The Transformation of Deep Water Wave Hindcasts to Shallow Water

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How I Created a Frankenstein Hindcast Out of Your Wonderful Deep Water Hindcast

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Background

- Long-term shallow water wave data important for design and operation of coastal facilities and the safe performance of human activities in coastal areas.
- Clients during the Preliminary Feasibility Studies want the “60 sec” answer.
- Engineers are usually confronted with insufficient or no wave data → use of wave hindcasts.
- Wave hindcasts are often available in deep water and away from our project site.
- The “ideal” wave hindcast for coastal projects:
  - Long-term (i.e. 30 years, every 3 hours)
  - Shallow water
  - Include “Basin” scale events (long period swell of distant storms)
  - Include “Local” wind-generated, short period waves
  - Account for wave transformation process due to the bathymetry and the presence of islands, headland, shoals and protective structures.
- The “ideal” shallow water wave hindcast is often expensive or impractical for many projects.

Methodology

To achieve the “Ideal” wave hindcast:

1. Deep water wave hindcast including “basin” and “local” events.
2. Wave transformation model, with a deep water boundary, to model the wave propagation to shallow water.
3. Linear wave theory assumption
   a) $H_{sw} = H_{dw} \times f(T_{dw}, MWD_{dw}, \text{Water Level}) \leftarrow Ct = Cr \times Cs \times Cd$
   b) $MWD_{sw} = f(T_{dw}, MWD_{dw}, \text{Water Level})$
   c) $T_{sw} = T_{dw}$
4. A data fitting technique to develop transfer functions between deep and shallow water wave conditions.

Methodology (cont.)

1. Select from a Frequency of Occurrence (or Probability) Table ranges of most likely deep water T and MWD and define an input wave condition matrix for modeling
2. Define maximum, minimum and mean sea levels.
3. Compute, with a wave transformation model, wave heights and directions at location of interest for deep water H=1, T and MWD (from matrix) and water levels.
4. Compute Ct and MWD_{sw} transfer functions with splines.
5. Compute the shallow water hindcast by applying the transfer functions to each point of the deep water hindcast.

Applicable to seas and swell independently and compute the resultant “total” wave height as:

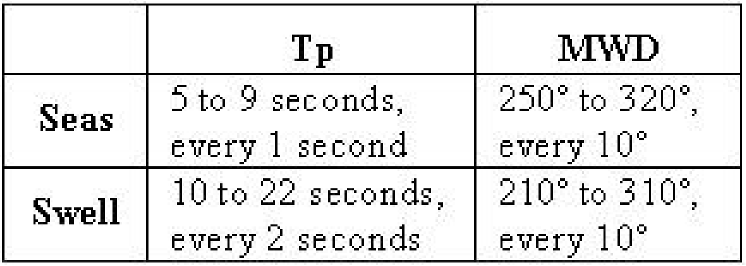
$$H_{total} = (H_{swell}^2 + H_{seas}^2)^{1/2}$$

Dominant wave period and direction based on the highest wave height fo the two components.

Case Study

- Imperial Beach, California.
- Coastal Data Information Program (CDIP) pressure array wave measurements from 1983 to 1996 in 10.4 m (MLLW).
- Bathymetry digitized from charts by Continental Shelf Data Systems (1971).
- Water levels from CDIP measurements.
- GROW deep water wave hindcast (seas and swell partitions, every 3 hours).
- MIKE 21 PMS (parabolic mild slope) wave transformation model.

Model Setup



	Tp	MWD
Seas	5 to 9 seconds, every 1 second	250° to 320°, every 10°
Swell	10 to 22 seconds, every 2 seconds	210° to 310°, every 10°

Water Levels (MLLW)	
Maximum	2.4m
Mean	0.90m
Minimum	-0.53 m

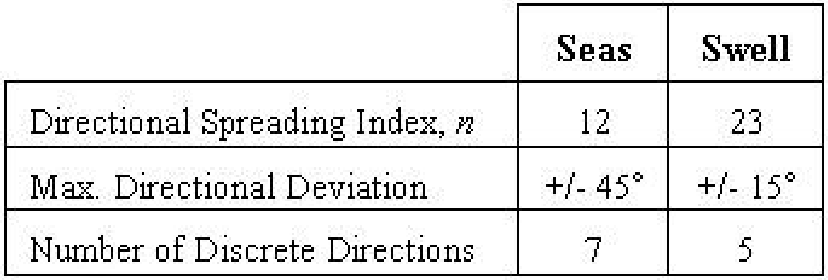
5 Tp x 11 MWD x 3 WL = **165 seas conditions**

7 Tp x 8 MWD x 3 WL = **168 swell conditions**

Spectra

Seas = Pierson-Moskowitz

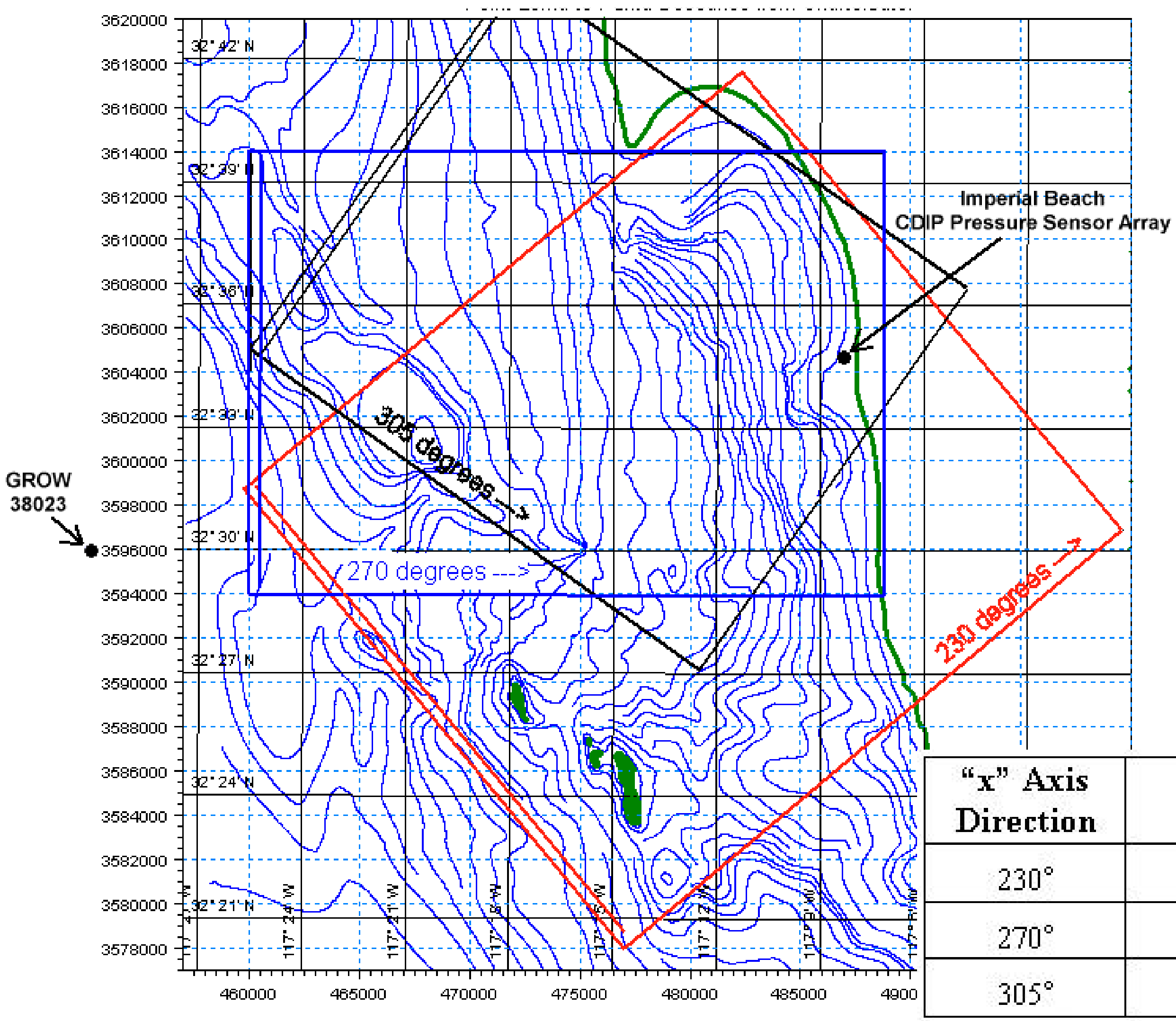
Swell = Jonswap, $g = 9$



	Seas	**Swell**
Directional Spreading Index, *n*	12	23
Max. Directional Deviation	+/- 45°	+/- 15°
Number of Discrete Directions	7	5

Input wave conditions matrix covered approximately 80% of occurrences.

Model Grids



$\Delta x = \Delta y = 10$ m

"x" Axis Direction	Seas Directions	Swell Directions
230°	250°	210° to 250°
270°	260° to 280°	260° to 280°
305°	290° to 320°	290° to 310°

Modeling Results

Deep Water MWD	Tp (sec)								
	2	3	4	5	6	7	8	9	10
240	0.99	0.96	0.92	0.89	0.86	0.83	0.81	0.78	0.76
250	1.00	0.97	0.95	0.91	0.88	0.85	0.83	0.81	0.78
260	1.00	0.97	0.94	0.92	0.89	0.86	0.84	0.83	0.81
270	1.00	0.96	0.94	0.93	0.90	0.87	0.85	0.84	0.82
280	1.00	0.99	0.96	0.93	0.90	0.88	0.86	0.84	0.82
290	1.00	0.95	0.92	0.87	0.84	0.82	0.81	0.80	0.79
300	0.97	0.89	0.85	0.80	0.76	0.74	0.73	0.72	0.71
310	0.94	0.84	0.75	0.68	0.64	0.61	0.61	0.61	0.61
320	0.72	0.64	0.58	0.54	0.50	0.48	0.47	0.48	0.49
330	0.55	0.50	0.46	0.42	0.37	0.36	0.34	0.34	0.34

Ct at MSL for seas

Deep Water MWD	Tp (sec)								
	8	10	12	14	16	18	20	22	24
200	0.30	0.29	0.29	0.35	0.51	0.64	0.60	0.62	0.66
210	0.44	0.42	0.40	0.48	0.59	0.64	0.60	0.61	0.62
220	0.56	0.56	0.56	0.64	0.68	0.65	0.59	0.56	0.54
230	0.63	0.69	0.73	0.77	0.75	0.70	0.65	0.61	0.56
240	0.78	0.79	0.81	0.77	0.73	0.72	0.73	0.72	0.71
250	0.78	0.80	0.82	0.83	0.86	0.92	0.92	0.94	0.95
260	0.79	0.79	0.80	0.82	0.86	0.88	0.92	0.96	0.97
270	0.84	0.81	0.78	0.79	0.86	0.91	0.95	0.97	0.99
280	0.84	0.84	0.85	0.85	0.86	0.89	0.95	0.96	0.98
290	0.84	0.86	0.88	0.93	0.98	1.02	1.04	1.05	1.07
300	0.66	0.73	0.80	0.85	0.94	1.02	1.10	1.17	1.25
310	0.56	0.61	0.67	0.71	0.77	0.88	1.00	1.13	1.23
320	0.46	0.51	0.59	0.60	0.63	0.76	0.86	1.01	1.14

Ct at MSL for swell

Deep Water MWD	Tp (sec)								
	2	3	4	5	6	7	8	9	10
240	201	215	225	234	240	245	247	249	251
250	230	237	242	247	251	254	257	260	260
260	269	267	265	265	266	267	268	268	268
270	270	270	271	272	272	273	273	273	273
280	270	273	276	278	279	279	278	278	278
290	295	293	292	291	289	287	285	283	281
300	299	298	297	295	293	290	288	286	284
310	303	300	300	298	296	294	291	289	286
320	305	304	303	301	299	297	294	291	288
330	310	308	306	305	302	300	296	294	290

MWD at MSL for seas

Deep Water MWD	Tp (sec)								
	8	10	12	14	16	18	20	22	24
200	240	247	253	258	261	263	264	264	265
210	244	250	255	259	262	263	264	265	265
220	250	254	258	261	263	264	265	265	265
230	254	257	260	262	264	265	266	266	266
240	257	260	263	264	265	266	267	267	267
250	261	264	266	267	268	268	269	269	269
260	268	269	269	270	270	270	270	271	271
270	273	273	272	272	271	271	271	271	271
280	278	277	276	275	274	274	273	273	273
290	283	281	279	277	277	276	275	275	275
300	287	285	282	279	278	277	277	277	276
310	290	287	284	282	280	279	278	277	276
320	293	289	286	284	281	280	279	278	276

MWD at MSL for swell

Water Level Effects (Max-Min)

Deep Water MWD	Tp (sec)				
	5	**6**	**7**	**8**	**9**
250	3%	3%	4%	2%	1%
260	4%	5%	2%	2%	1%
270	3%	2%	2%	1%	1%
280	2%	2%	1%	0%	0%
290	4%	2%	2%	0%	-1%
300	4%	3%	3%	1%	-1%
310	6%	6%	5%	2%	-2%
320	6%	6%	4%	2%	0%

? Ct for seas

Deep Water MWD	Tp (sec)				
	5	**6**	**7**	**8**	**9**
250	-1	-2	-2	-3	-3
260	-2	-2	-2	-3	-2
270	-1	-1	-1	-1	-1
280	0	-1	-1	0	0
290	0	0	0	0	0
300	1	1	1	1	1
310	1	2	2	2	2
320	1	2	2	2	2

?MWD (deg) for seas

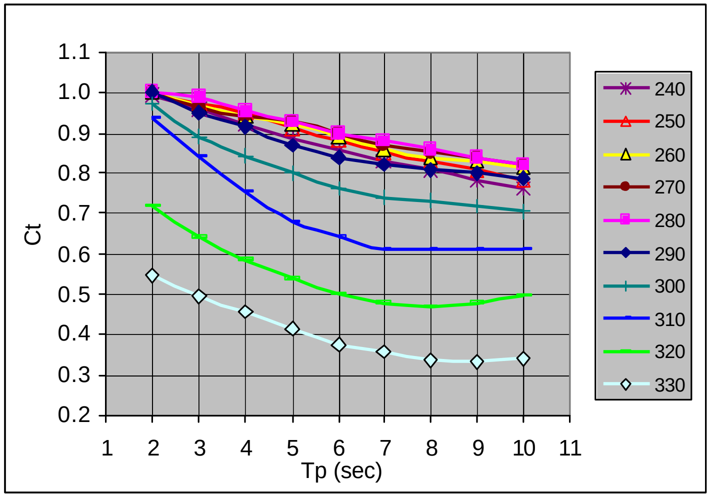
MWD	10	12	14	16	18	20	22
210	13%	5%	-8%	-5%	-2%	2%	-3%
220	5%	-2%	-3%	0%	-2%	0%	-7%
230	1%	-1%	-4%	-3%	-4%	-3%	-5%
240	0%	-1%	0%	-3%	-3%	-3%	-3%
250	0%	-1%	-1%	-1%	-3%	0%	-4%
260	-1%	-4%	-5%	-6%	-5%	-5%	
270	1%	0%	-1%	-2%	-4%	-4%	-4%
280	0%	-1%	0%	1%	0%	-1%	-3%
290	-3%	-3%	-4%	-4%	-3%	-1%	0%
300	-4%	-5%	-6%	-6%	-6%	-5%	
310	-5%	-6%	-4%	-8%	-9%	-9%	-8%

?Ct for swell

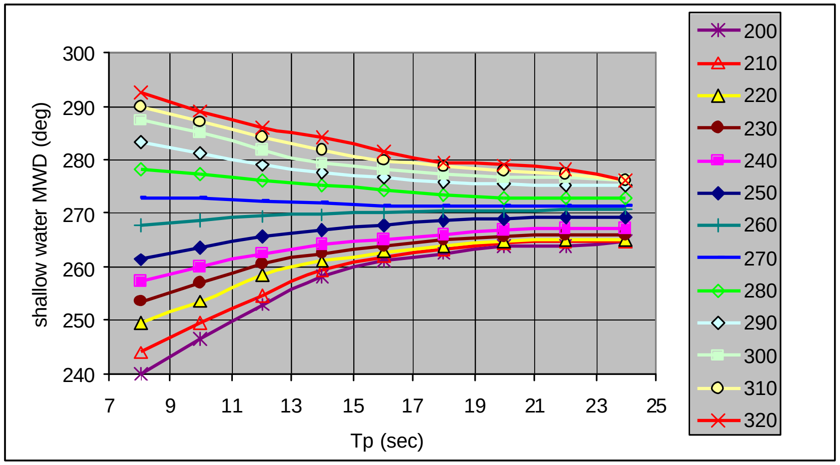
Deep Water MWD	Tp (sec)						
	10	**12**	**14**	**16**	**18**	**20**	**22**
210	-7	-6	-5	-3	-3	-4	-4
220	-6	-5	-4	-4	-3	-3	-3
230	-5	-4	-4	-3	-3	-3	-3
240	-4	-3	-3	-3	-3	-3	-3
250	-2	-2	-2	-2	-2	-2	-2
260	-2	-2	-2	-2	-2	-2	
270	-6	-5	-4	-3	-3	-3	-3
280	-1	-1	-1	-1	-1	-1	-1
290	0	0	0	-1	-1	-1	0
300	1	1	0	0	-1	0	
310	2	1	1	1	0	0	0

?MWD (deg) for swell

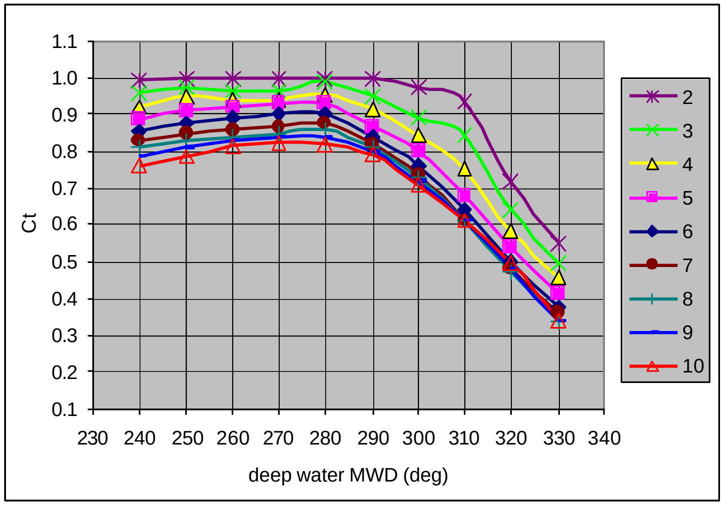
Transfer Functions



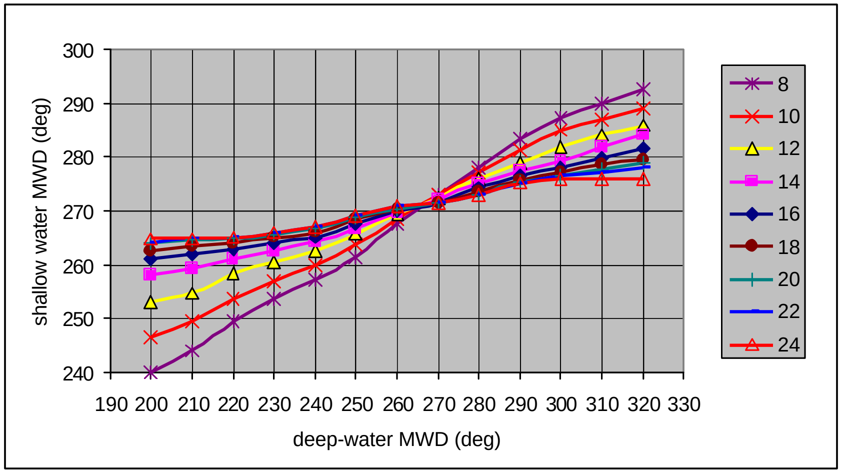
Ct versus Tp for seas at MSL



MWD versus Tp for swell at MSL



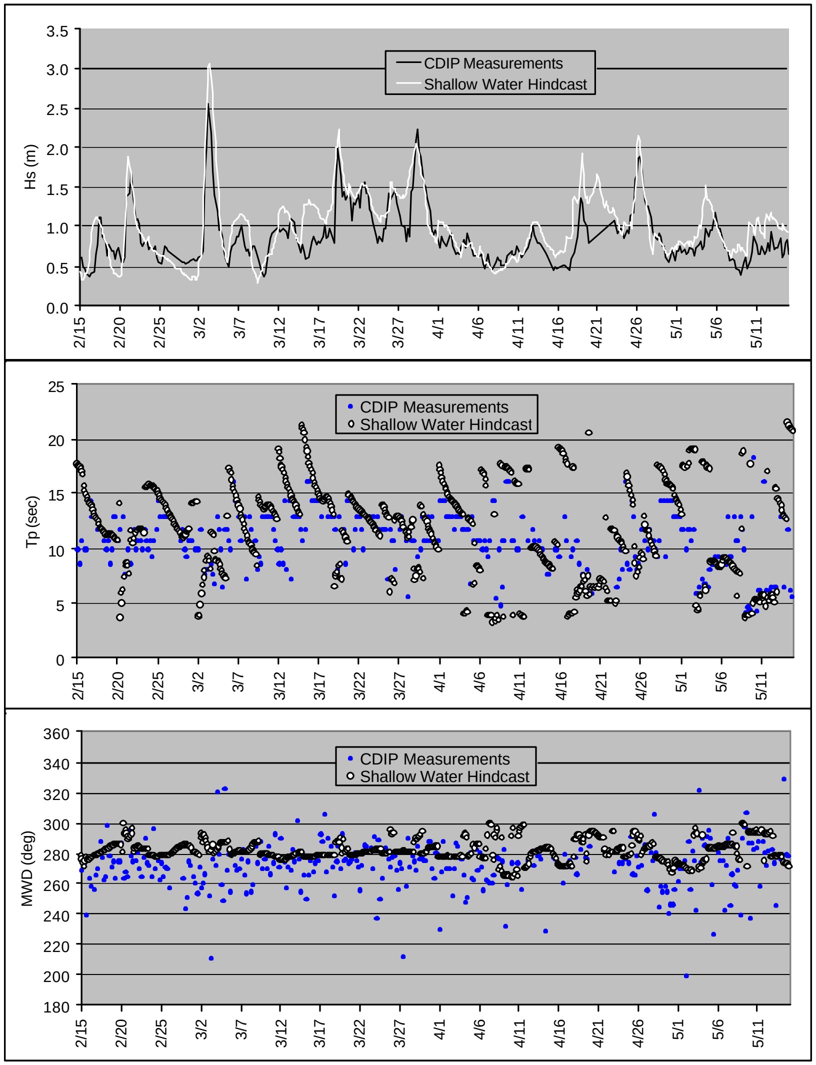
Ct versus deep water MWD for seas at MSL



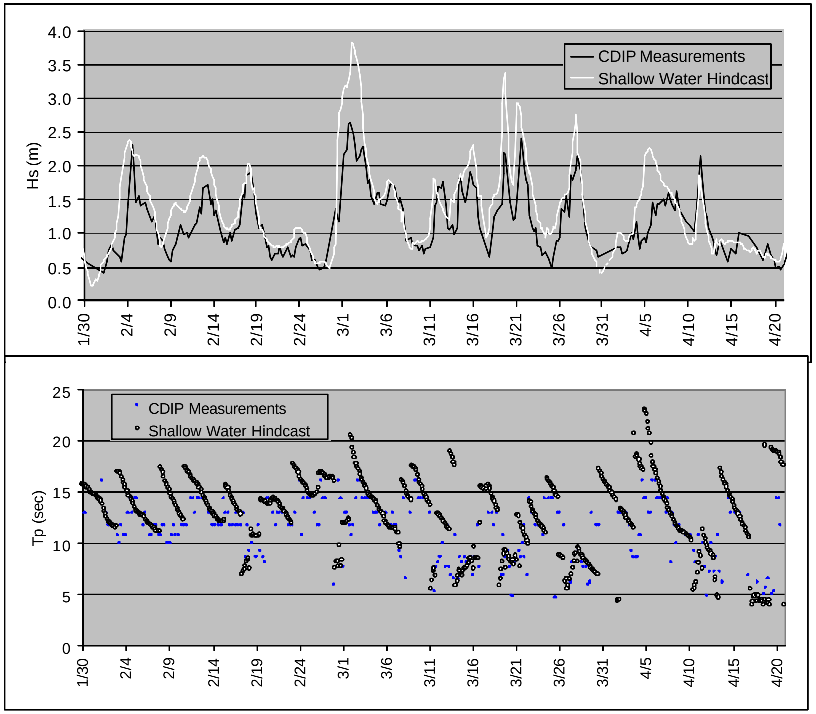
MWD versus deep water MWD for swell at MSL

Storm Time Series

March-May 1985

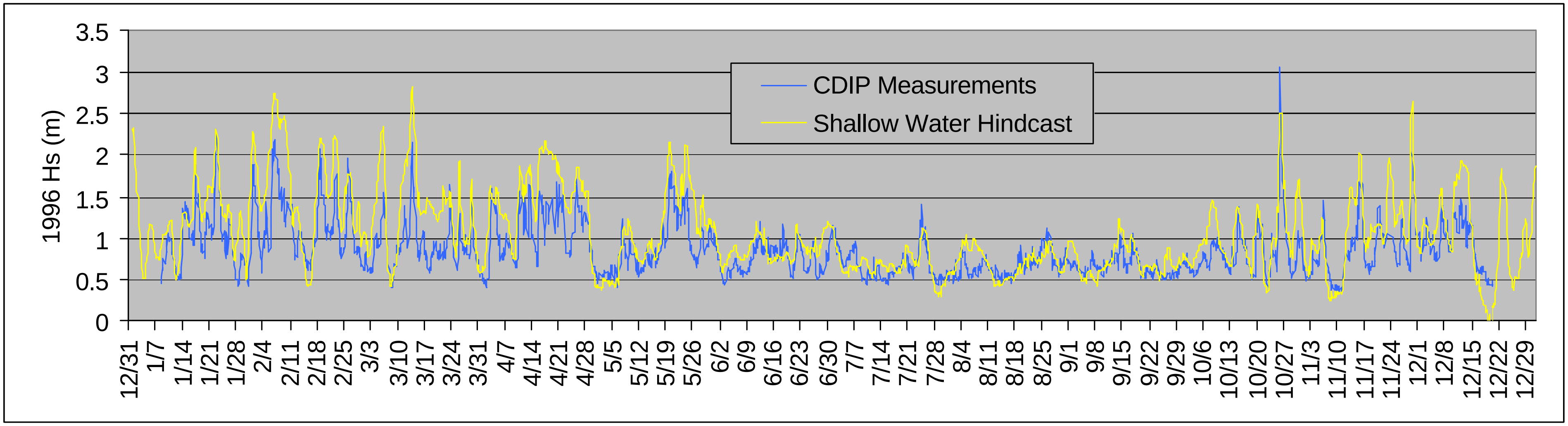


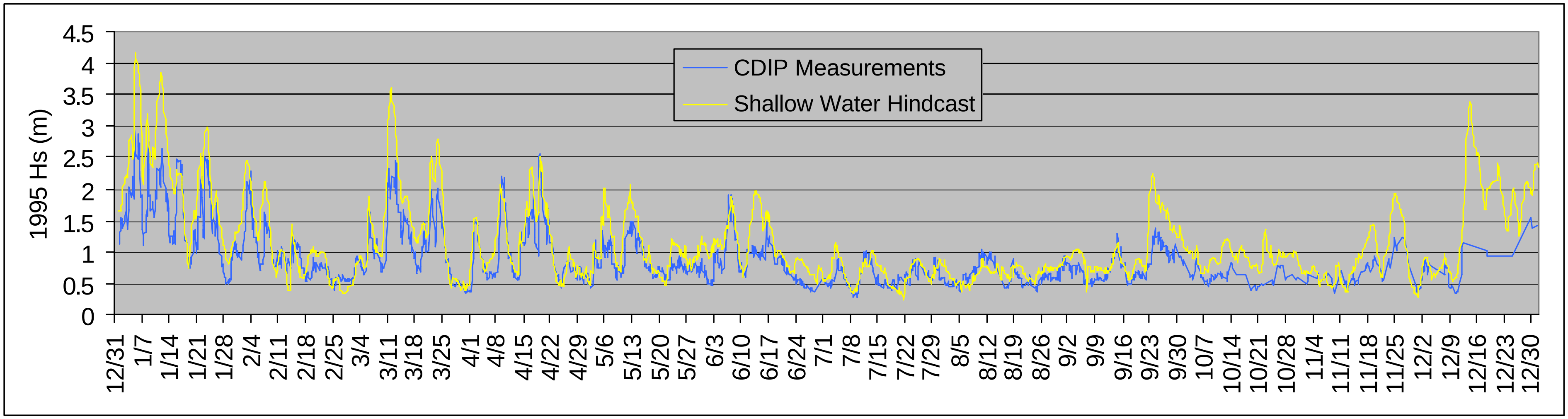
January-April 1991



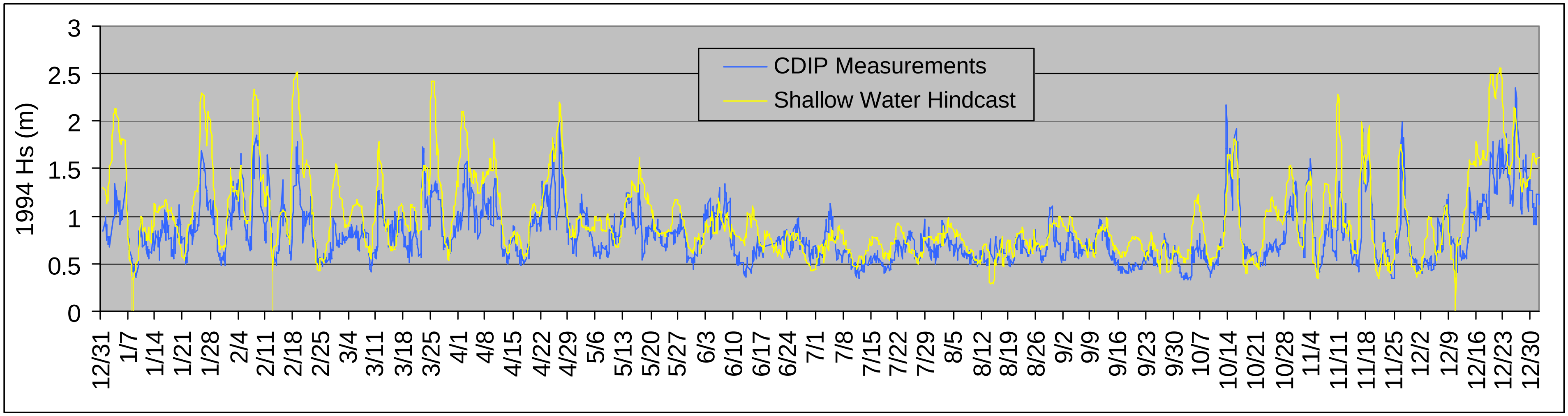
- **Good in general, but not good match for high waves.**
- **Long period waves arriving before storms, maximum measured Tp ~ 18 sec, discrepancies high when hincast Tp is high.**
- **Measured wave direction noisy, no conclusion about hindcast.**

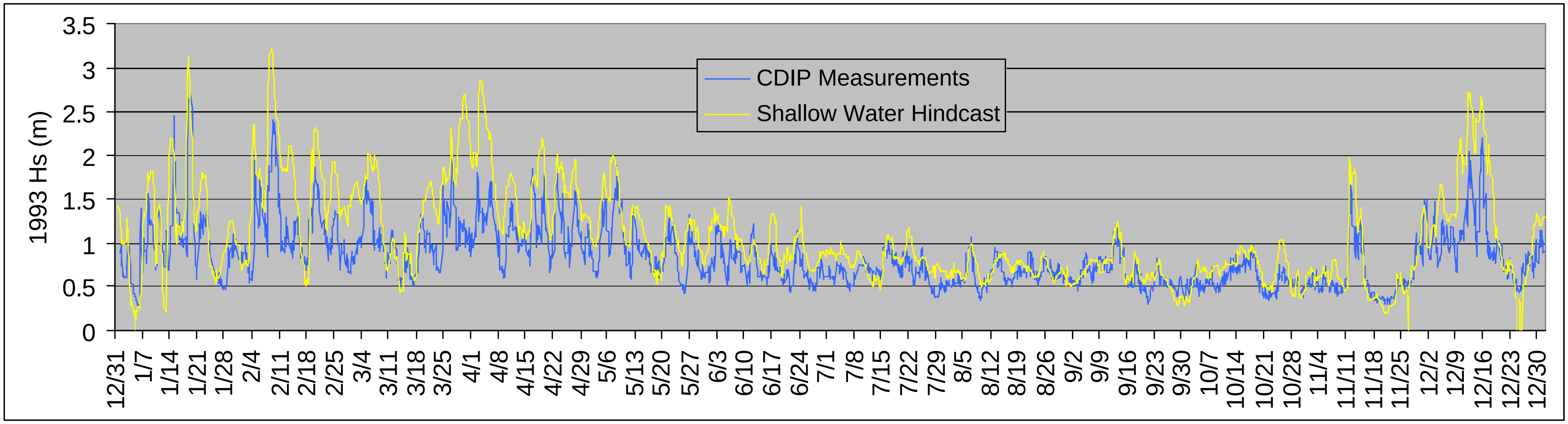
Annual H_s Time Series



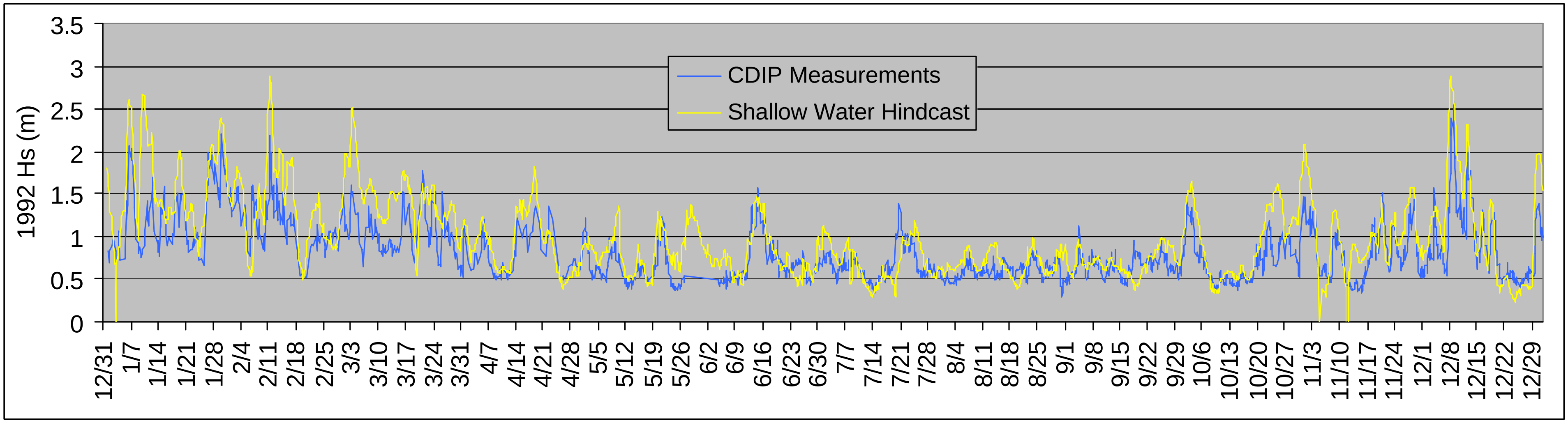


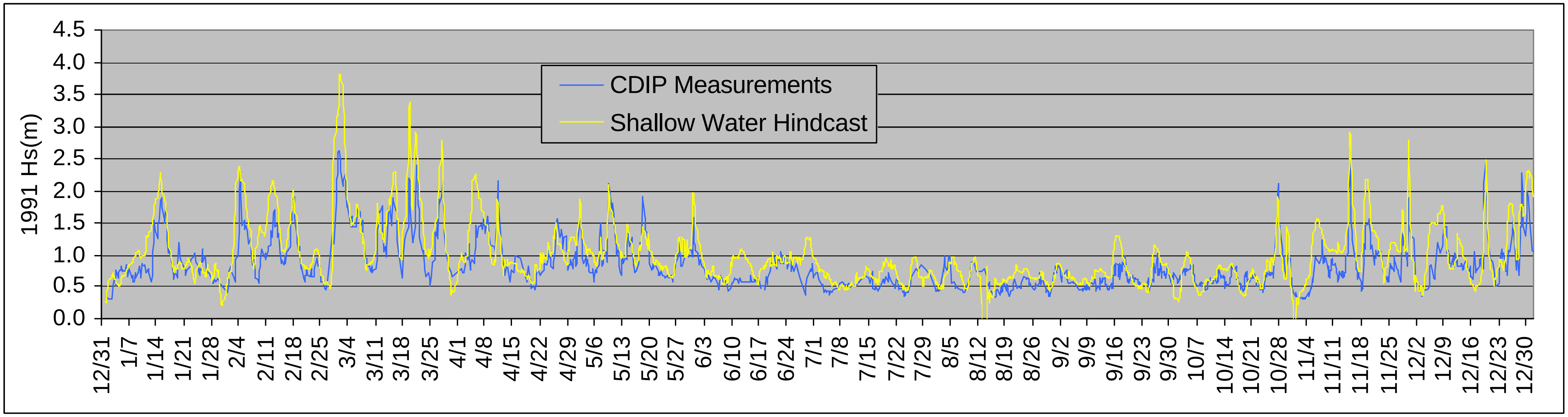
Annual H_s Time Series (cont.)



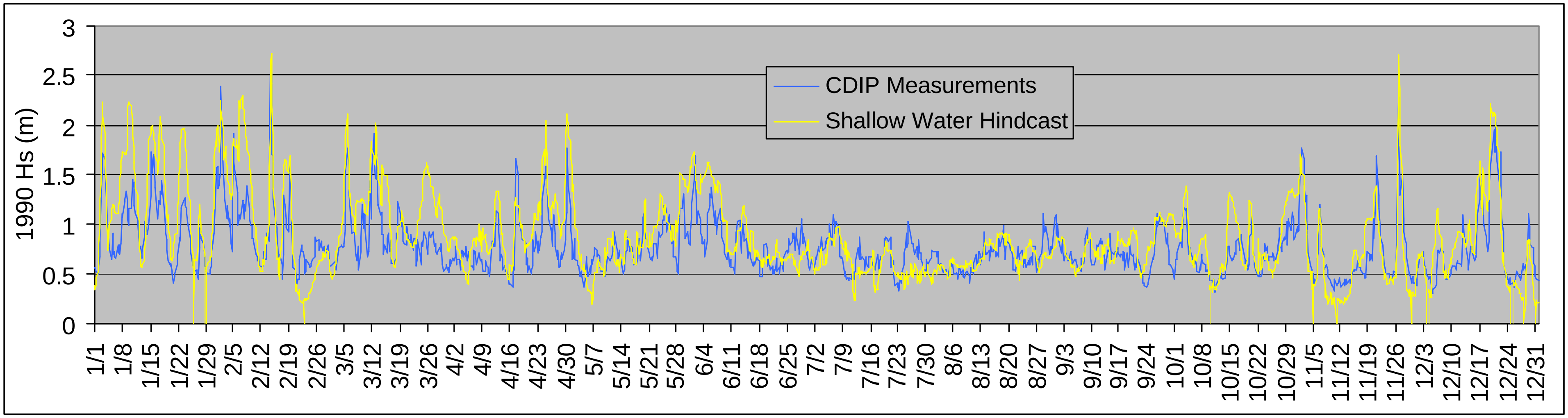


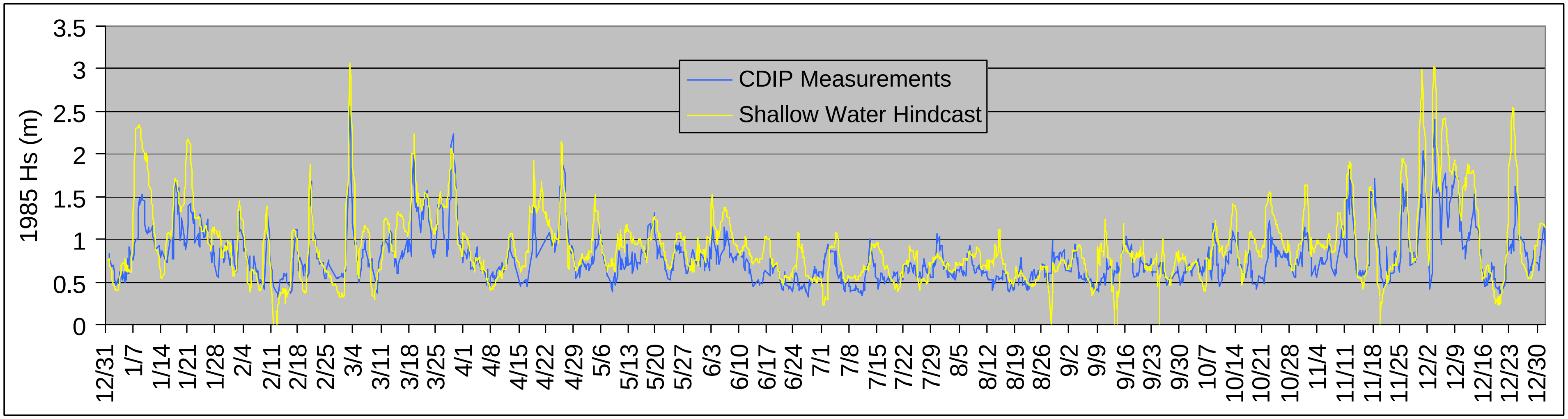
Annual H_s Time Series (cont.)



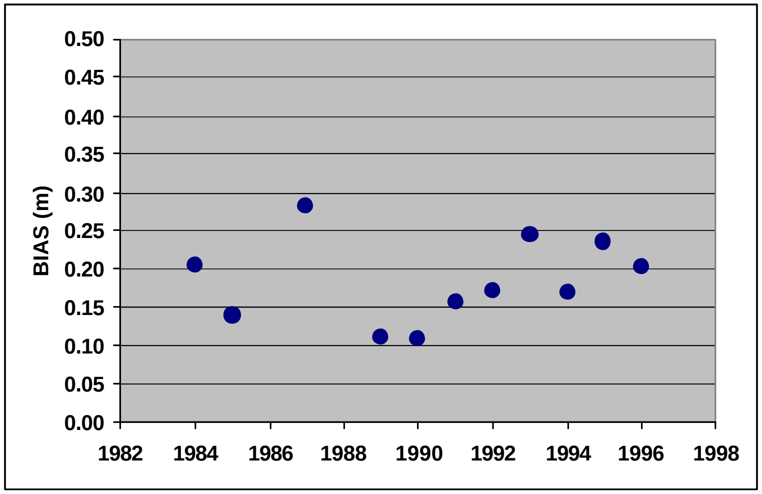


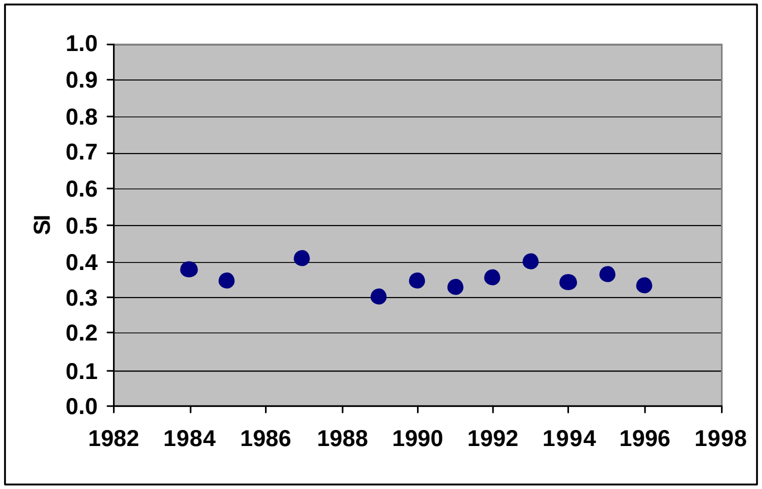
Annual H_s Time Series (cont.)

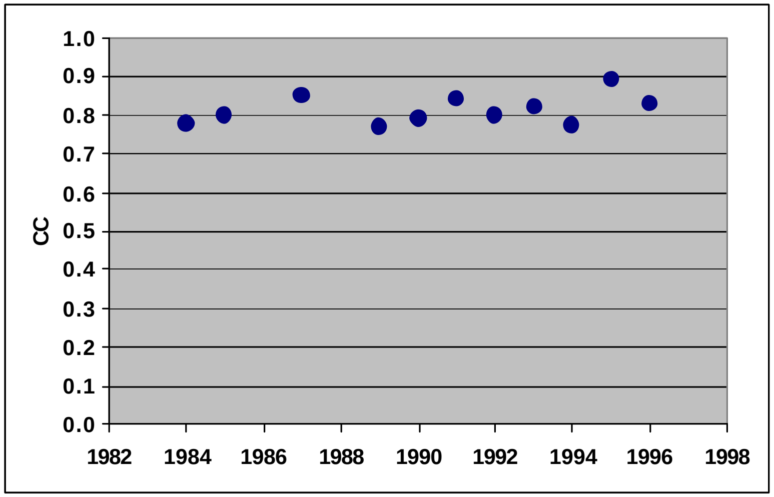


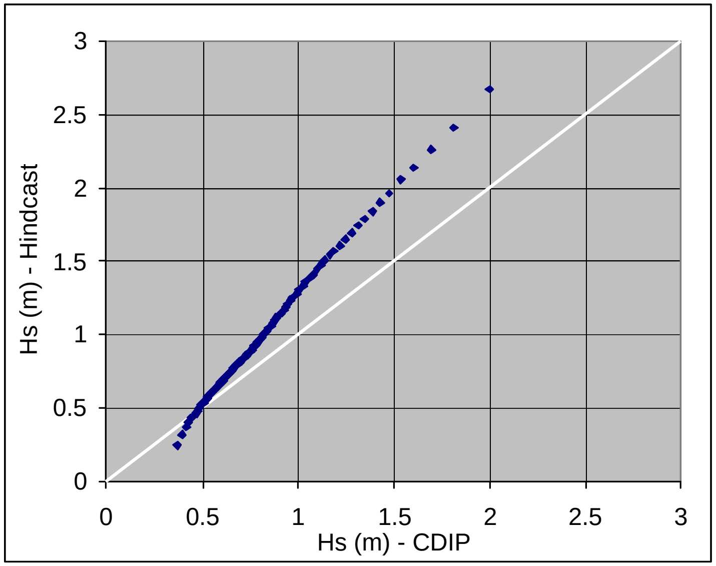


Bias, SI, CC and Q-Q Plots









A Hypothesis for Discrepancies

- Tested sensitivity of spectral shape, directional spreading, bottom friction….. Few % differences.

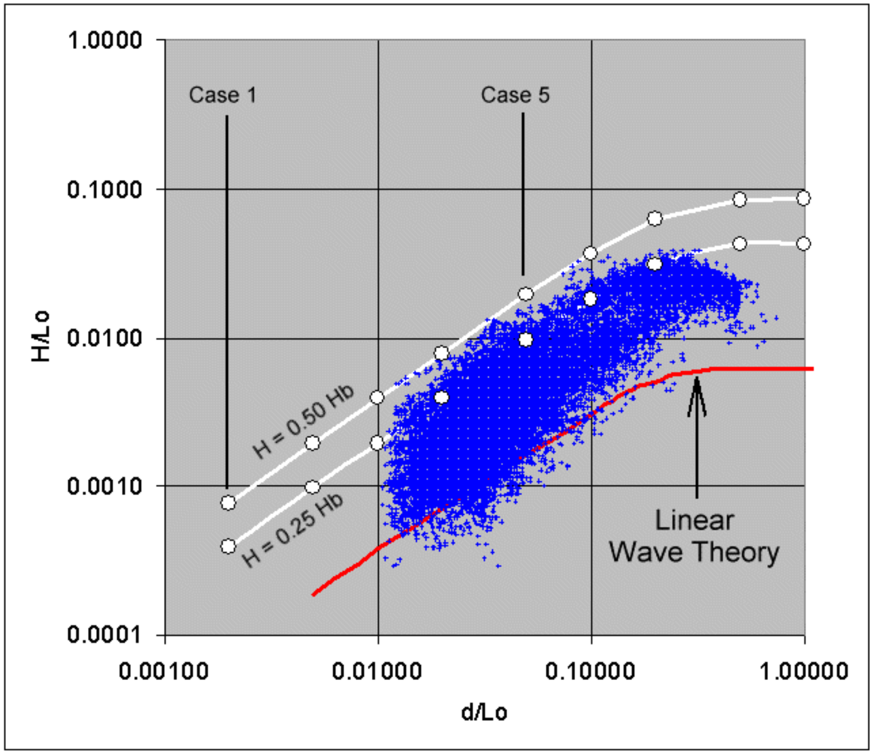
Some observations from the analysis:

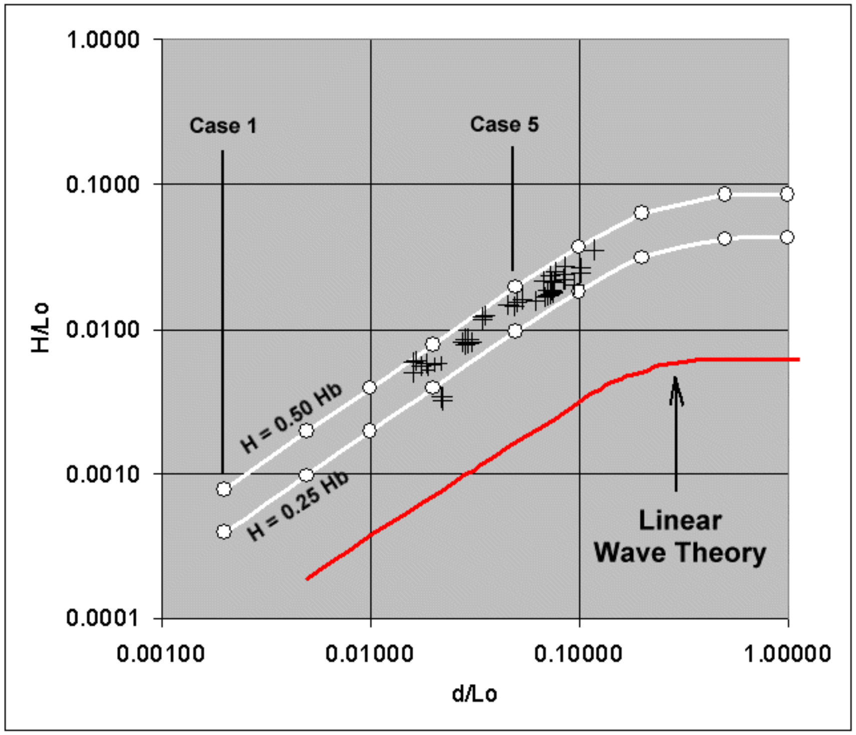
1. H_s time series indicated discrepancies during high, long period wave events.
2. Q-Q plots indicated discrepancies increased with wave height
3. Shallow water hindcast featured longer periods

Could the source the discrepancies be attributed to the fact that Linear Wave Theory, in the frequency domain was used to derive the shallow water non-linear waves from pressure measurements?

Non-Linear Waves Analysis

- Shallow water wave hindcast in terms of wave steepness, H/Lo, and relative depth, d/Lo
- Linear Wave Theory and Stream Function Theory limits (SPM, 1984 and Dean, 1974)





Wave Height Discrepancy

$$h = \frac{p}{r\,g\,K_p}$$

where ***p*** is the dynamic pressure and K_p is the transfer function

$$K_p = \frac{1}{\cosh\left(\frac{2p}{L}\,d\right)}$$

Literature is abundant in studies suggesting a “correction” to LWT approach is required

$$h = N\,\frac{p}{r\,g\,K_p}$$

$1 \leq \mathrm{N} \leq 1.35$

H/Hb = 0.25

		Cases				
	Units	**3**	**4**	**5**	**6**	**7**
d/Lo	-	0.01	0.02	0.05	0.1	0.2
H/Lo	-	0.0019	0.0039	0.0098	0.0183	0.0313
Lo	m	1042	521	208.4	104.2	52.1
T	sec	25.8	18.3	11.6	8.2	5.8
H	m	2.03	2.03	2.03	1.91	1.63
p/ r g	m	1.77	1.76	1.66	1.33	0.75
Kp	-	0.969	0.938	0.847	0.705	0.459
$H_{\text{Linear Wave}}$	m	1.83	1.88	1.95	1.88	1.64
$(H_{\text{Linear Wave}} - H) / H$		**-10%**	**-8%**	**-4%**	**-1%**	**1%**

H/Hb = 0.5

		Cases				
	Units	**3**	**4**	**5**	**6**	**7**
d/Lo	-	0.01	0.02	0.05	0.1	0.2
H/Lo	-	0.0039	0.0078	0.0195	0.0366	0.0625
Lo	m	1042	521	208.4	104.2	52.1
T	sec	25.8	18.3	11.6	8.2	5.8
H	m	4.05	4.05	4.06	3.82	3.26
p/ r g	m	3.14	3.11	3.02	2.54	1.53
Kp	-	0.969	0.938	0.847	0.705	0.459
$H_{\text{Linear Wave}}$	m	3.24	3.32	3.56	3.61	3.33
$(H_{\text{Linear Wave}} - H) / H$		**-20%**	**-18%**	**-12%**	**-6%**	**2%**

Wave Period Discrepancy

Frequency domain analysis with LWT transfer function K_p underpredicts the long period waves, shifting the Tp to lower periods

H/Hb = 0.25

		Cases				
	Units	**3**	**4**	**5**	**6**	**7**
d/Lo	-	0.01	0.02	0.05	0.1	0.2
H/Lo	-	0.0019	0.0039	0.0098	0.0183	0.0313
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p/ r g	m	1.77	1.76	1.66	1.33	0.75
Kp	-	0.969	0.938	0.847	0.705	0.459
$H_{\text{Linear Wave}}$	m	1.83	1.88	1.95	1.88	1.64
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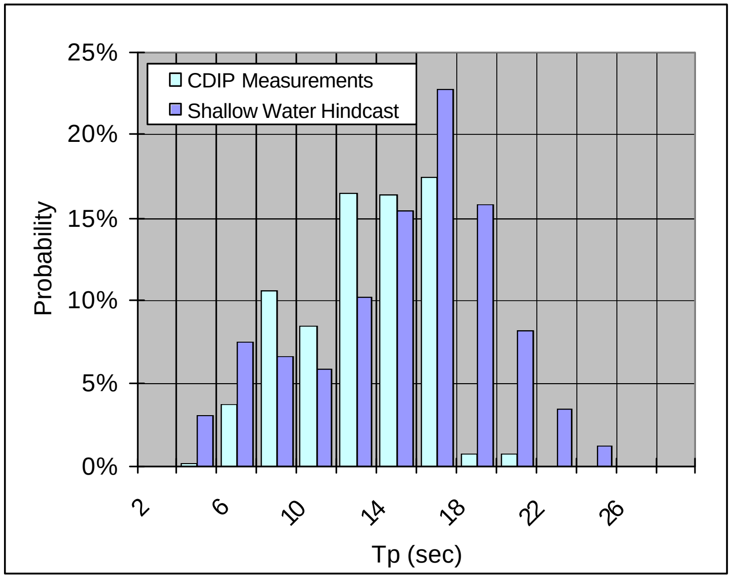


Previous studies have suggested that “Wave-by-Wave” analysis of measured waves may be more appropriate. The application of LWT K_p in the frequency domain to the non-linear waves is not correct since harmonics are not dispersive.

Case Study Conclusions

1. The shallow water wave hindcast showed good agreement for small wave height, short period waves…..
2. But not consistent with the measurements for high, long period waves.
3. Discrepancies attributed to the inherent limitations of LWT transfer function to compute *h* from *p*
4. The assumptions made for the spectral and directional spreading characteristics of NE Pacific Ocean seas and swell seem adequate.

Conclusions About the Method

- Simple and fast ("60 sec answer")
  - wave transformation, 5 days
  - transfer functions and shallow water hindcast, 1 day
- Derived hindcasts include "basin" and "local" events provided that input deep water wave hindcast is derived with a global wave model
- Transfer functions, no preliminary knowledge of functional form
- "Physics" in the transfer functions depend on the features of the wave transformation model used, i.e.:
  - refraction, diffraction, shoaling, white capping, wave breaking
  - bottom friction
  - wave-wave and wave-current interaction
- Suitable for computation of operational scenarios, persistence, seasonal effects, downtime, sediment transport, wave setup and runup…. response based methods in general.

Future Work

- Currently working in developing N-dimensional spline transfer functions
  - 1^{st} step, 4D transfer functions for H_{sw} and $MWD_{sw} = f(H_{dw}, T_{dw}, MWD_{dw}, \text{Water Level})$
  - 2^{nd} step, account for additional parameters such as spectral shape, directional spreading and spatial wave paremeter variations

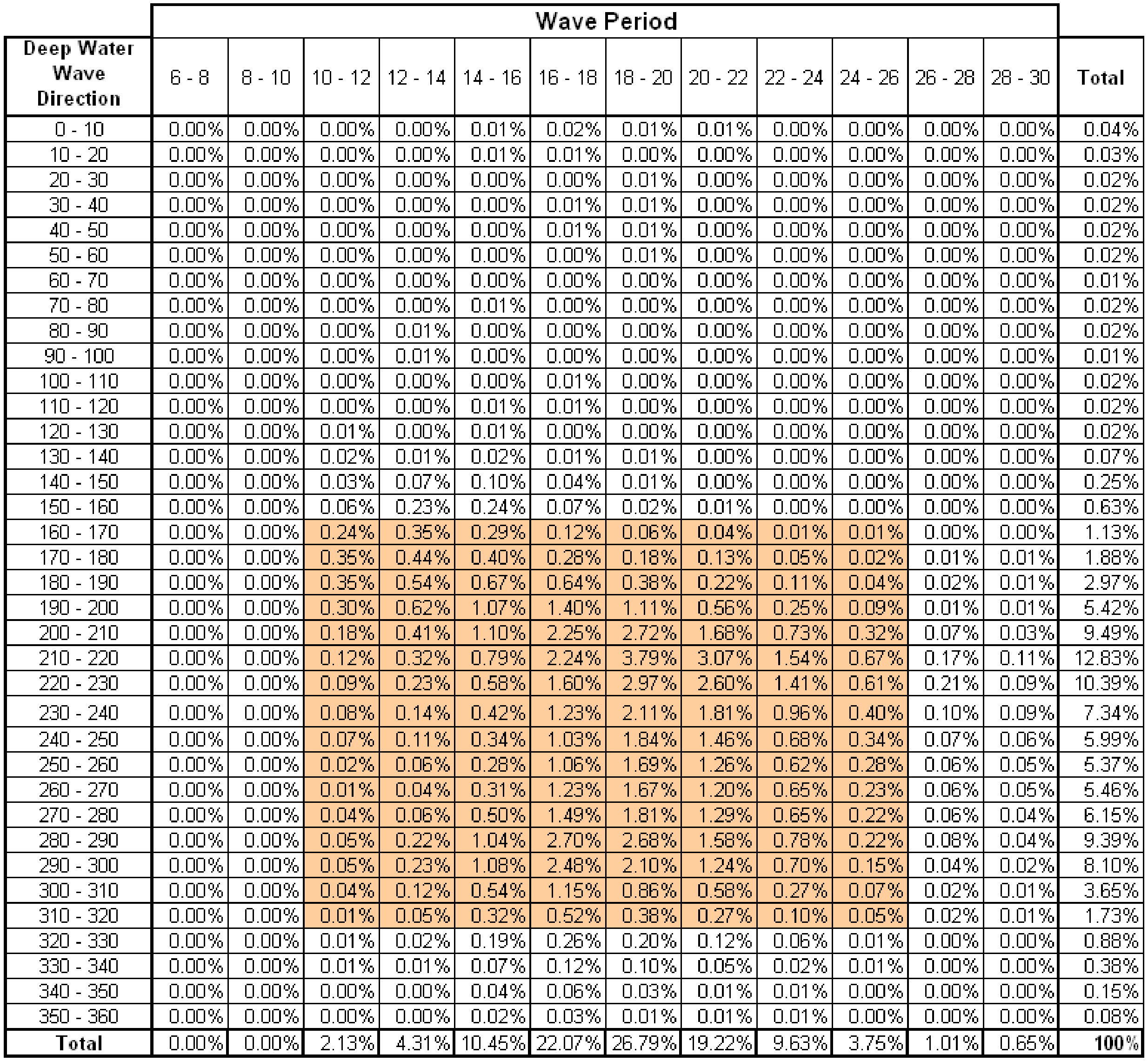
- Operational model: deep water buoy data input → transfer function → shallow water wave parameters.

- Validation…. Anybody with shallow water wave data and interested in collaborating in the validation please let me know!

Acknowledgements

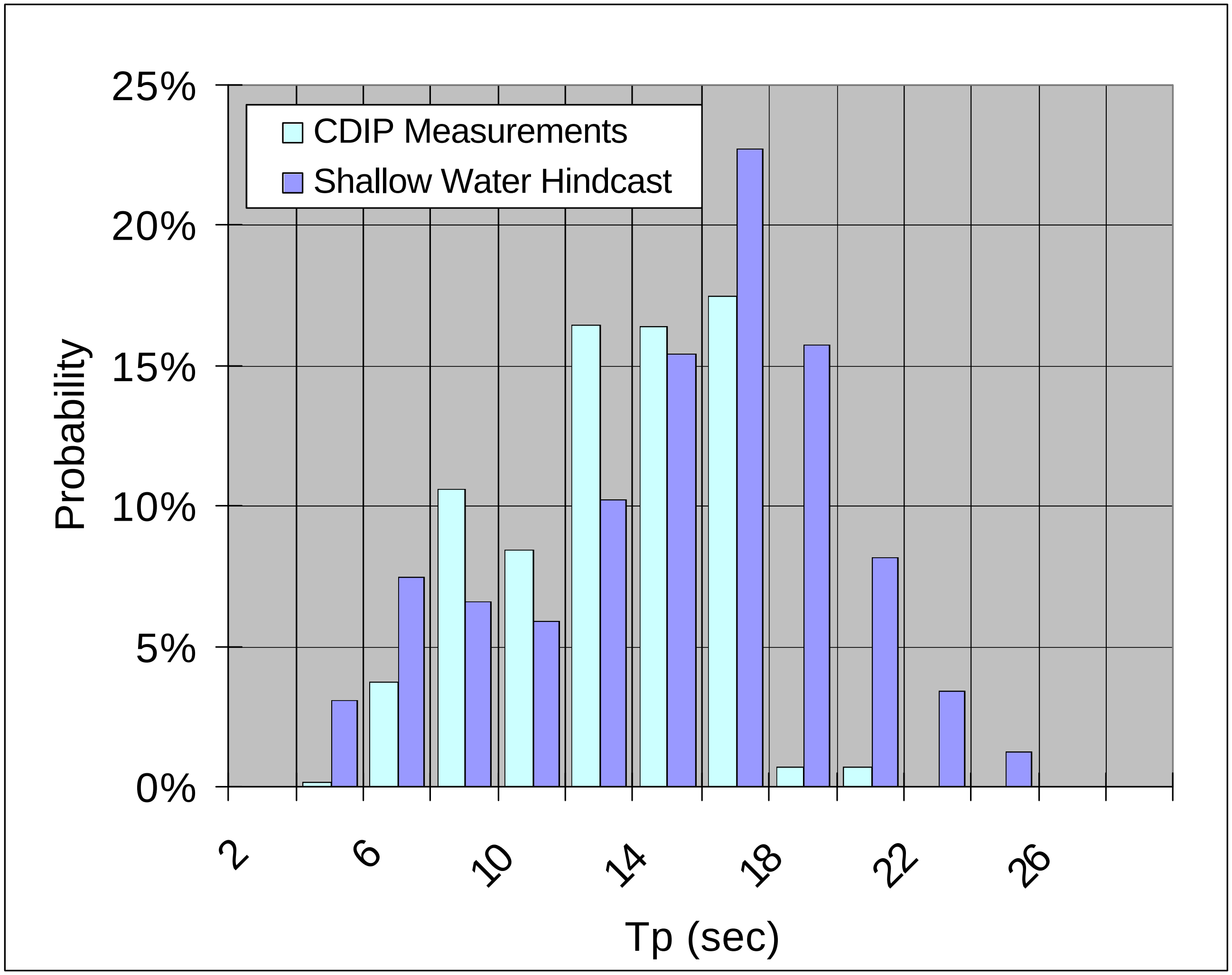
- Vince Cardone and Oceanweather, Inc. for the GROW deep water wave hindcast and valueable comments and suggestions.
- J. Ian Collins for his assistance and guidance.
- BMT Scientific Marine Services Inc. for allowing the time and resources to attend this workshop.

Probability Table



	Wave Period											
Deep Water Wave Direction	6 - 8	8 - 10	10 - 12	12 - 14	14 - 16	16 - 18	18 - 20	20 - 22	22 - 24	24 - 26	26 - 28	28 - 30	**Total**
0 - 10	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.04%
10 - 20	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.03%
20 - 30	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%
30 - 40	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%
40 - 50	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%
50 - 60	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%
60 - 70	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
70 - 80	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%
80 - 90	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%
90 - 100	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
100 - 110	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%
110 - 120	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%
120 - 130	0.00%	0.00%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%
130 - 140	0.00%	0.00%	0.02%	0.01%	0.02%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.07%
140 - 150	0.00%	0.00%	0.03%	0.07%	0.10%	0.04%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%
150 - 160	0.00%	0.00%	0.06%	0.23%	0.24%	0.07%	0.02%	0.01%	0.00%	0.00%	0.00%	0.00%	0.63%
160 - 170	0.00%	0.00%	0.24%	0.35%	0.29%	0.12%	0.06%	0.04%	0.01%	0.01%	0.00%	0.00%	1.13%
170 - 180	0.00%	0.00%	0.35%	0.44%	0.40%	0.28%	0.18%	0.13%	0.05%	0.02%	0.01%	0.01%	1.88%
180 - 190	0.00%	0.00%	0.35%	0.54%	0.67%	0.64%	0.38%	0.22%	0.11%	0.04%	0.02%	0.01%	2.97%
190 - 200	0.00%	0.00%	0.30%	0.62%	1.07%	1.40%	1.11%	0.56%	0.25%	0.09%	0.01%	0.01%	5.42%
200 - 210	0.00%	0.00%	0.18%	0.41%	1.10%	2.25%	2.72%	1.68%	0.73%	0.32%	0.07%	0.03%	9.49%
210 - 220	0.00%	0.00%	0.12%	0.32%	0.79%	2.24%	3.79%	3.07%	1.54%	0.67%	0.17%	0.11%	12.83%
220 - 230	0.00%	0.00%	0.09%	0.23%	0.58%	1.60%	2.97%	2.60%	1.41%	0.61%	0.21%	0.09%	10.39%
230 - 240	0.00%	0.00%	0.08%	0.14%	0.42%	1.23%	2.11%	1.81%	0.96%	0.40%	0.10%	0.09%	7.34%
240 - 250	0.00%	0.00%	0.07%	0.11%	0.34%	1.03%	1.84%	1.46%	0.68%	0.34%	0.07%	0.06%	5.99%
250 - 260	0.00%	0.00%	0.02%	0.06%	0.28%	1.06%	1.69%	1.26%	0.62%	0.28%	0.06%	0.05%	5.37%
260 - 270	0.00%	0.00%	0.01%	0.04%	0.31%	1.23%	1.67%	1.20%	0.65%	0.23%	0.06%	0.05%	5.46%
270 - 280	0.00%	0.00%	0.04%	0.06%	0.50%	1.49%	1.81%	1.29%	0.65%	0.22%	0.06%	0.04%	6.15%
280 - 290	0.00%	0.00%	0.05%	0.22%	1.04%	2.70%	2.68%	1.58%	0.78%	0.22%	0.08%	0.04%	9.39%
290 - 300	0.00%	0.00%	0.05%	0.23%	1.08%	2.48%	2.10%	1.24%	0.70%	0.15%	0.04%	0.02%	8.10%
300 - 310	0.00%	0.00%	0.04%	0.12%	0.54%	1.15%	0.86%	0.58%	0.27%	0.07%	0.02%	0.01%	3.65%
310 - 320	0.00%	0.00%	0.01%	0.05%	0.32%	0.52%	0.38%	0.27%	0.10%	0.05%	0.02%	0.01%	1.73%
320 - 330	0.00%	0.00%	0.01%	0.02%	0.19%	0.26%	0.20%	0.12%	0.06%	0.01%	0.00%	0.00%	0.88%
330 - 340	0.00%	0.00%	0.01%	0.01%	0.07%	0.12%	0.10%	0.05%	0.02%	0.01%	0.00%	0.00%	0.38%
340 - 350	0.00%	0.00%	0.00%	0.00%	0.04%	0.06%	0.03%	0.01%	0.01%	0.00%	0.00%	0.00%	0.15%
350 - 360	0.00%	0.00%	0.00%	0.00%	0.02%	0.03%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.08%
Total	0.00%	0.00%	2.13%	4.31%	10.45%	22.07%	26.79%	19.22%	9.63%	3.75%	1.01%	0.65%	**100%**

Measured vs. Predicted Tpeak



LNG Terminal