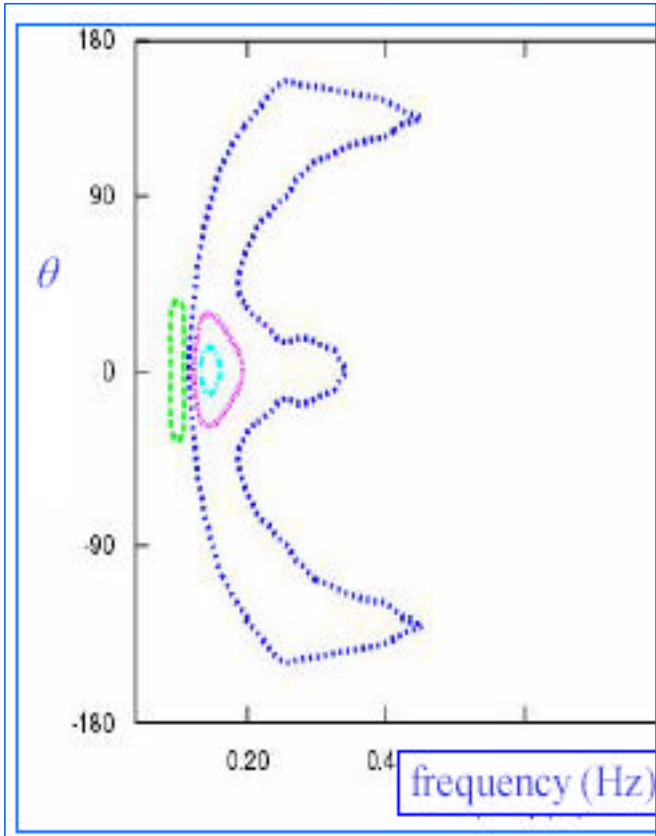
Simple peak spectrum



2-d energy

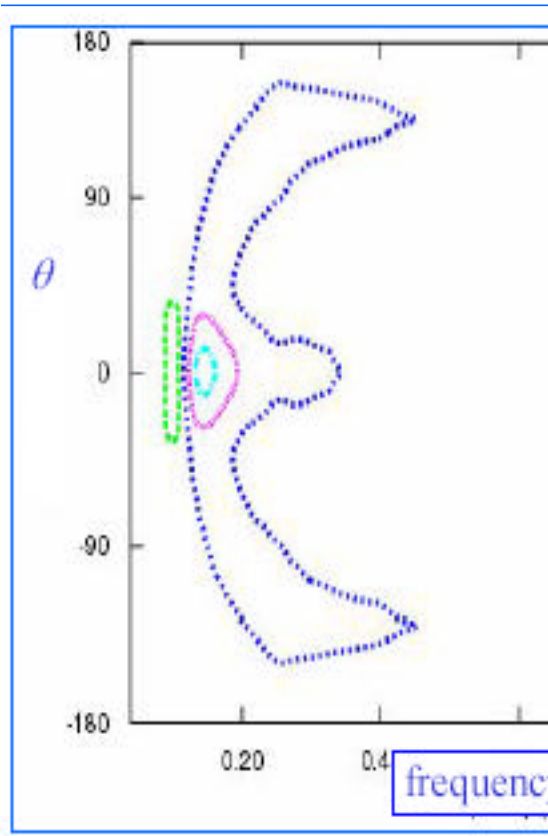


DIA-WW3

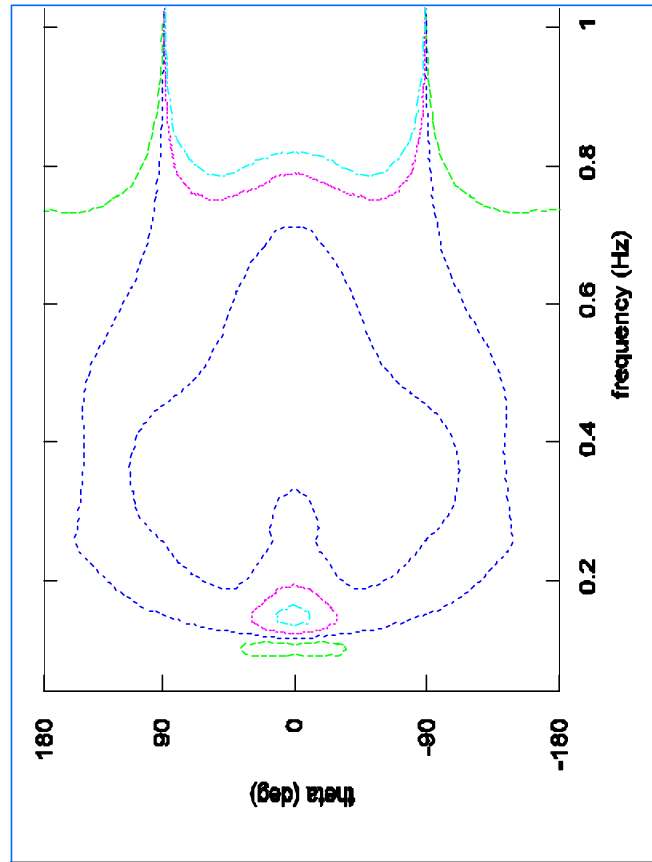


RTW-WW3

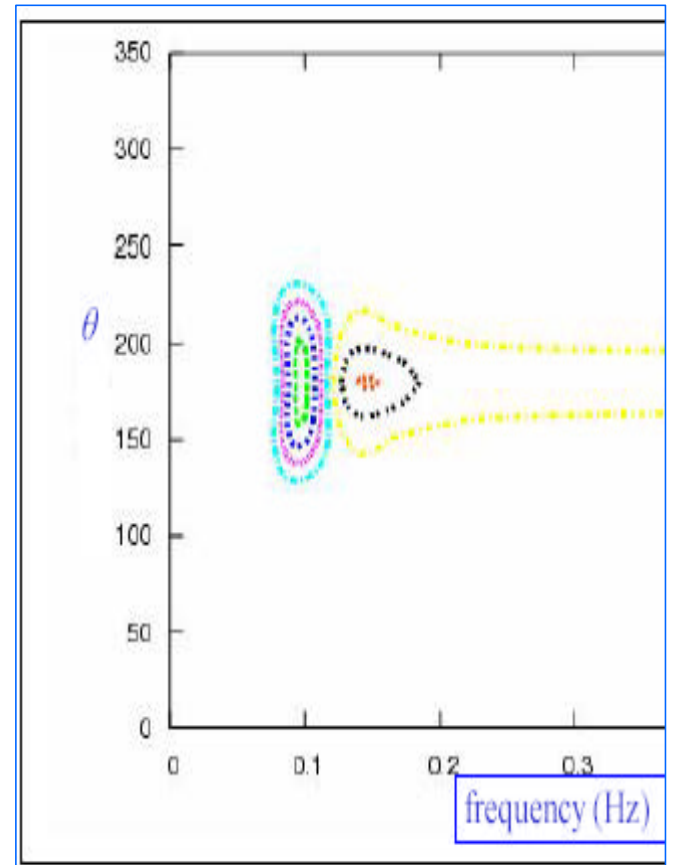
Simple peak spectrum



RTW-WW3



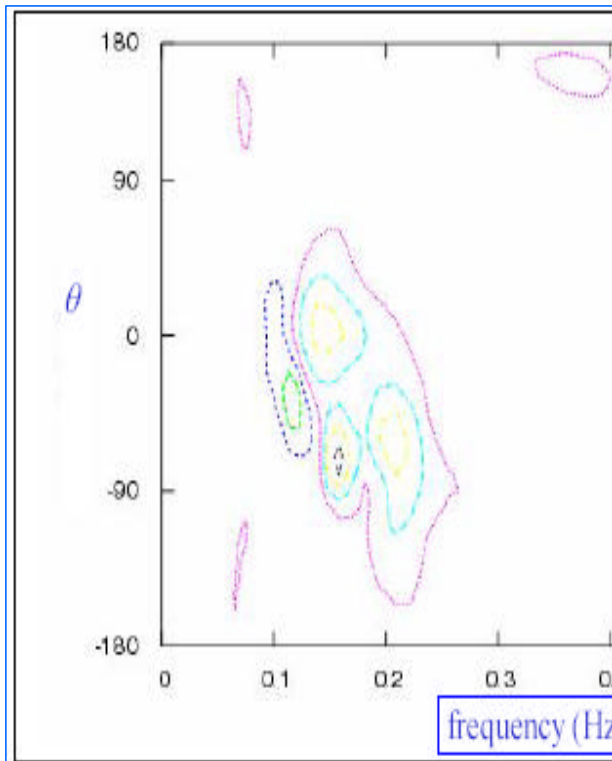
RTW-WW3
No tail



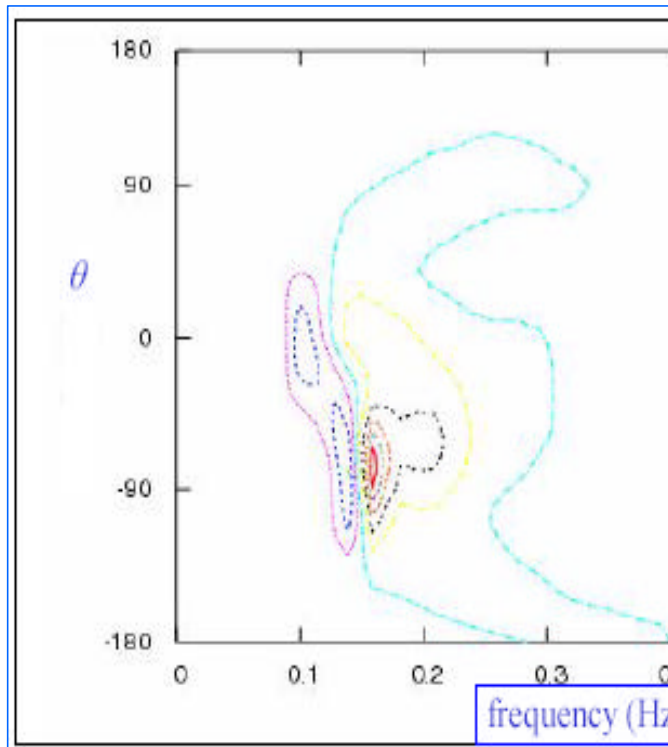
RTW-erdc

intensity and a tail

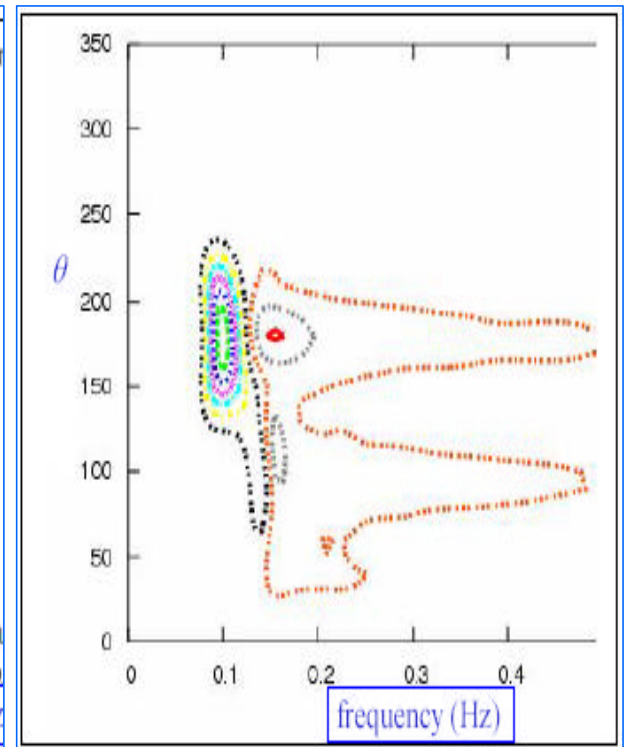
Double-peaked spectrum



DIA-WW3



RTW-WW3



RTW-erdc

intensity and a tail !

Review

2. energy – flux balances

$$T(k_1, k_3) = \oint ds J^{-1} C^2 D(n_1, n_2, n_3, n_4) \theta$$

Flux past ?_A :

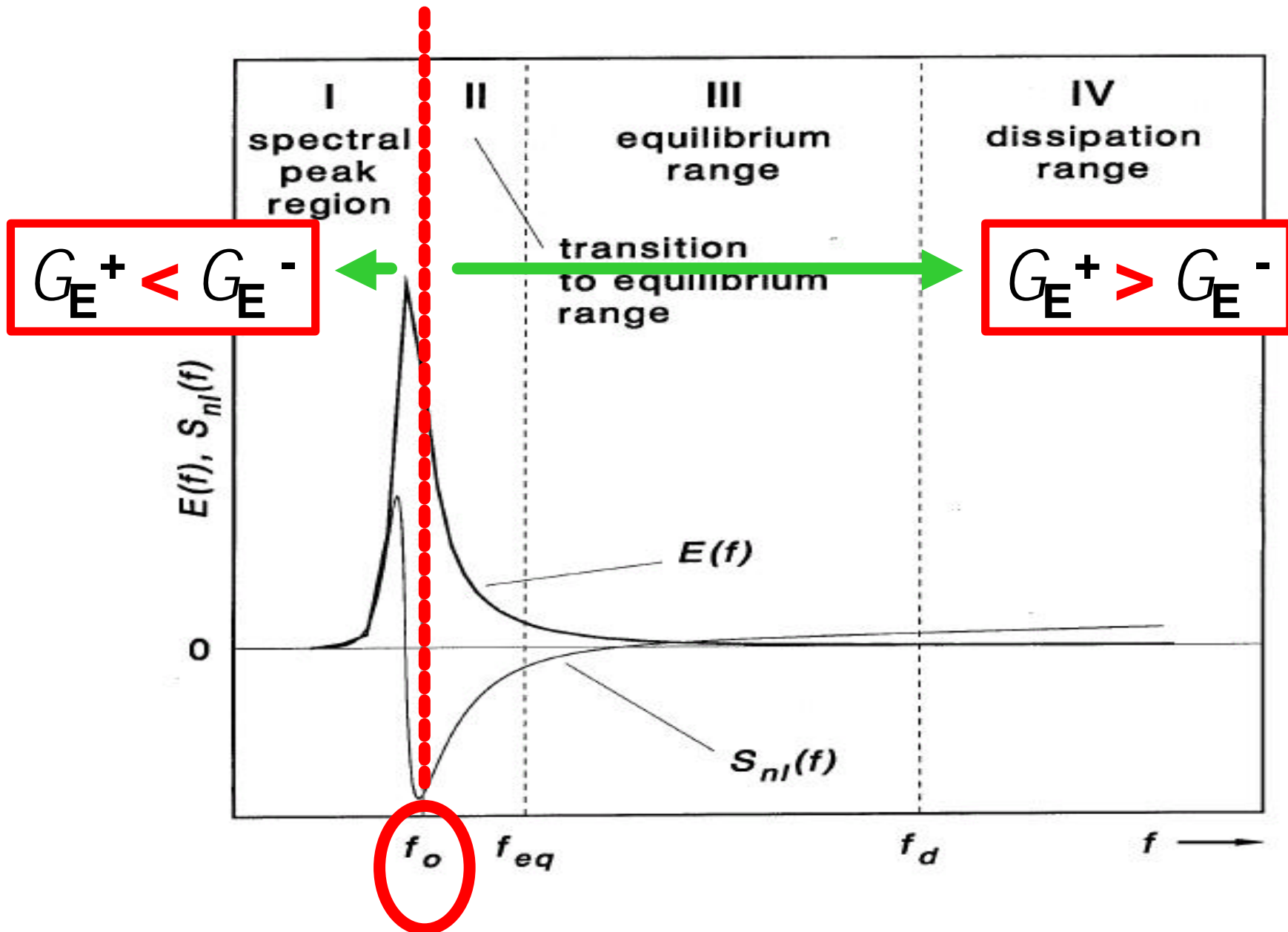
$$G(?_A) = \int \oint ds \dots \dots H_{3A} H_{A1} dk_3 dk_1$$

Heaviside functions

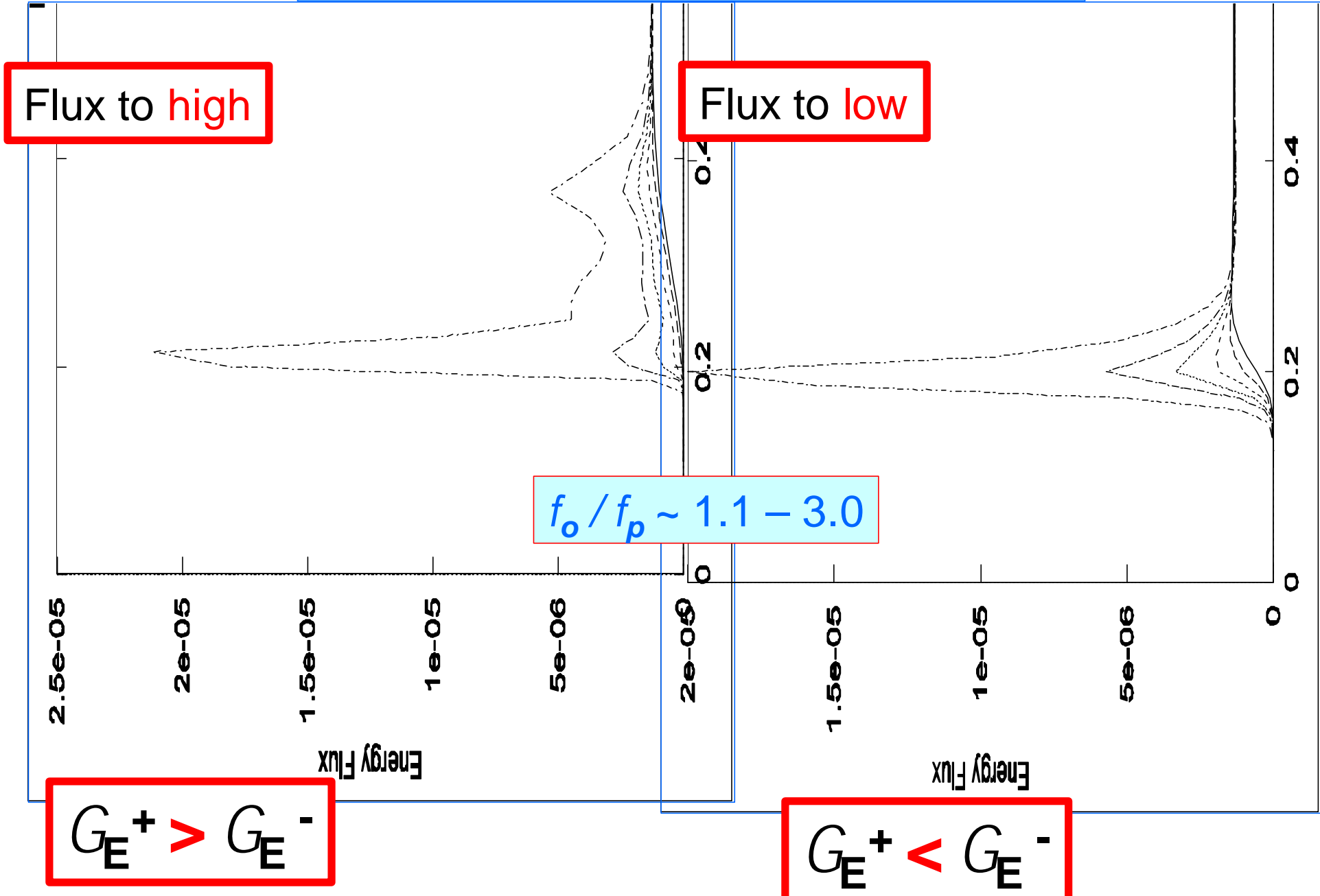
$$H(x) = \begin{cases} 1 & \text{when } x \geq 0 \\ 0 & \text{when } x < 0 \end{cases}$$

$$H_{3A} = H(|k_3| - |k(?_A)|) \dots \text{Etc.}$$

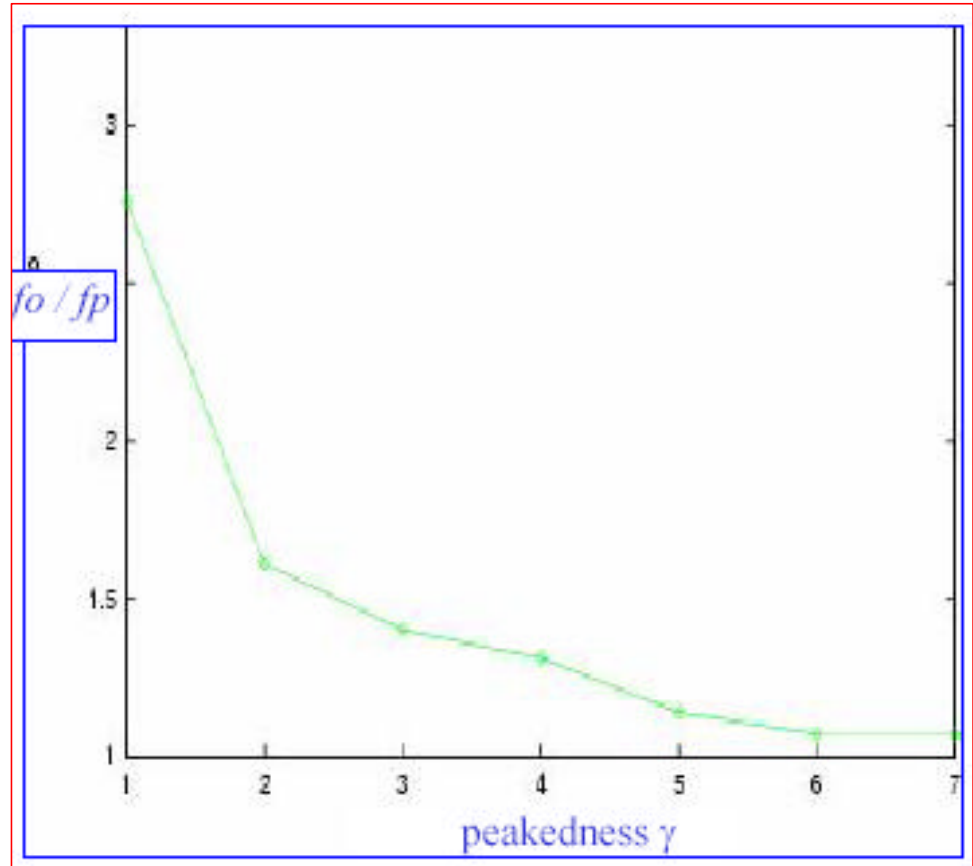
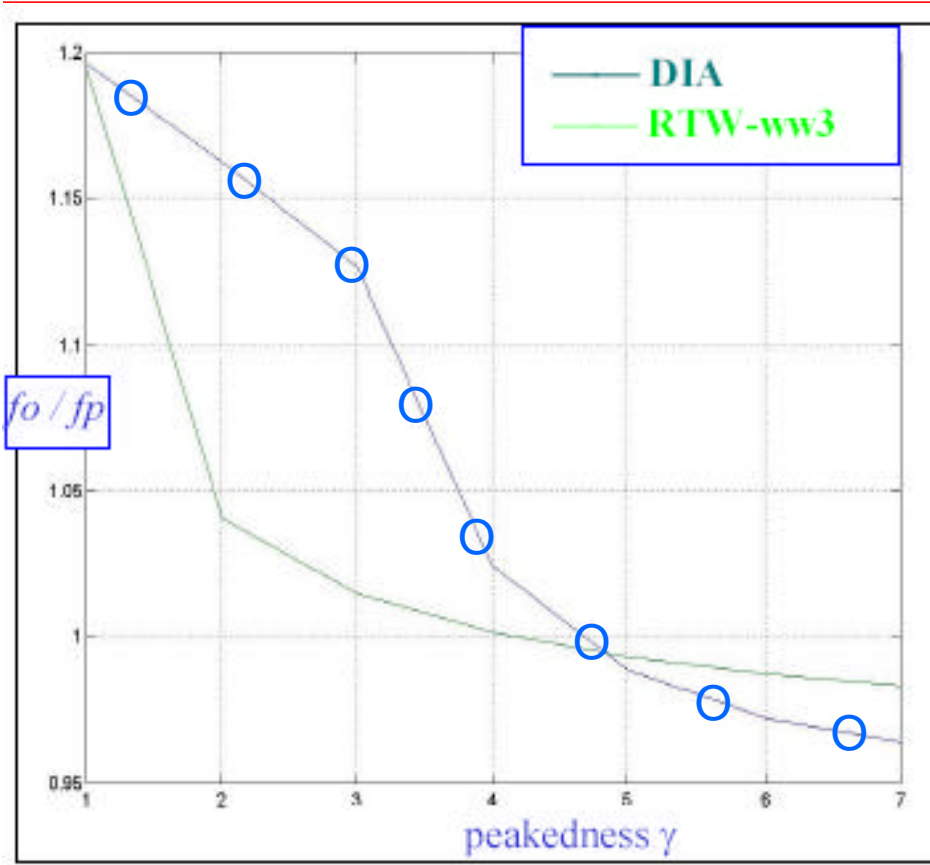
Spectral regions ...



Wave maturity $0.65 < \gamma < 7.0$



Variation of f_o with wave maturity

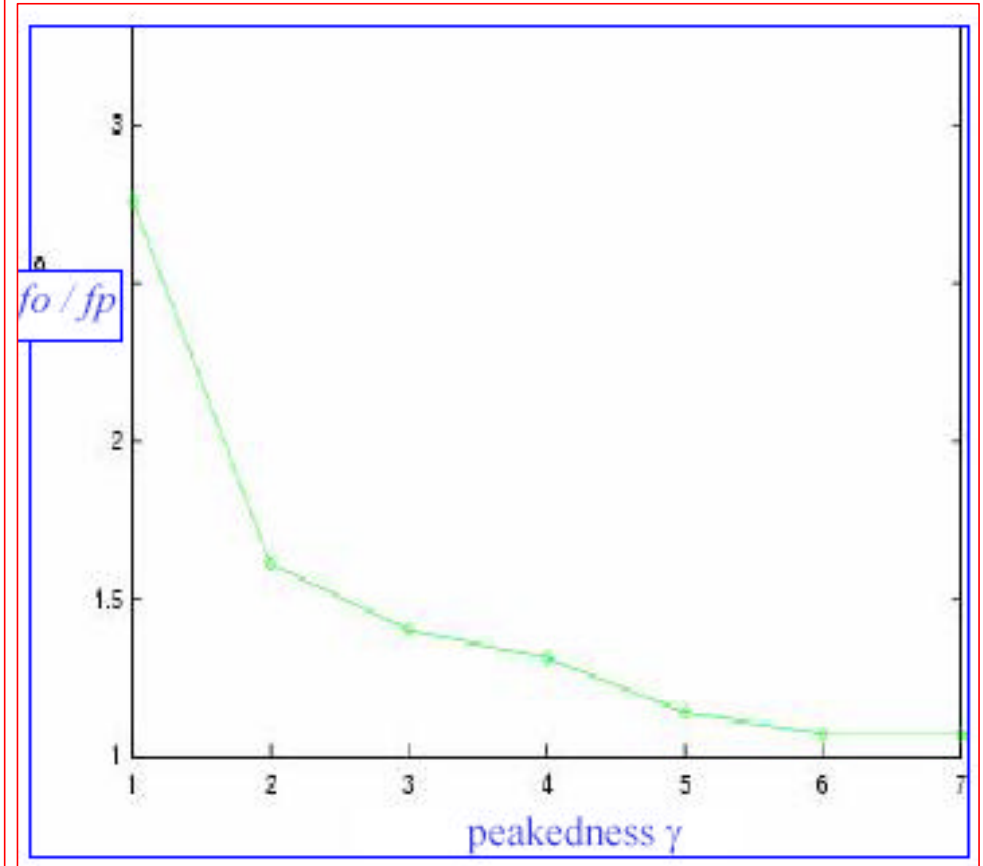
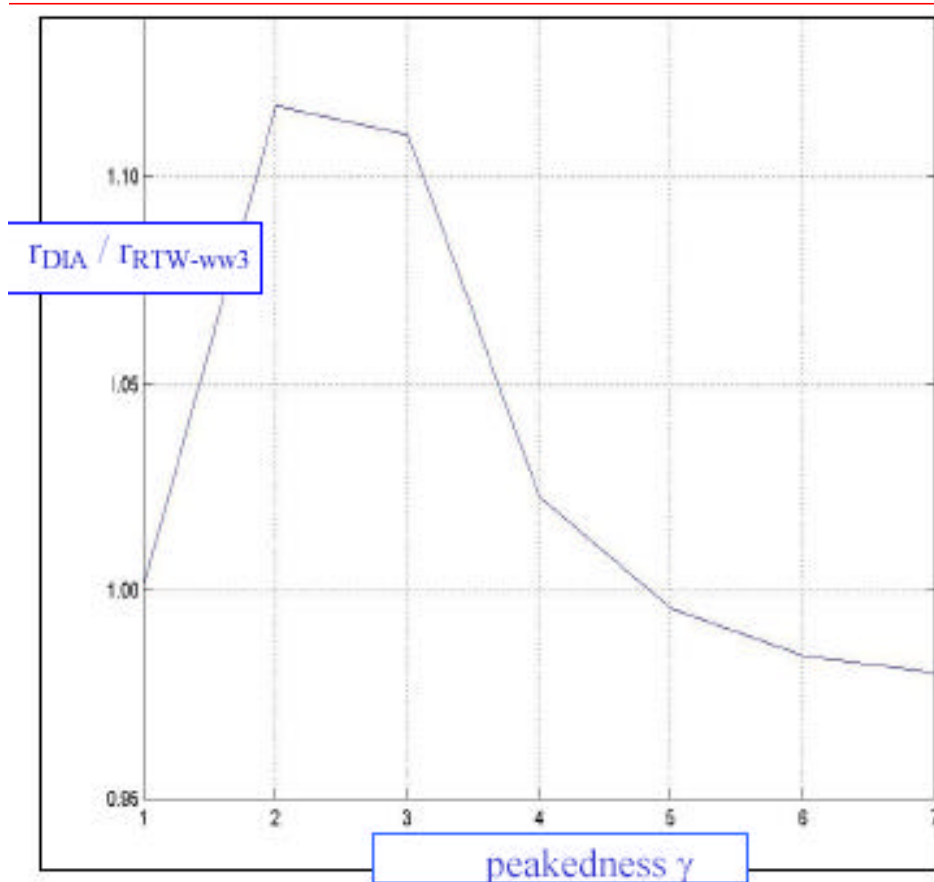


RTW-ww3 —
DIA-ww3 —○

RTW-erdc

always > 1

Variation of f_o with wave maturity

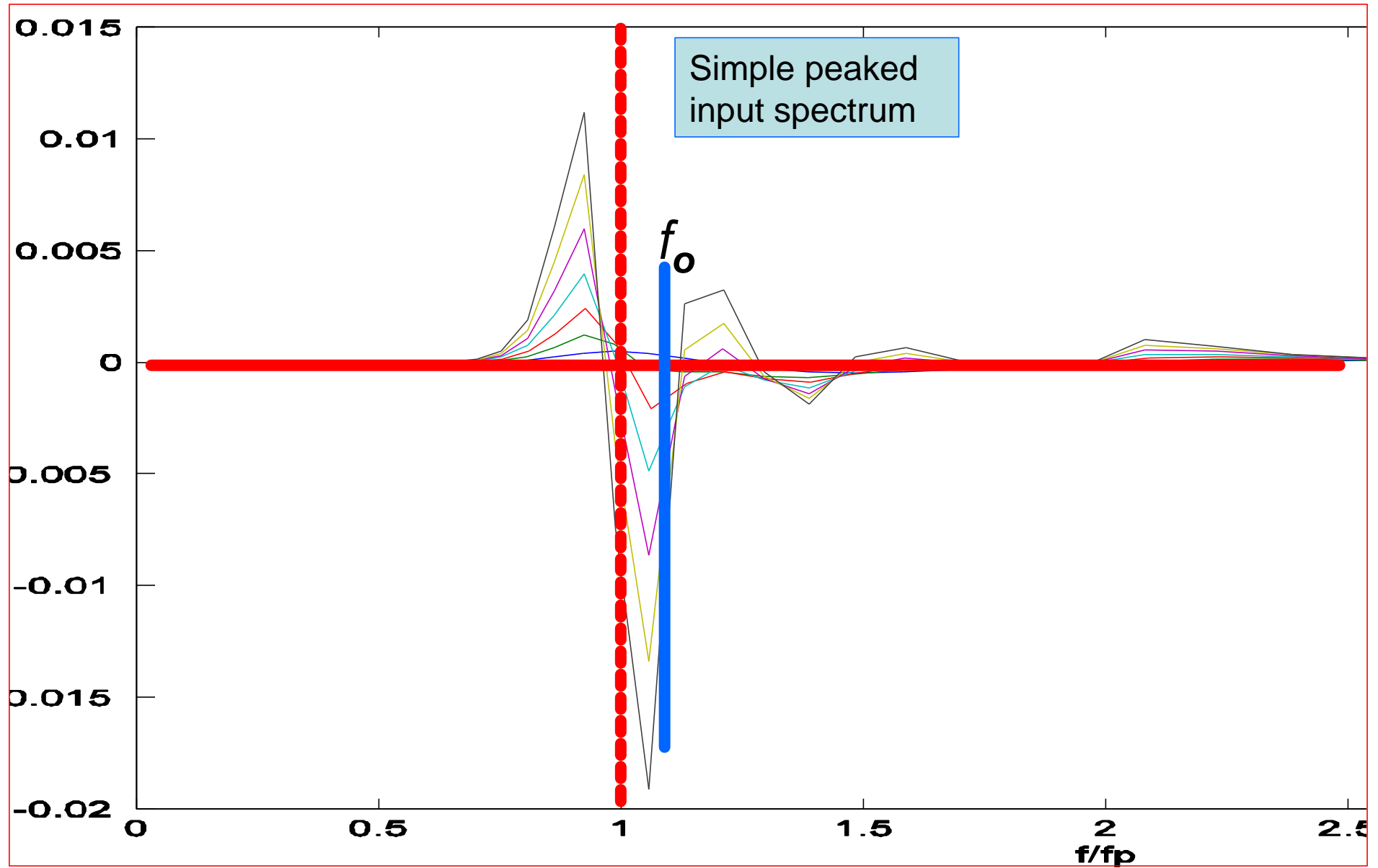


$$r_{DIA} / r_{RTW-ww3} < r_{DIA} / r_{RTW-erdc}$$

RTW-erdc

always > 1 !

DIA does not allow enough energy to be retained \rightarrow Sin and Sds must compensate



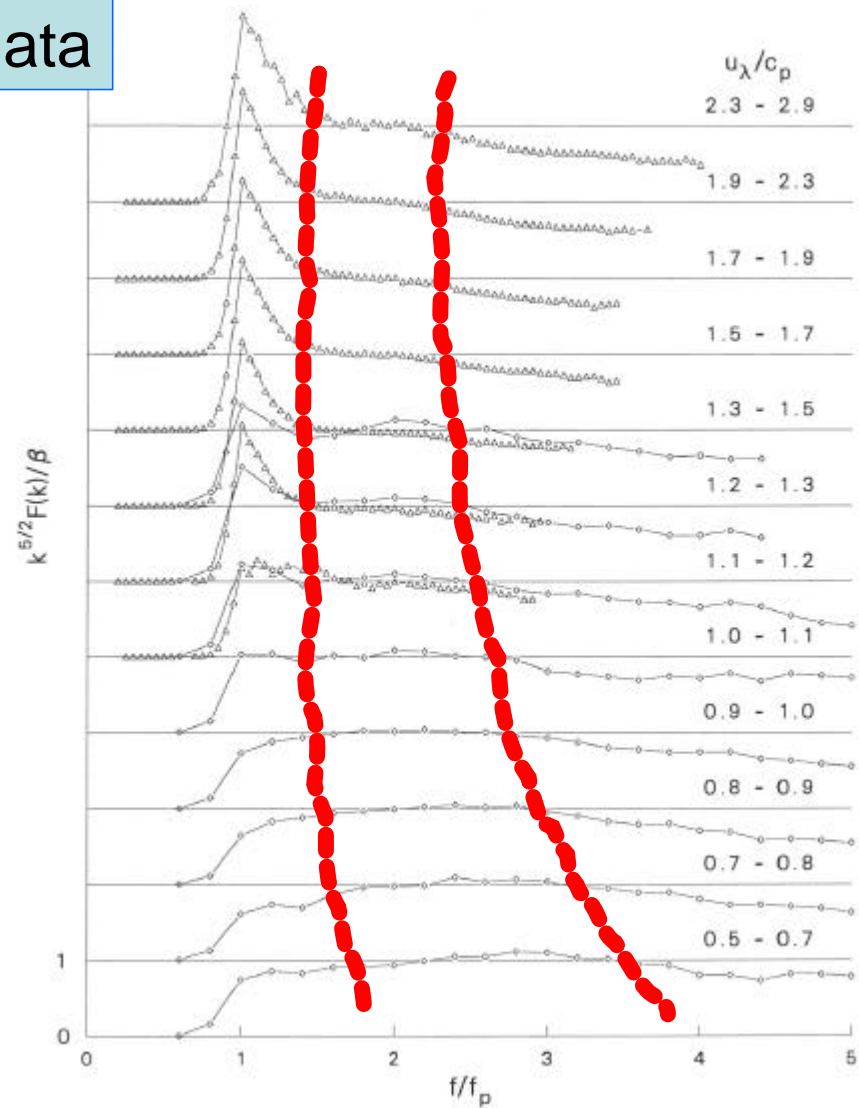
3. relation to source term parameterizations

Detailed balance in observed data

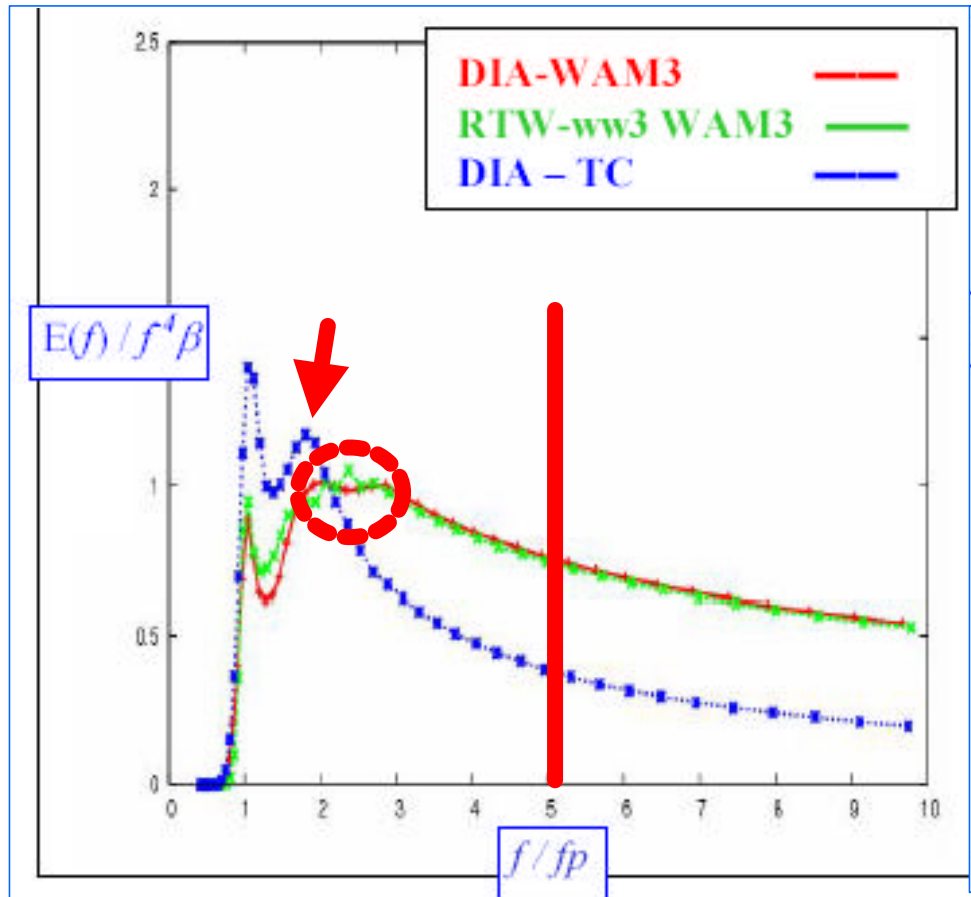
Source terms allow equilibrium range formation

Relation to $f_o \dots ?$

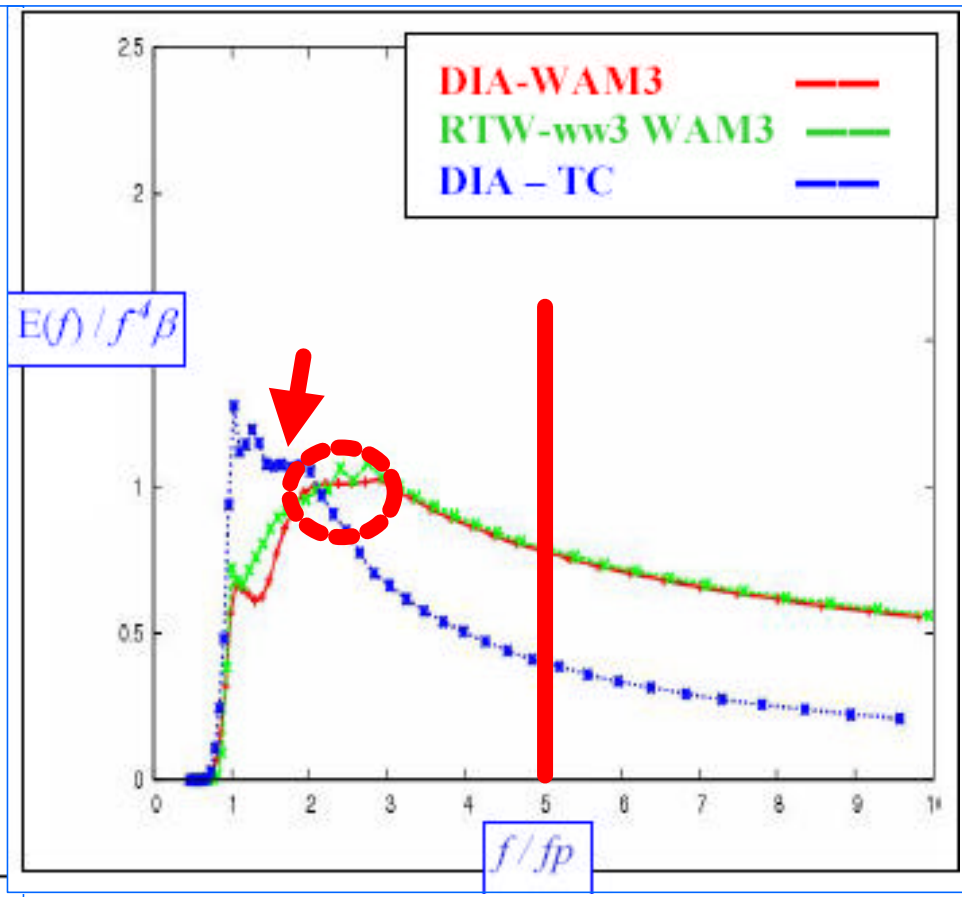
Resio et al. 2004



Evaluation using standard source terms



After 5 hr simulation

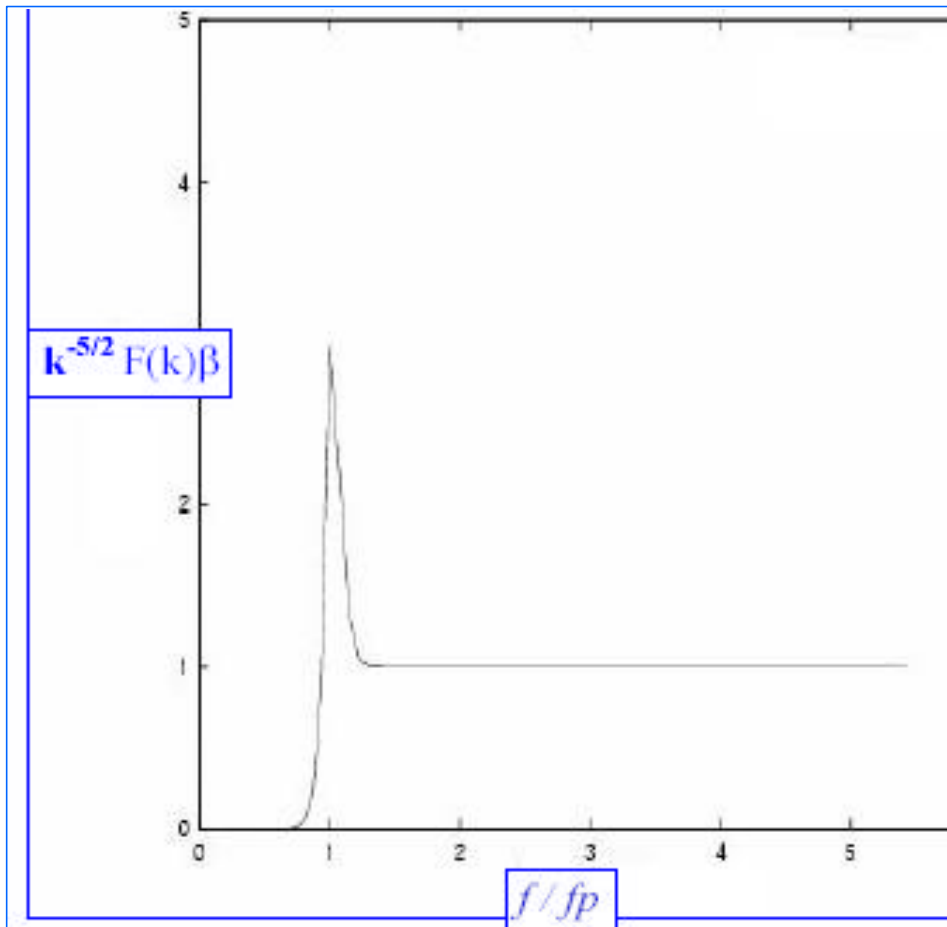


After 30 hr simulation

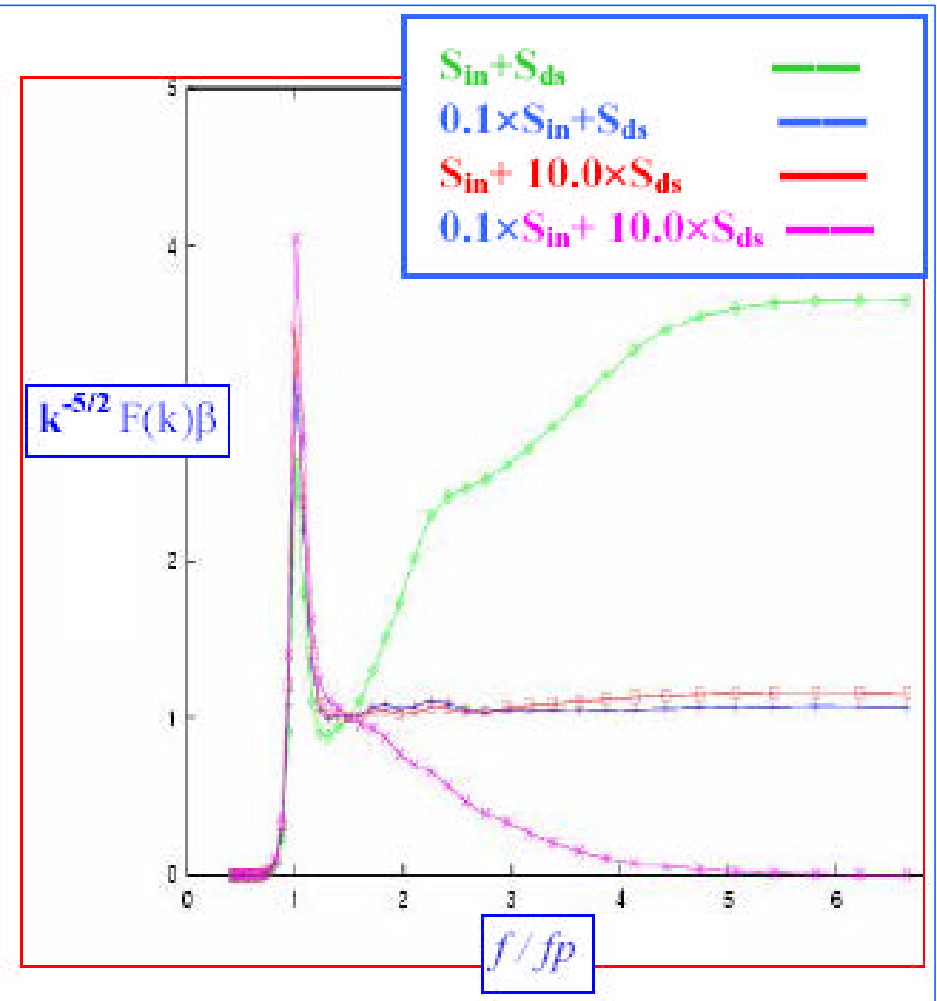
“dip” at $\sim 1.2 f_p$

limited equilibrium range ...!

Comparison with RTW-erdc integrations



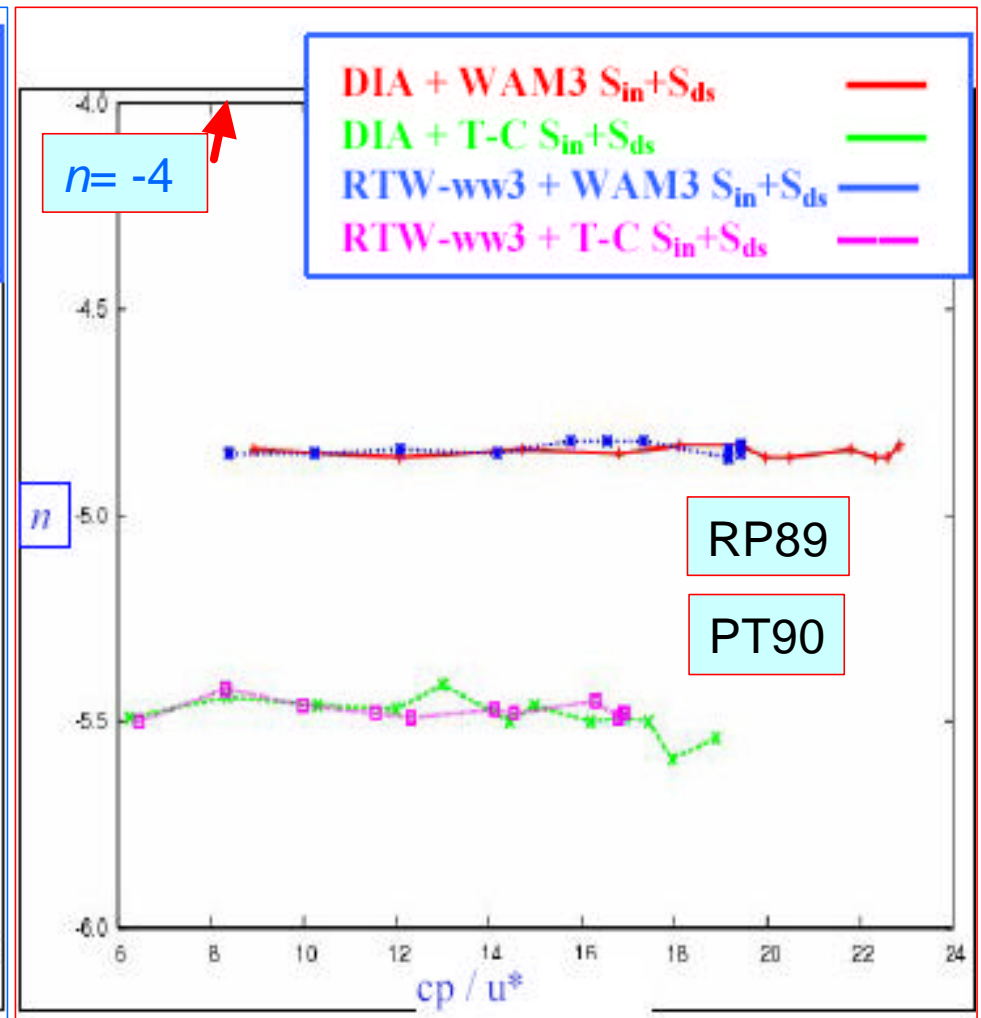
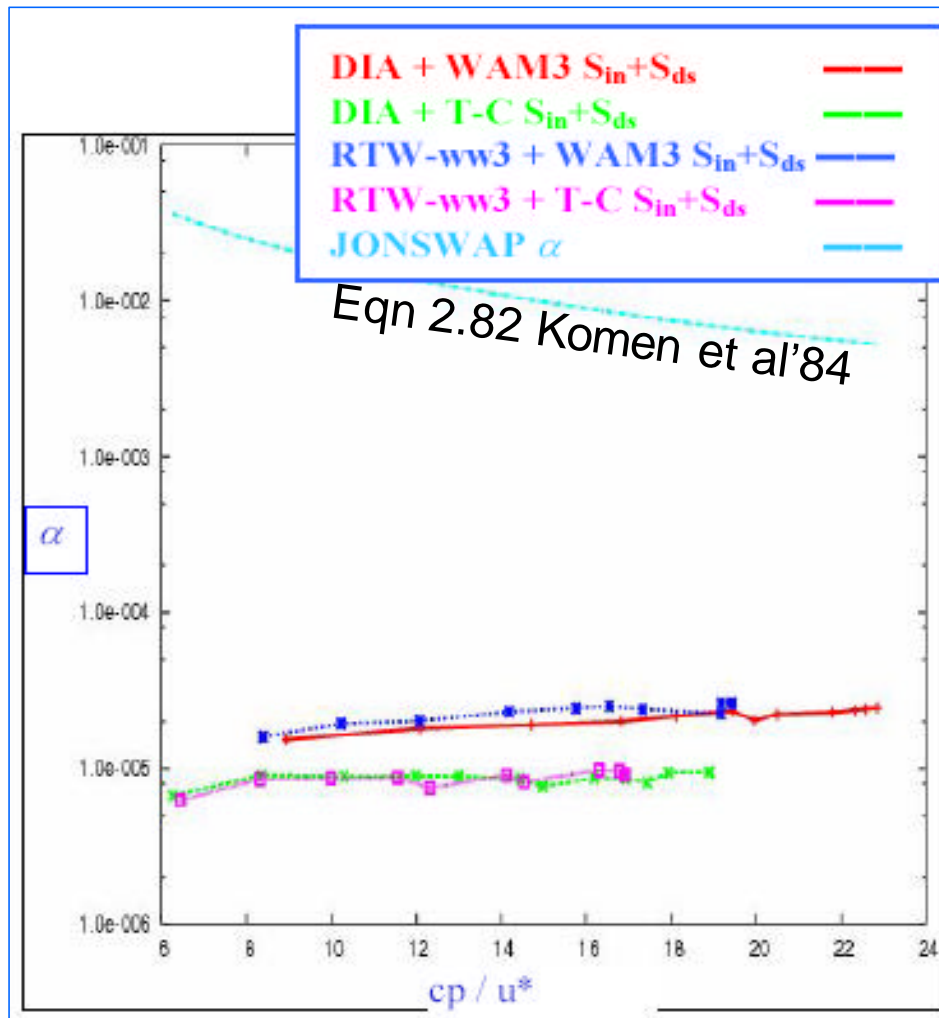
Normalized $F(k)$ with JONSWAP-type input:
 $a = 0.01$, $U_{10} = 15$ m/s, $f_p = 0.1$, $\gamma = 3$



RTW-erdc for S_{ni} ; WAM3 $S_{in} + S_{ds}$
1-hr simulation

source terms adjustments ... !

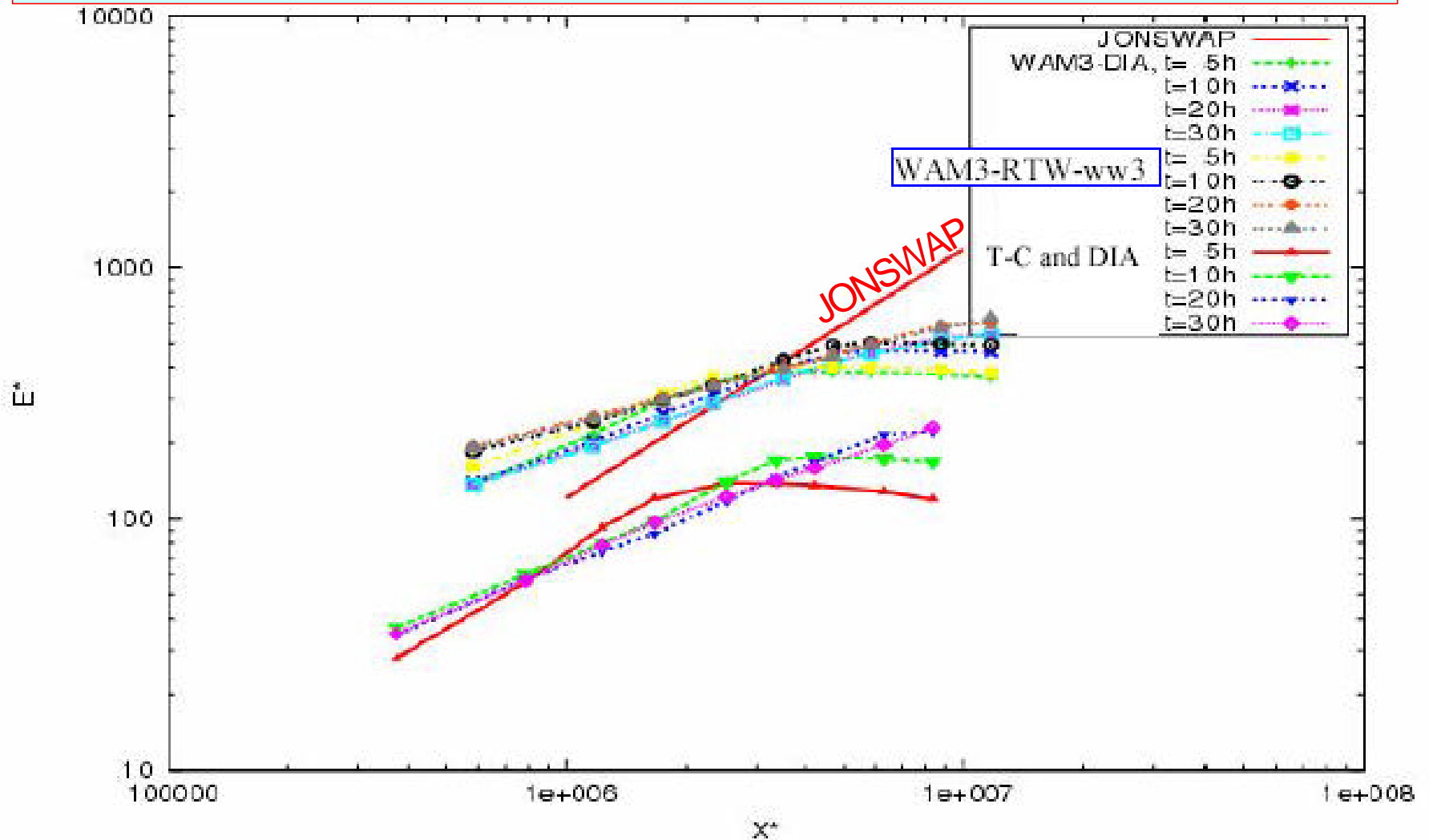
4. challenges for improvements



For a and “ n ” results ... **issues!**

WAM3 $S_{in} + S_{ds}$ give one family of curves, T-C give another; results not dependent on DIA or RTW-ww3

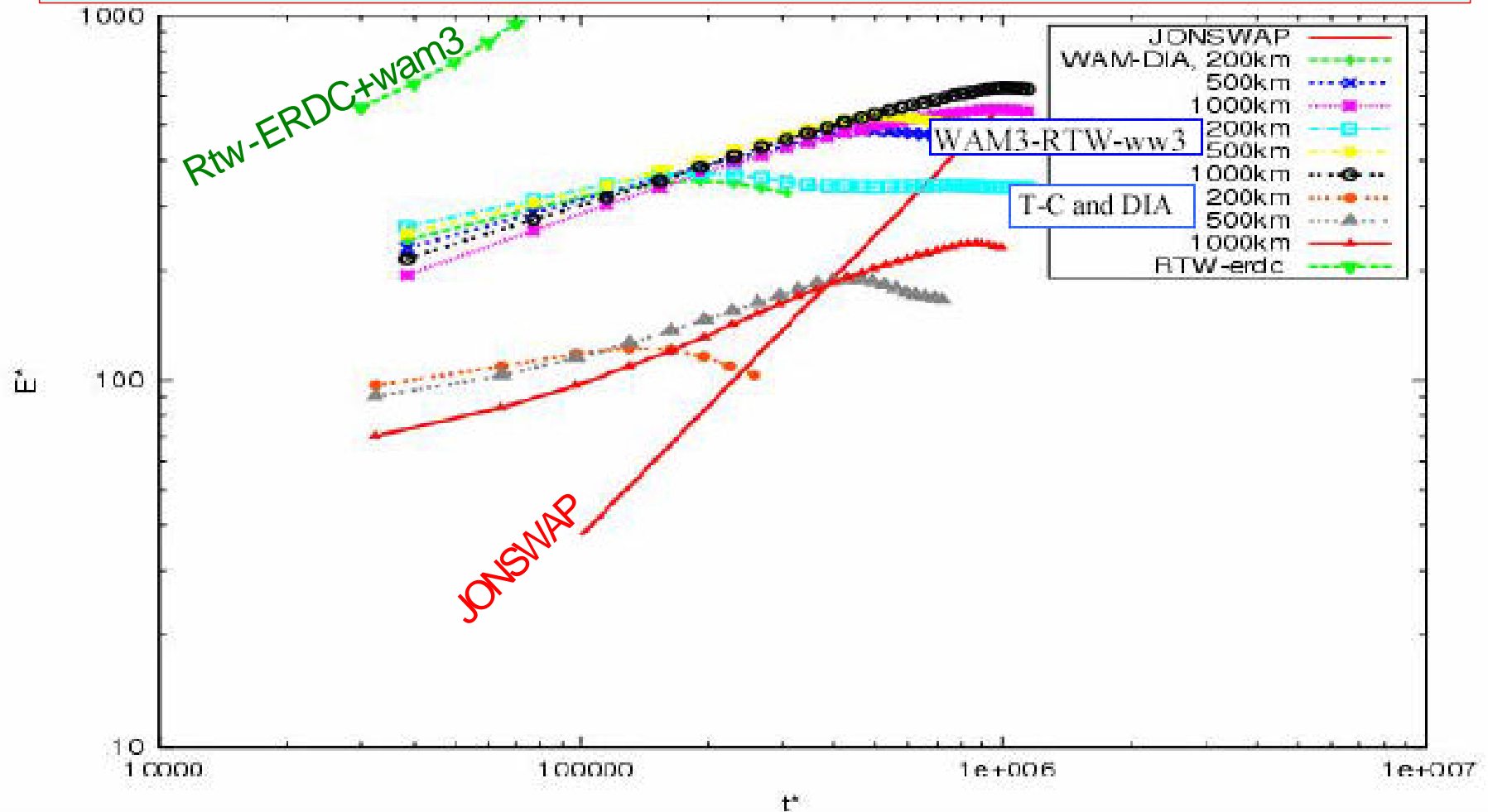
Parametric balance; E^* vs. x^*



WAM3 Sin + Sds give one family of curves, T-C give another;
results not dependent on DIA or RTW-ww3

DIFFERENCES in slopes compared to JONSWAP, as a function of t ... !

Parametric balance; E^* vs. t^*



WAM3 Sin + Sds give one family of curves, T-C give another;
results not dependent on DIA or RTW-ww3

DIFFERENCES in slopes wrt JONSWAP, as a function of x ... !

Offset for RTW-erdc + WAM3 source terms

conclusions

1. Serious limitations in DIA (RTW-ww3) for simple peaked or double peaked / sheared spectra (tail, low- f distortion)
2. DIA impacts f_o – the null frequency for equilibrium range – which defines energy retained, detailed balance...
3. Detailed balance – WAM3 and T-C source terms can support equilibrium range, with tuning factors
4. T-C and WAM3 Sin+ Sds determine values for a and “ n ”, not DIA or RTW-ww3: and differ from JONSWAP / Toba
5. Fetch / duration curves dominated by WAM3 or T-C source terms, not DIA or RTW-ww3 formulation
6. “ “ curves too shallow and not self-similar.
7. RTW-erdc + WAM3 has improved slope, but offset