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WEAK TURBULANT FLUXES ESTIMATION FOR SURFACE WATER WAVE SPECTRUM

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Introduction

The traditional method of wind wave estimation is based on the equation of wave spectrum energy evolution.

For the first time the attitude for wind wave estimation based on the wind wave balance equation was proposed by V. M. Makkaviev in 1937.

It was one of the first attempts to describe mathematically a wind wave development, which found its practical application. In its simplest form the equation describes evolution of wind wave energy depending on average wind speed and dissipation.

The works published after 1956 (Longuet—Higgins 1957; Longuet—Higgins et al. 1960, 1961, 1962, 1964: Phillips, 1957, 1958;Miles 1957,1960; Hasselmann, 1960, 1962, 1963, Zakharov 1968, etc.) founded the principles of modern physical notion of wind wave development.

The component of the wind wave energy input *Gin* is usually determined with the help of the relation based on the model of averaged airflow interaction with wave (Miles, 1960). Although it was proposed in 1957, this model is still used nowadays. The mechanism specified by using full-scale observation data (Snyder et al., 1981) can be described as follows:

$$G_{in}(w,b) =$$
$$\max\left\{0;\, 0.25\, a_1 \frac{r_a}{r_w} w \times \left(a_2 \frac{U_{10}}{c} \cos(b - b_U) - 1 \right) S(w,b) \right\},$$

where: *U*10 is the wind speed at 10 m level; is the angle between the wind speed and the direction of wave spectral component propagation; *a*1 and *a*2 are the parameters with the value of about 1.0. The wind energy is supplied to the wave spectrum range at .

$$a_2 (U_{10} / c) \cos(\beta - \beta_U) > 1$$

The accurate numerical modeling of the statistical structure of atmospheric boundary layer above sea surface based on the numerical solution of the Reynold two-dimensional equations is described in papers (Chalikov, 1986; Burgers&Makin, 1992, Chalikov&Belevich, 1995; Belevich&Neelov, 1999; Makin&Kudriavtsev, 2002; Kudriavtsev& Makin, 2004). It is shown that the wind wave energy input term can be expressed as:

$$G_{in}(w, b) = B_U w\, S(w, b)$$

where Bu is a non-dimensional parameter of wind-wave interaction. Its value become negative for wave propagating faster than wind speed.

The wave dissipation used in the WAM model (The WAM model, 1988; Komen et al., 1994) connected with wave breaking is accepted in the form of the quasi-linear approximation, as it is suggested by G.Komen (1984) on the basis of the Hasselmann model:

$$G_{ds}(w,b) = -c_{ds1}\overline{w}\left(\frac{w}{\overline{\overline{w}}}\right)\left(\frac{\overline{a}}{a_{PM}}\right)^{m} S(w,b)$$

where *c*, *n* and *m* are the model parameters; $\overline{\omega}$ is the mean frequency of the wave spectrum; αpm is the constant of the Pierson-Moskovits spectrum;

One of the most important mechanisms in wind wave spectrum formation is non-linear energy transfer, based on the kinetic equation (Hasselman, 1962,1963; Zakharov, 1968). In terms of wave action spectrum the non-linear energy transfer function is as follows:

$$Gnl = \iiint T(\vec{k},\vec{k}_1,\vec{k}_2,\vec{k}_3) d(\vec{k}+\vec{k}_1-\vec{k}_2-\vec{k}_3) d(s+s_1-s_2-s_3)\times$$

$$\times\left\{N_2 N_3 \left(N+N_1\right)-N_1 N\left(N_2+N_3\right)\right\} d\vec{k}_1 d\vec{k}_2 d\vec{k}_3$$

where $N_i = N(\vec{k}_i)$ is the spectral density of wave action; $T(\vec{k},\vec{k}_1,\vec{k}_2,\vec{k}_3)$ is the core function of non-linear interaction between waves; $\delta(\vec{k})$ and $\delta(\sigma)$ are the Diraque delta-function describing the resonance interaction between four wave components:

$$\vec{k}+\vec{k}_1=\vec{k}_2+\vec{k}_3 \qquad\qquad \sigma+\sigma_1=\sigma_2+\sigma_3$$

The most important property of the kinetic equation is a preservation of three following integral values: total wave action, energy and momentum.

$$N=\int N(\vec{k})\,d\vec{k} \qquad E=\int w(\vec{k})\,N(\vec{k})\,d\vec{k} \qquad \vec{K}=\int \vec{k}\,N(\vec{k})\,d\vec{k}$$

Stationary solution of the kinetic equation

The simplest Kolmogorov weak-turbulent solution of the kinetik equation was obtained by Zakhorov (Zakharov & Filonenko, 1966; Zakharov &Zaslavskii, 1982, 1983) for isotropic case. It is interpreted as frequency spectrum defined by energy and wave action fluxes correspondingly

$$S_1(w) = C_1 \sqrt[3]{P\, g^4 / w^{12}}$$

$$S_2(w) = C_2 \sqrt[3]{Q\, g^4 / w^{11}}$$

where *Q* is a wave action flux; *P* is a wave energy flux.

The first solution is interpreted as a model with energy input located at low frequency $w = 0$, and second one as wave action with input located at high frequency $w = 00$

In a more general case (Zakharov et al., 1992) the spectrum is defined by following fluxes: wave energy *P*, wave action *Q* and momentum *M* is as follows:

$$S(\omega,\beta)=\frac{g^{4/3}P^{1/3}}{\omega^4}F\left(\frac{\omega Q}{P},\frac{gM}{\omega P},\cos\beta\right)$$

The function F can be expanded into Taylor series

$$S(\omega,\beta)\cong\frac{g^{4/3}P^{1/3}}{\omega^4}\left(\alpha_0+\alpha_1\frac{gM\cos(\beta)}{\omega P}+\dots\right)$$

and the reliable estimations of the Kolmogorov constants are found out to be equal to $a_0=0.31\pm0.03$ and $a_1=0.24\pm0.03$

(Pushkarev, Resio, Zakharov, 2001; Lavrenov, Resio, Zakharov, 2002)

Motivations

- What is a ratio between wave energy, action and momentum fluxes directed into high and low frequency ranges for real spectrum?
- What is the value of wave energy dissipation estimated with the help of weak turbulence theory? Namely, the WAM model type dissipation covering main frequency domain or Zakharov's type located in the high frequency range.
- A question appears which type of dissipation is the most reliable from the physical point of view.

The new optimal algorithm of non-linear energy transfer is reduced to three dimensional integration over some undimentional variables

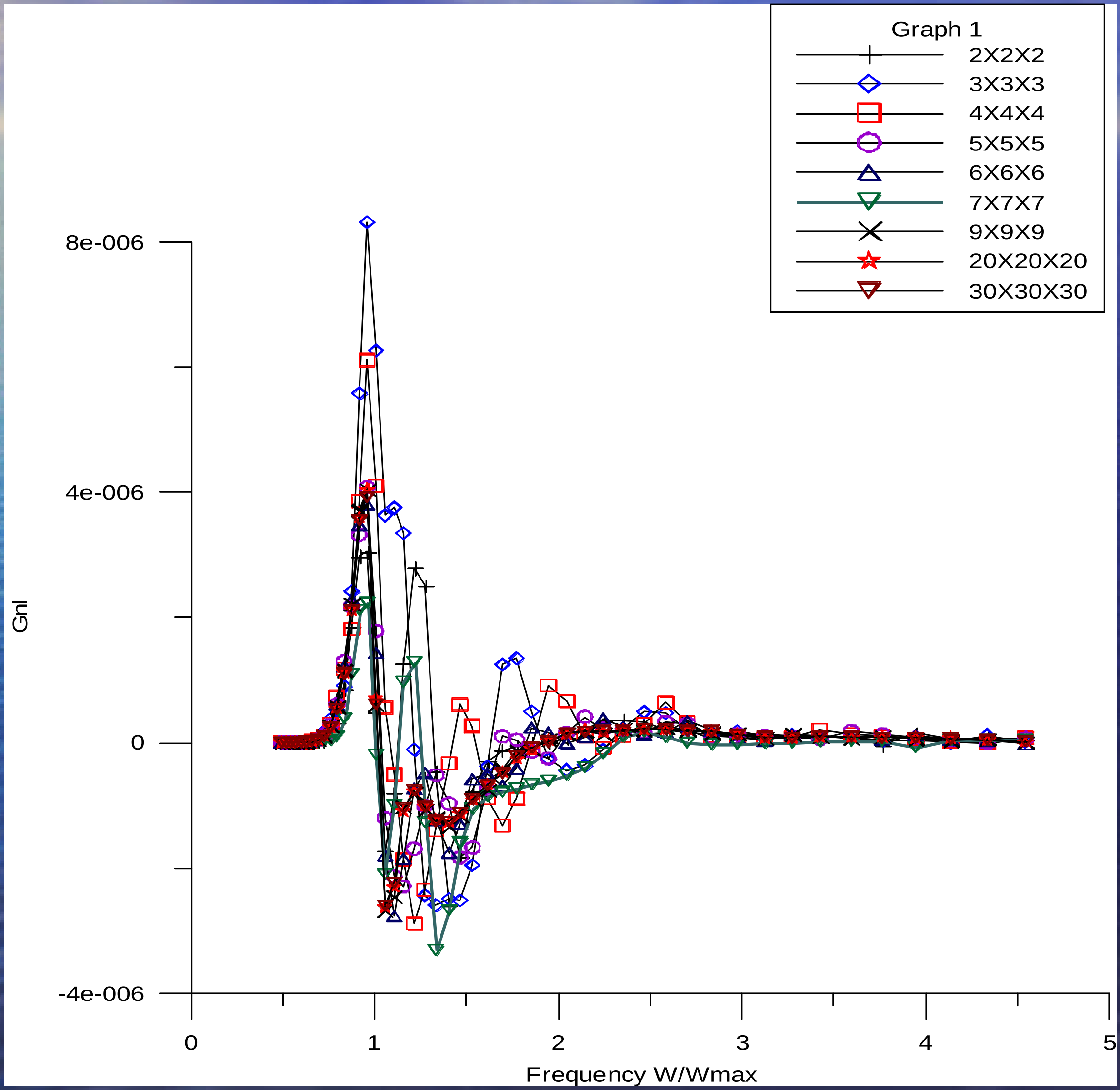
$$Gnl = \int_{0}^{\infty}\int_{-p}^{p}\int_{s_2} F(s_2, b_1, s_1)\, ds_2\, db_1\, ds_1 = \int_{-1}^{1}\int_{-1}^{1}\int_{1}^{1} F(\tilde{j}, \tilde{J}, \tilde{z}) \left(\frac{\partial(s_2, b_1, s_1)}{\partial(\tilde{j}, \tilde{J}, \tilde{z})} \right) \frac{\sqrt{(1-\tilde{y}^2)}\sqrt{|A-x|}}{\sqrt{(1\pm x)}}\, d\tilde{j}\, d\tilde{J}\, d\tilde{z}$$

Integrated function contains singularities. The integral is estimated with the help of Jakoby weight function or three dimensional cubature formula as follows

$$Gnl = \sum_{k=1}^{l}\sum_{i=1}^{n}\sum_{j=1}^{m} a_{ijk} F(S_{k,i,j})$$

CPU time of numerical integration is defined by number of summations

Non-linear energy transfer estimation for different number of summations

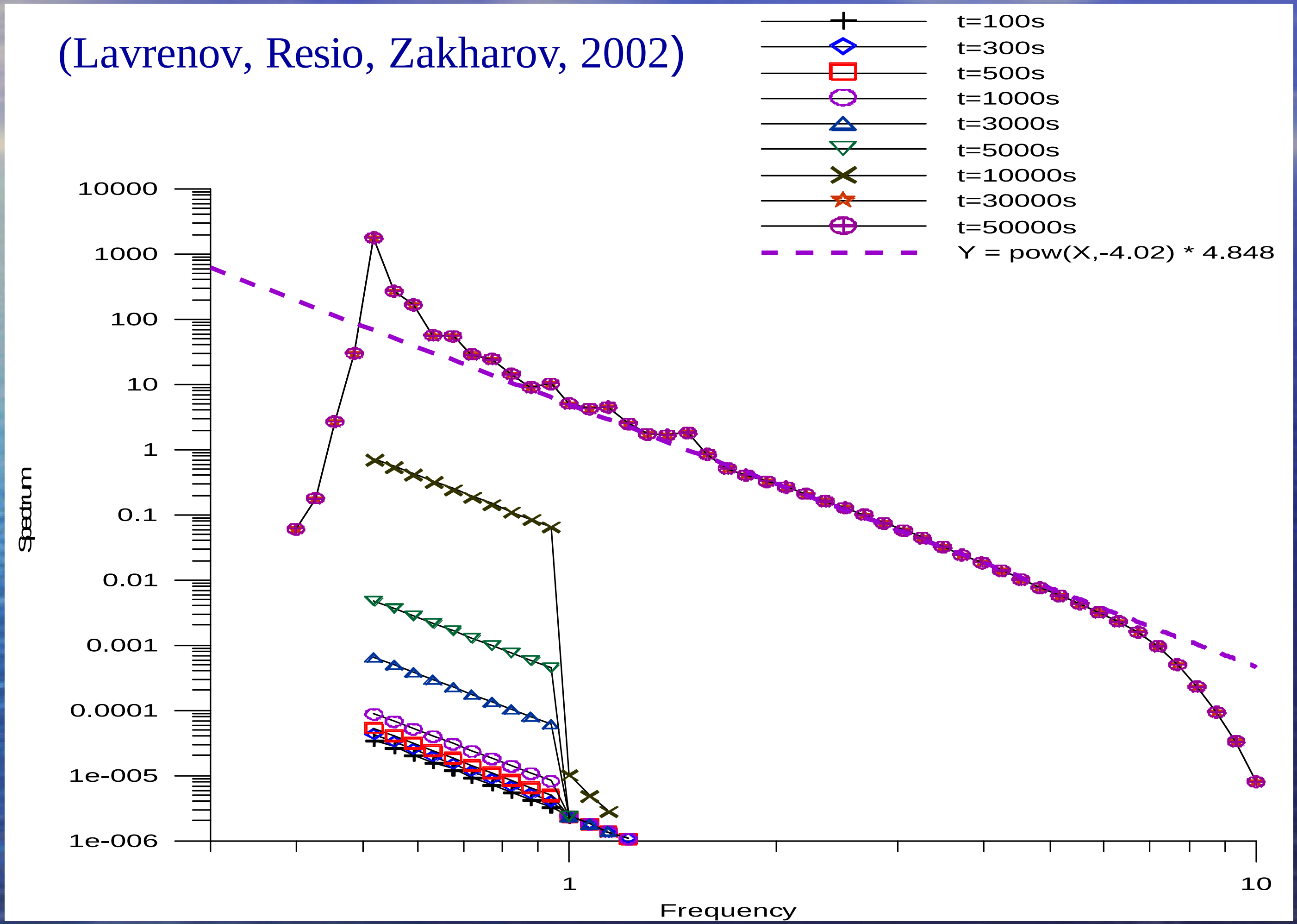


Estimation of numerical accuracy of wave energy, action and momentum preservation for various number of summation

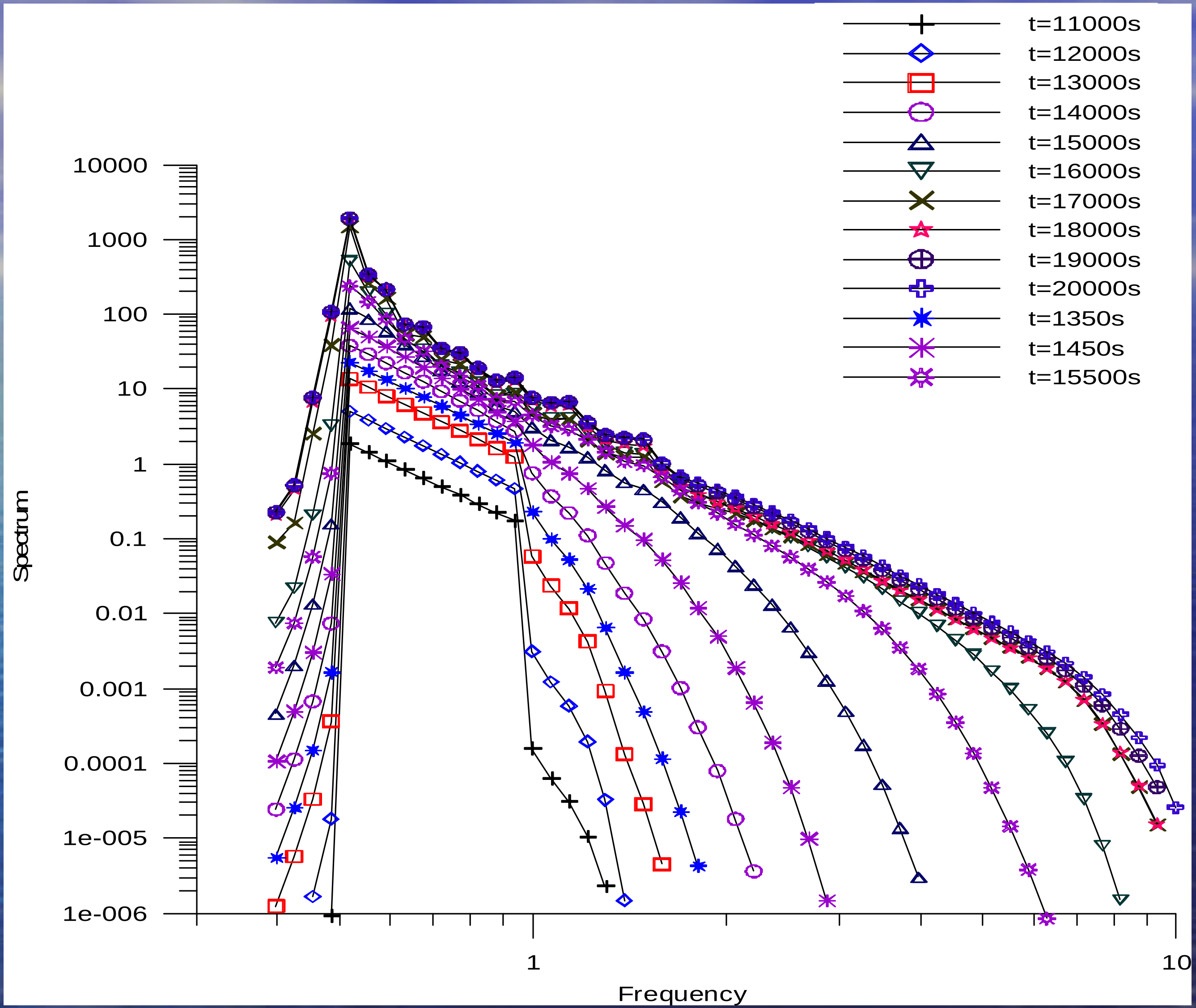
Gnl Conservativity

Imax	Jmax	Kmax	Total	Action	Energy	Momentum
2	2	2	16	0.243	0.191	0.0362
3	3	3	54	0.448	0.335	0.176
4	4	4	128	0.253	0.186	0.064
5	5	5	250	0.0487	0.0158	-0.073
6	6	6	432	0.042	0.008	-0.088
7	7	7	686	0.034	0.0077	-0.073
8	8	8	1024	0.0148	-0.0141	-0.099
9	9	9	1458	0.0205	-0.0099	-0.10
10	10	10	2000	0.00707	-0.0219	-0.111
11	11	11	2662	0.0087	-0.0102	-0.0997
20	20	20	16000	0.00203	-0.00112	-0.0101
30	30	30	54000	0.00164	-0.00141	-0.0103
		19	38	-0.914	-.953	-.987
		27	54	-0.259	-0.210	-0.434
		34	68	0.121	-0.0645	0.303

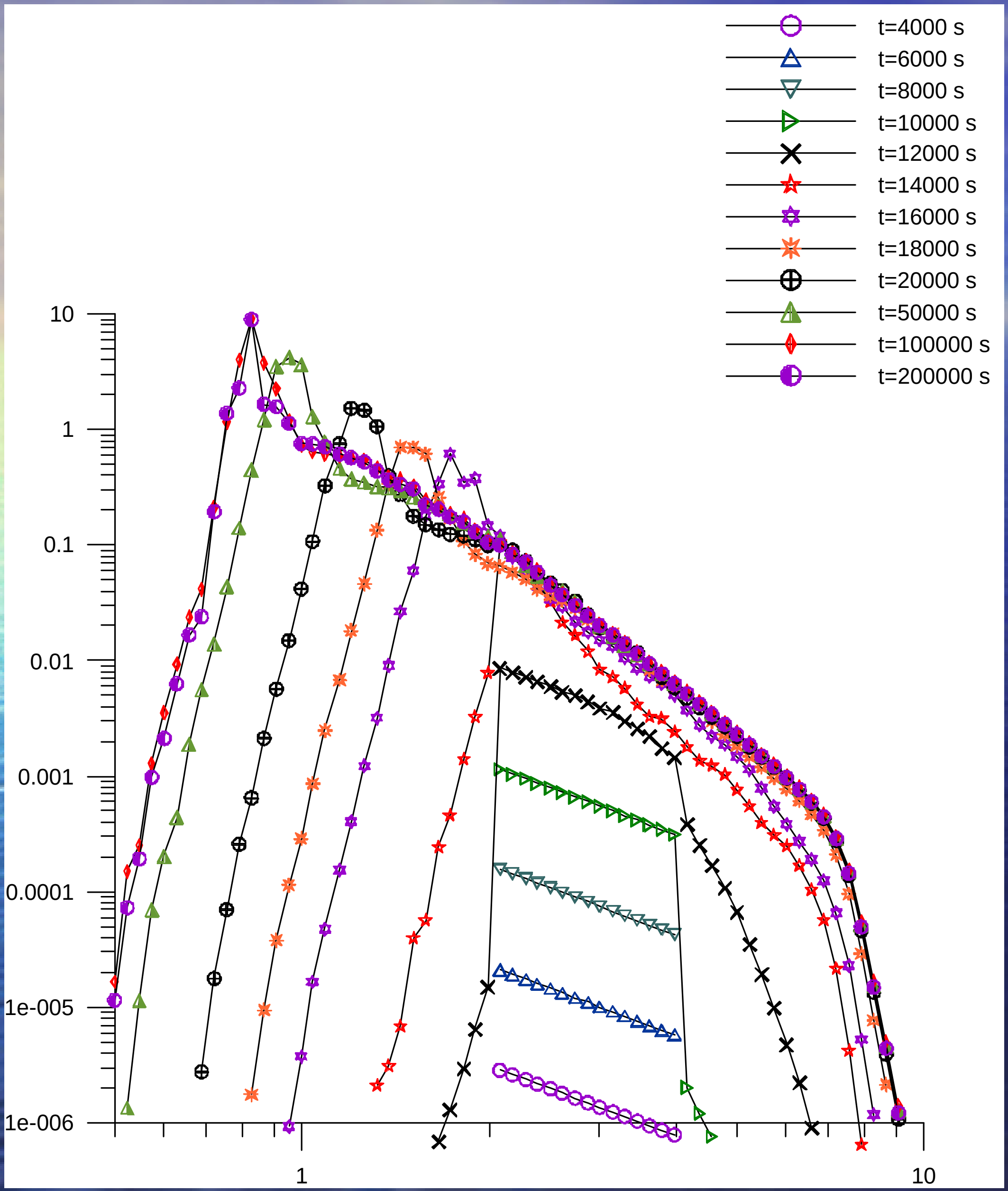
Numerical results for isotropic case



Frequency spectrum evolution within time interval : $100 \leq t \prec 50000\,\text{sec}$



Spectrum evolution within time interval : $11000 \le t \prec 15000$ sec



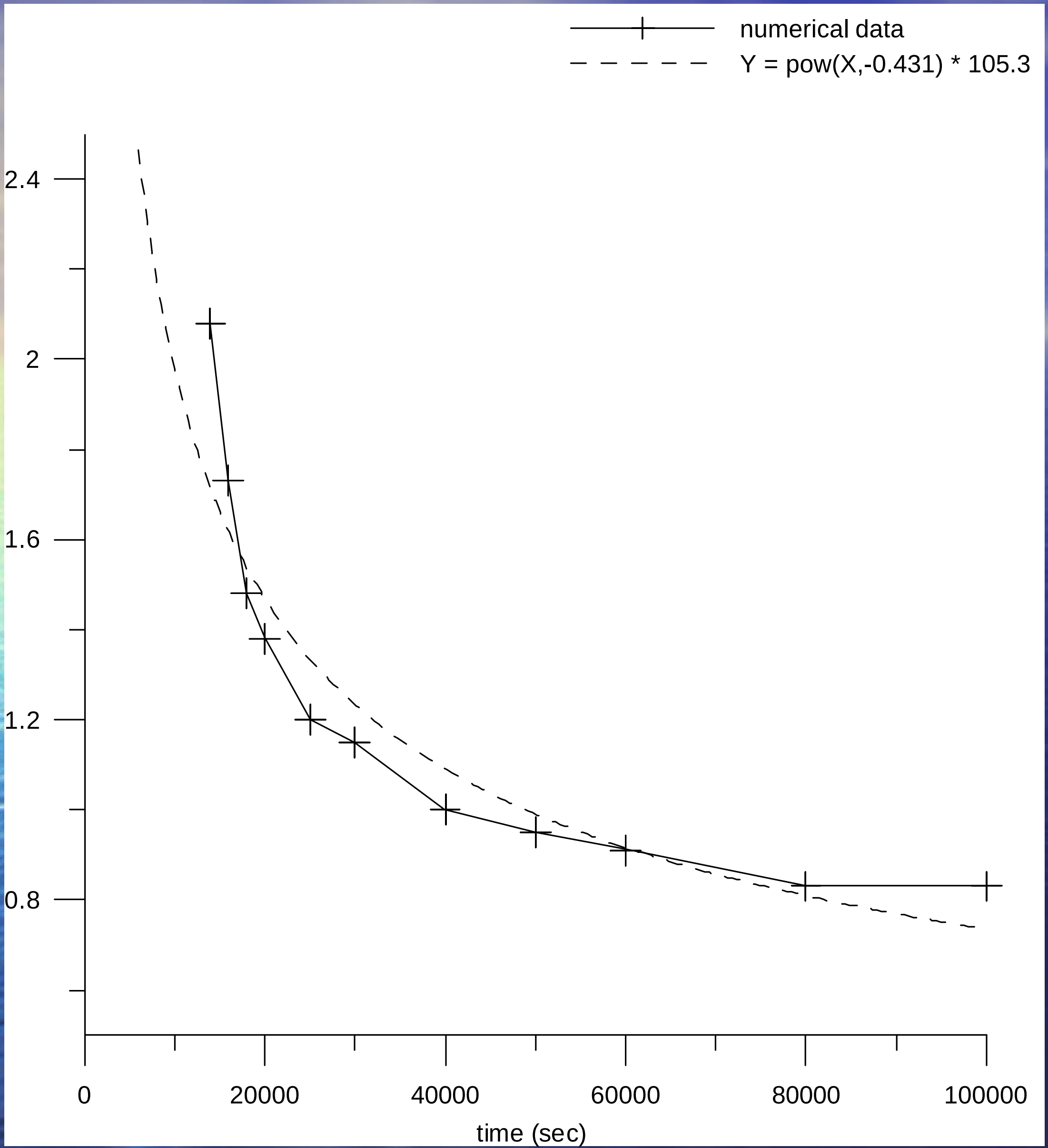
Frequency (rad/s)

Direct and Inverse Fluxes

Isotropic case

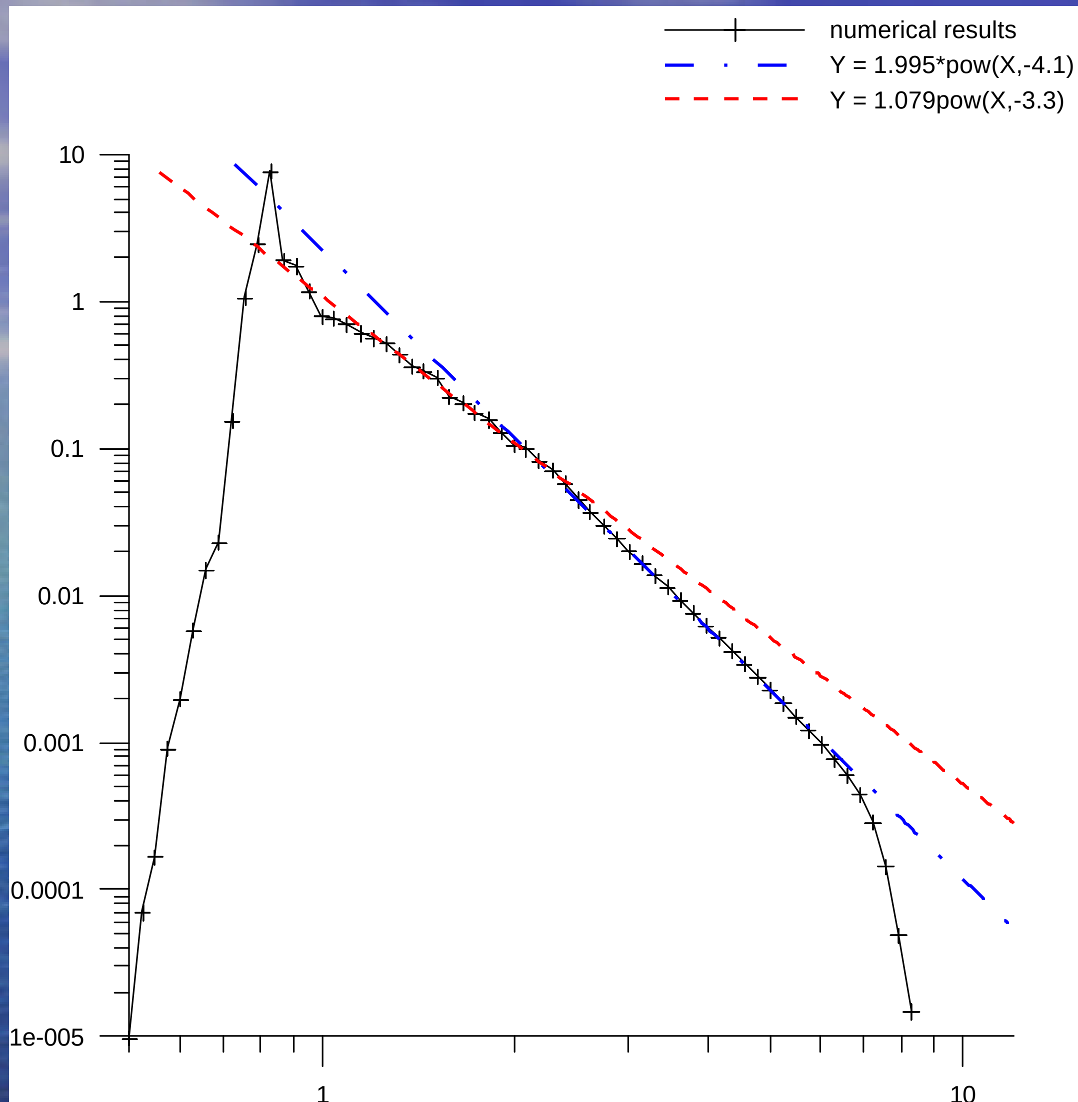
Spectrum evolution in time (isotropic case)
70 frequencies and 96 direction are used

There are three different stages in the wave spectrum evolution



Frequency of spectrum maximum evolution into low frequency range

Frequency of spectrum maximum is approximated as $\omega_{max} \approx t^{-0.4}$. This dependence is similar to one obtained with the help of experimental data (Davidan et al., 1985).

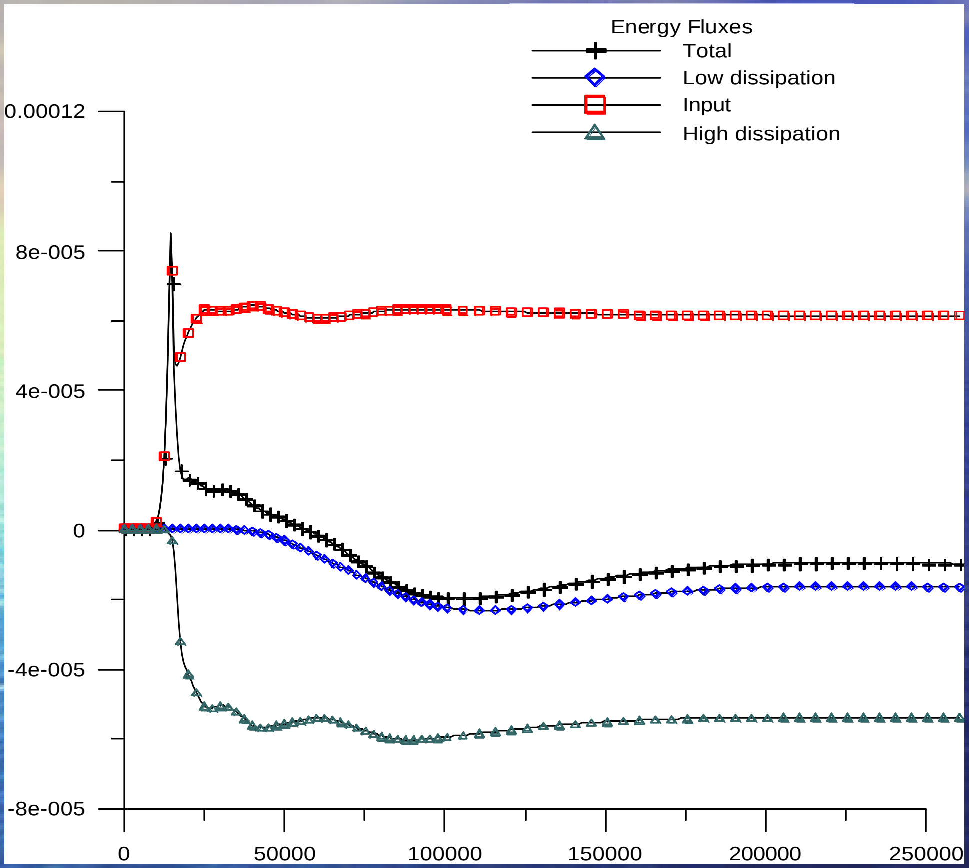


Frequency (rad/s)

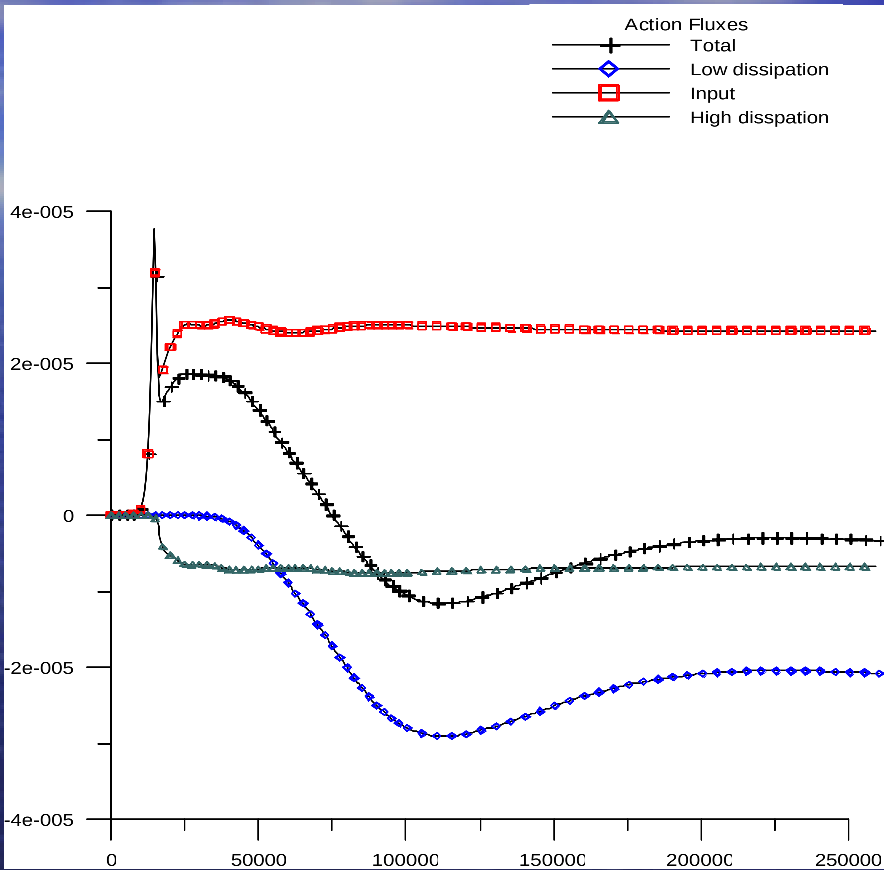
Limited spectrum with approximations for two ranges:

$1.0 \prec w \prec 2.0$ and $2.0 \prec w \prec 7.0$

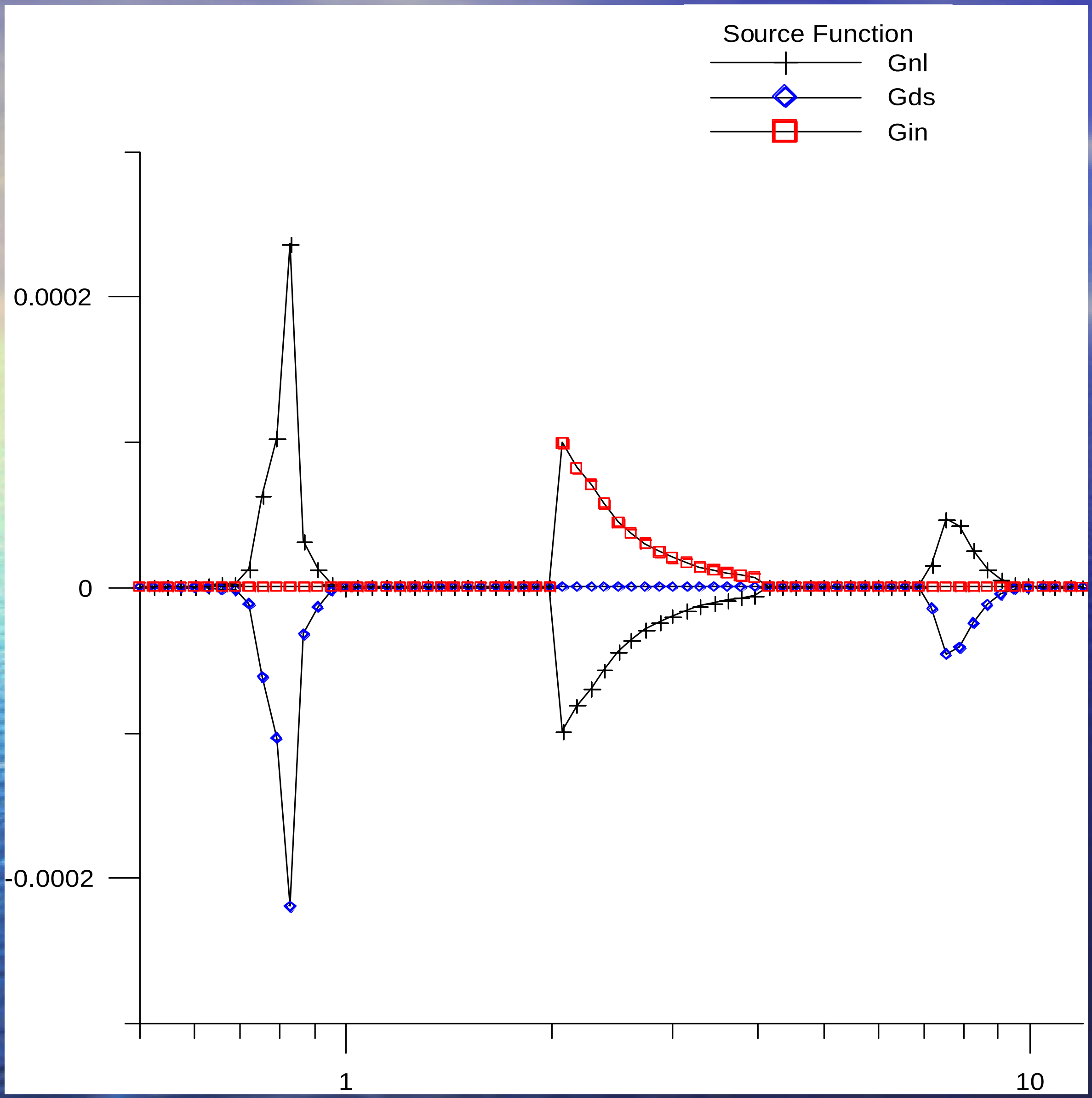
a)



b)



a) Wave energy fluxes time evolution: total, low dissipation, wave input, high dissipation
b) Wave action fluxes evolution



Frequency

Limited source function with:
Non-linear energy transfer
Energy input and Dissipation

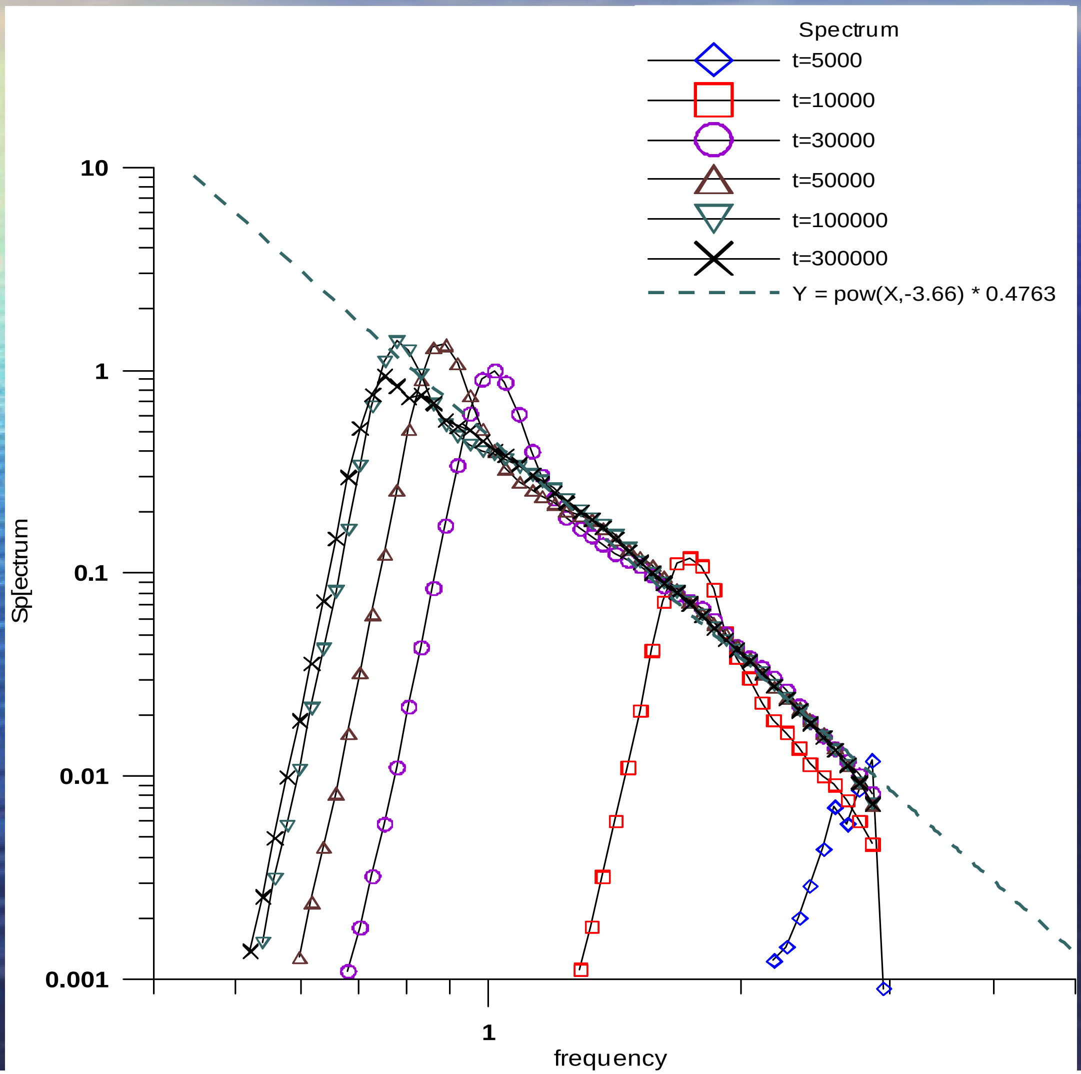
Estimation of relative limited fluxes of the energy, wave action and momentum into high and low frequency ranges (especial case)

Angular distribution	Energy flux to low freq	Energy flux to high freq	Wave action flux to low freq.	Wave action flux to high freq.	Momentum flux to low freq.	Momentum flux to high freq.
Isotropic	23.7 %	76.3 %	75.3 %	24.7 %		
$\cos^2(b)$	20.5 %	79.5 %	72.7 %	27.9 %	1.9 %	98.1 %

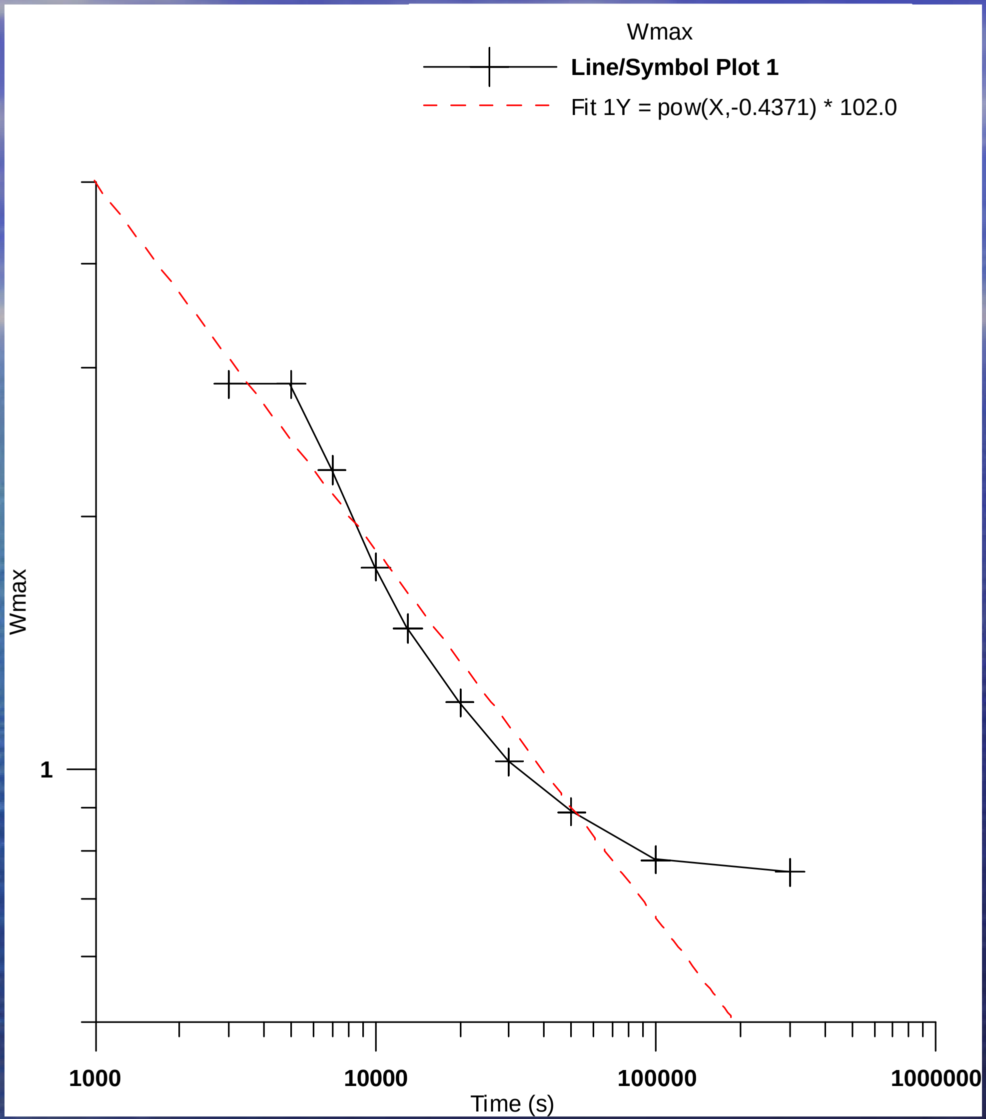
Reverse Flux to Atmosphere
due to Makin-Kudriavtcev's wind energy input

Spectrum Evolution in time

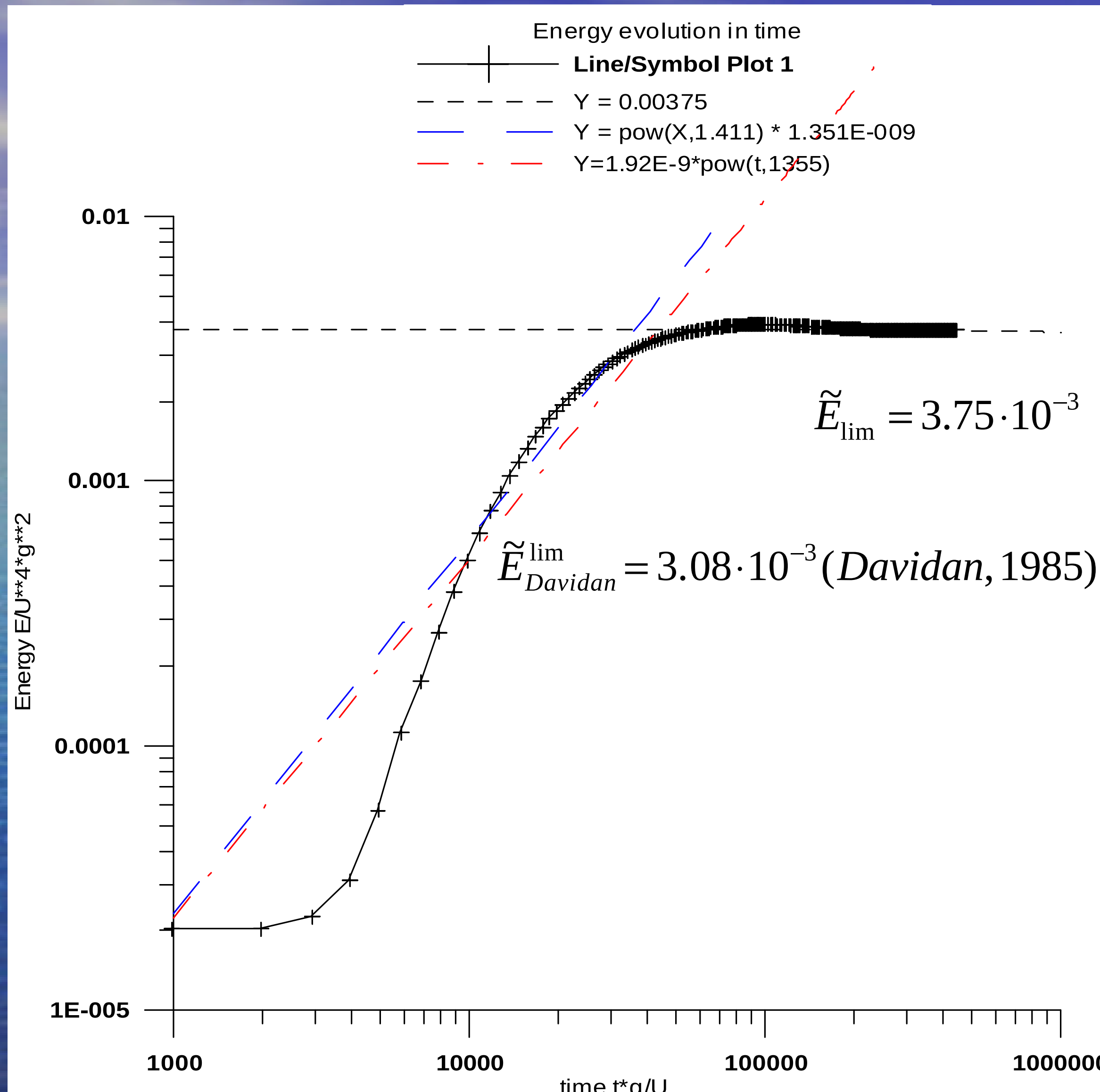
$$\frac{\partial N}{\partial t} = G$$



Frequency spectrum evolution in time

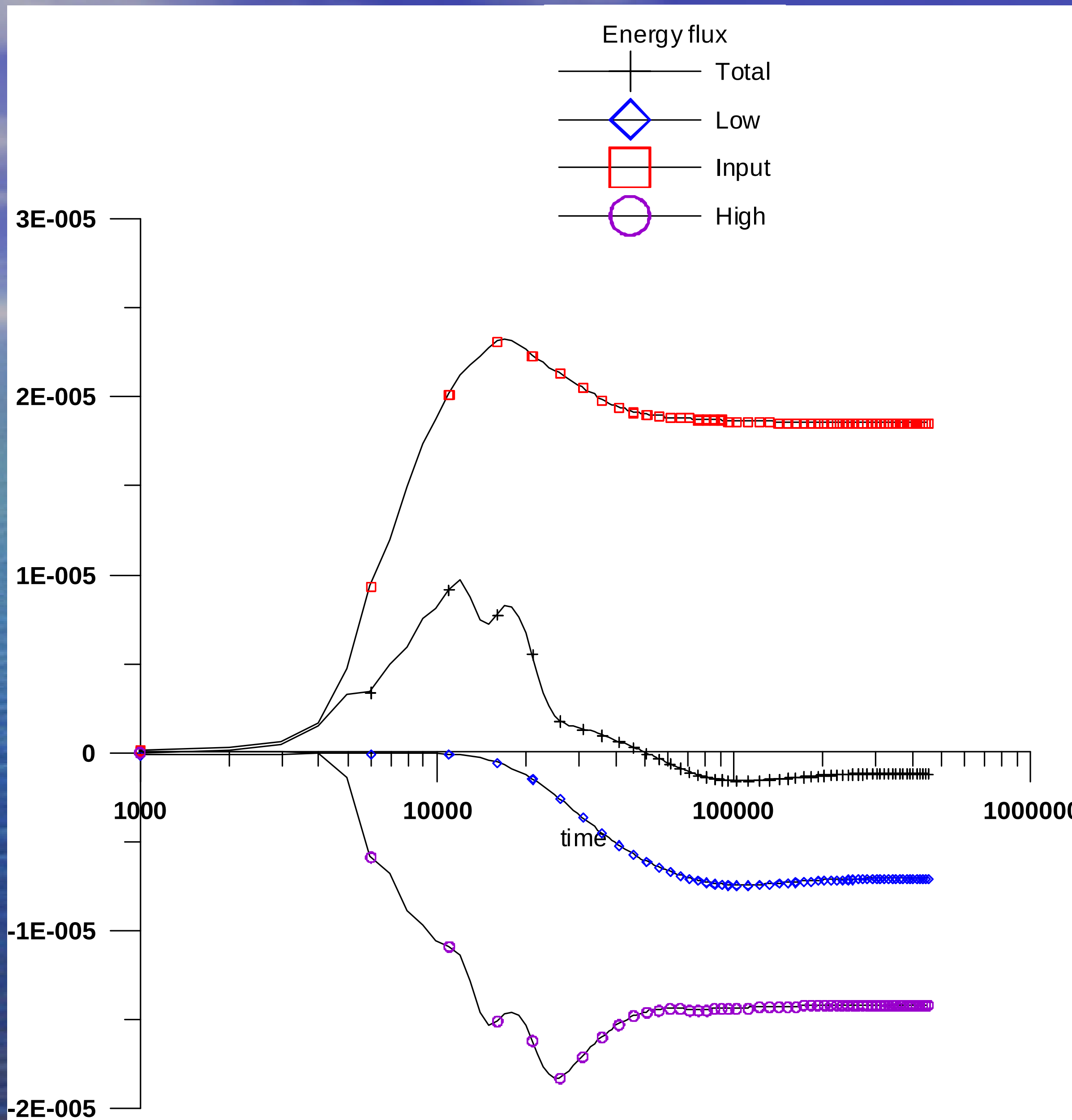


Spectrum frequency maximum evolution in time

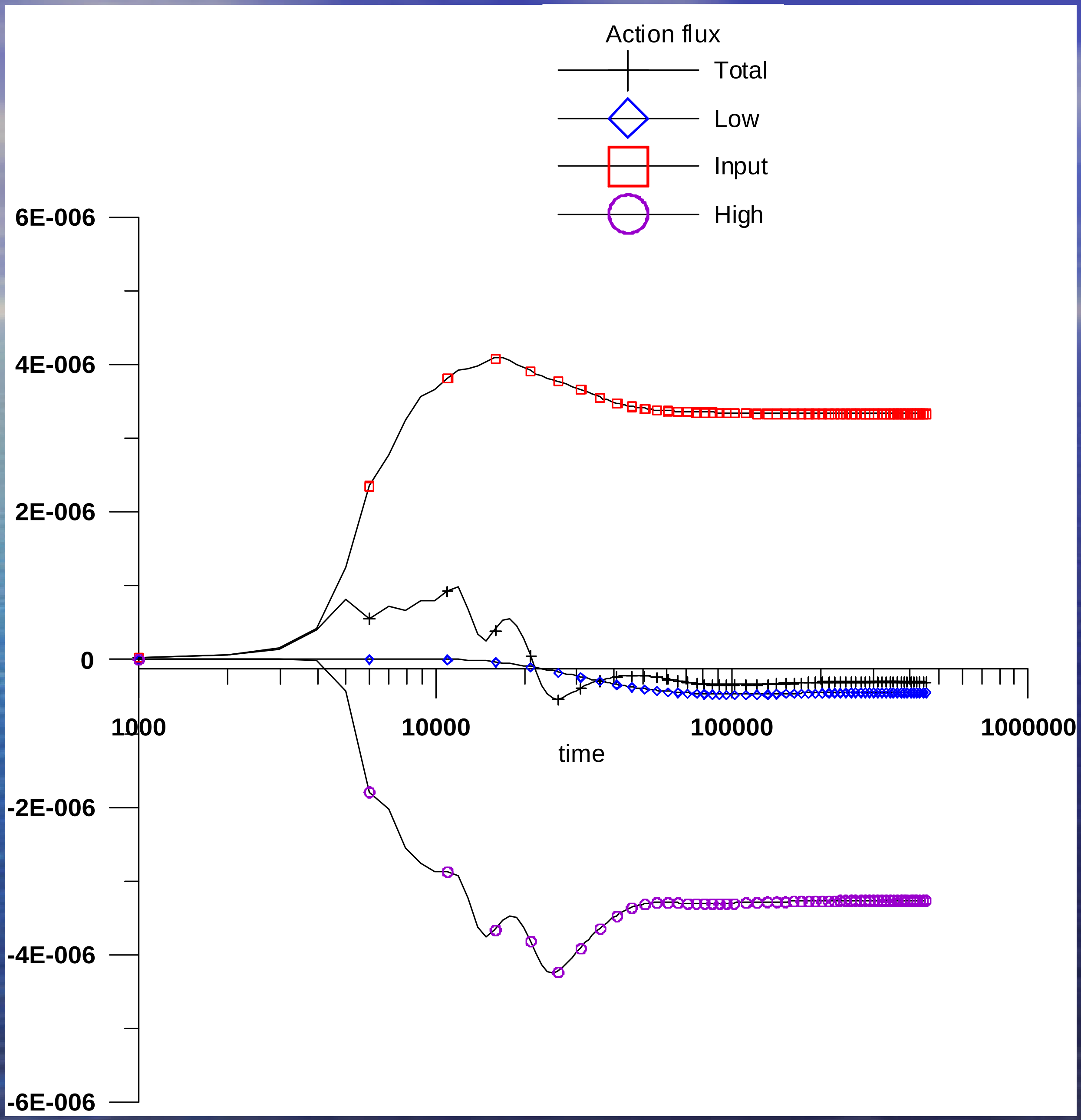


Undimensional energy evolution in time

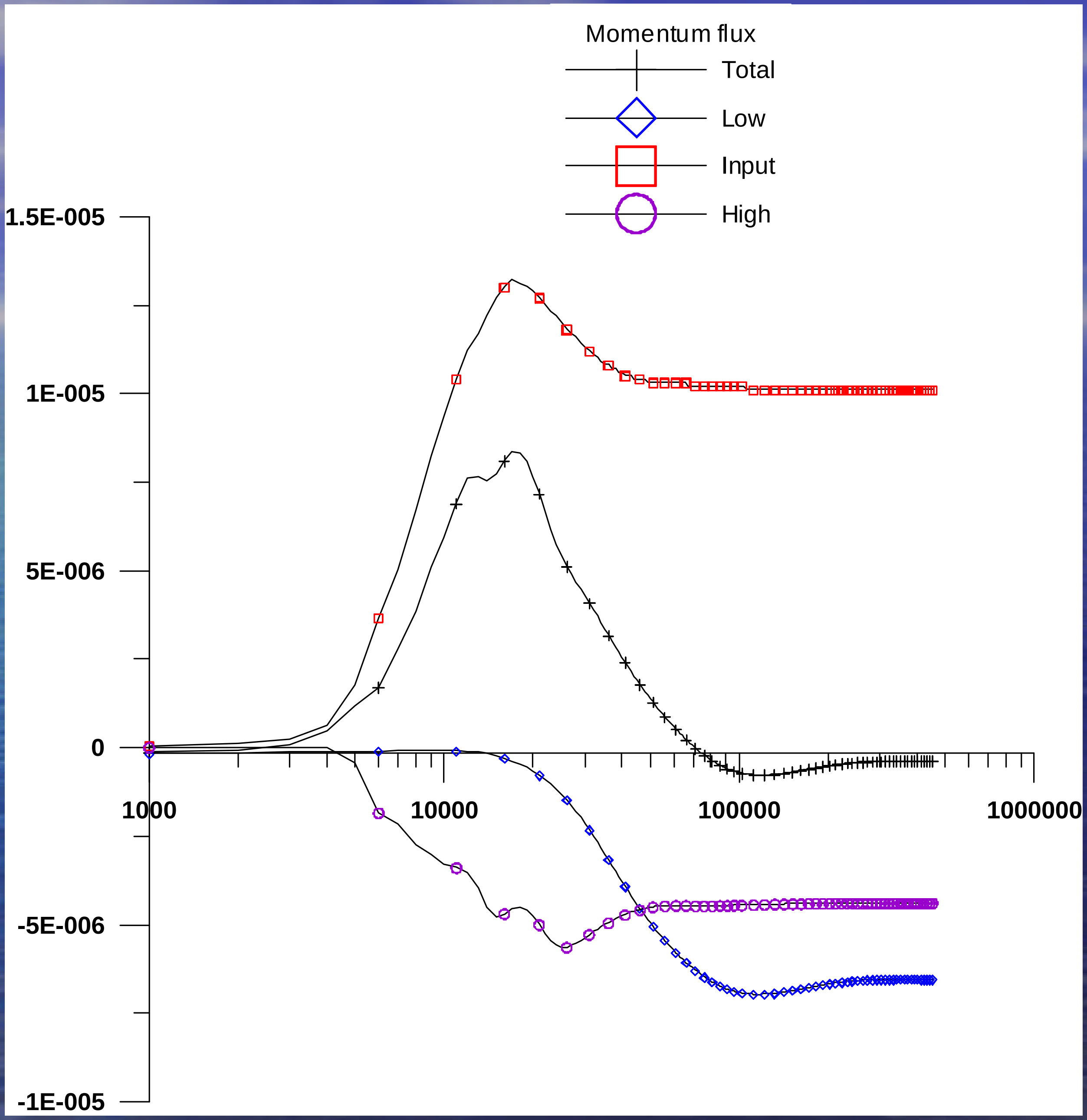
Nondimensional limited energy 0.00375. The field result analysis equal to 0.00303 (Davidan, 1985)



Evolution of energy fluxes in time

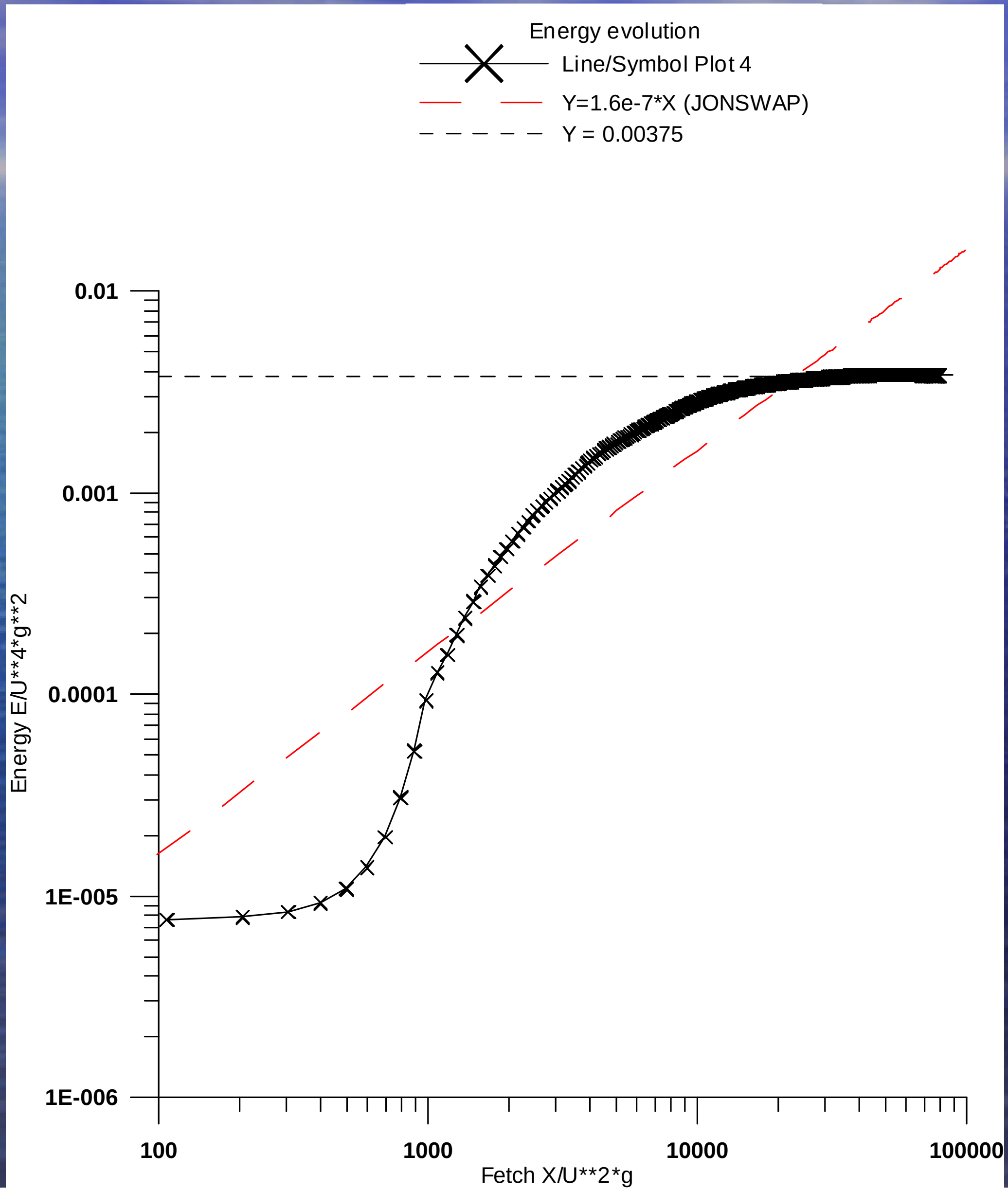


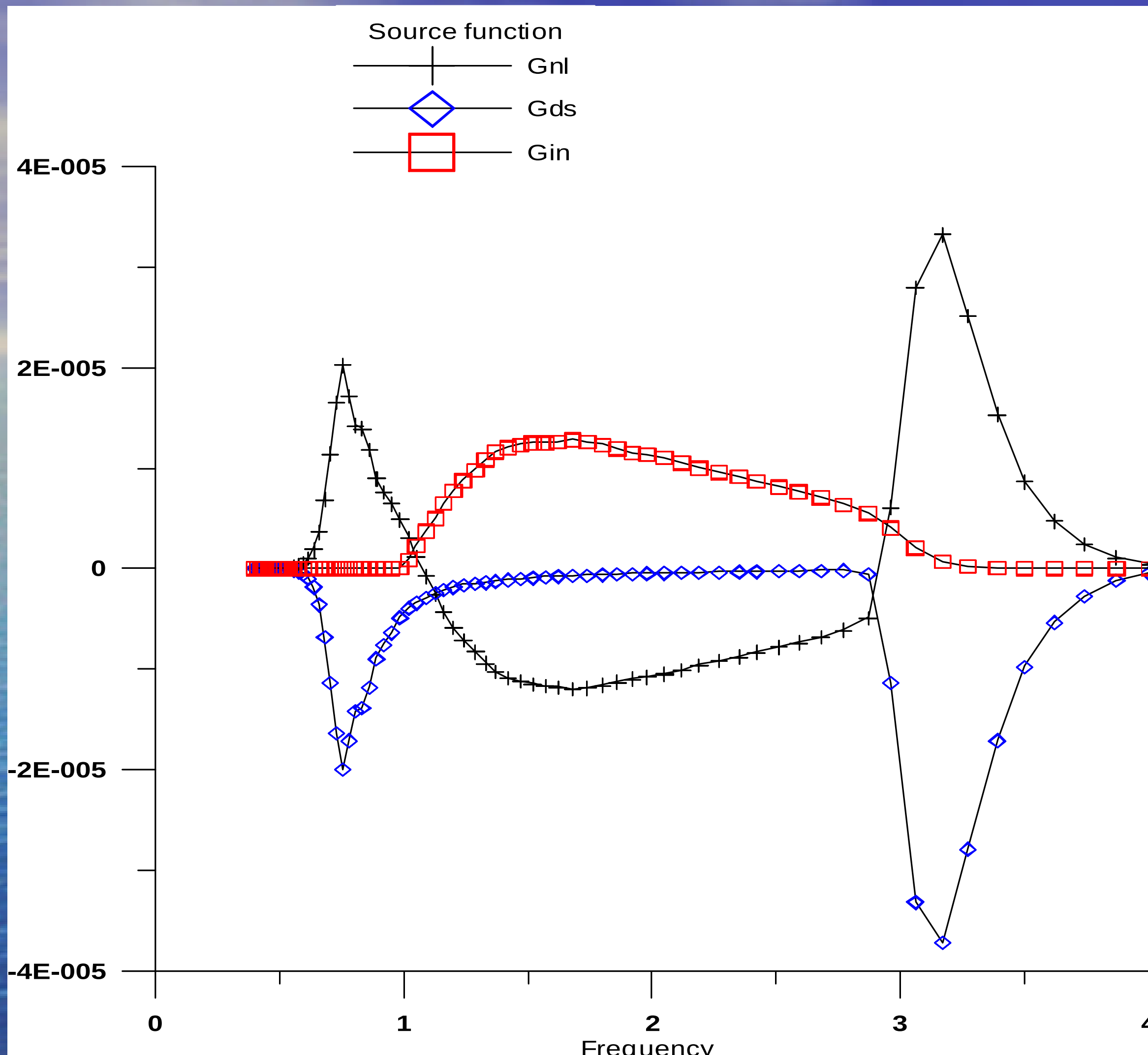
Evolution of wave action fluxes in time



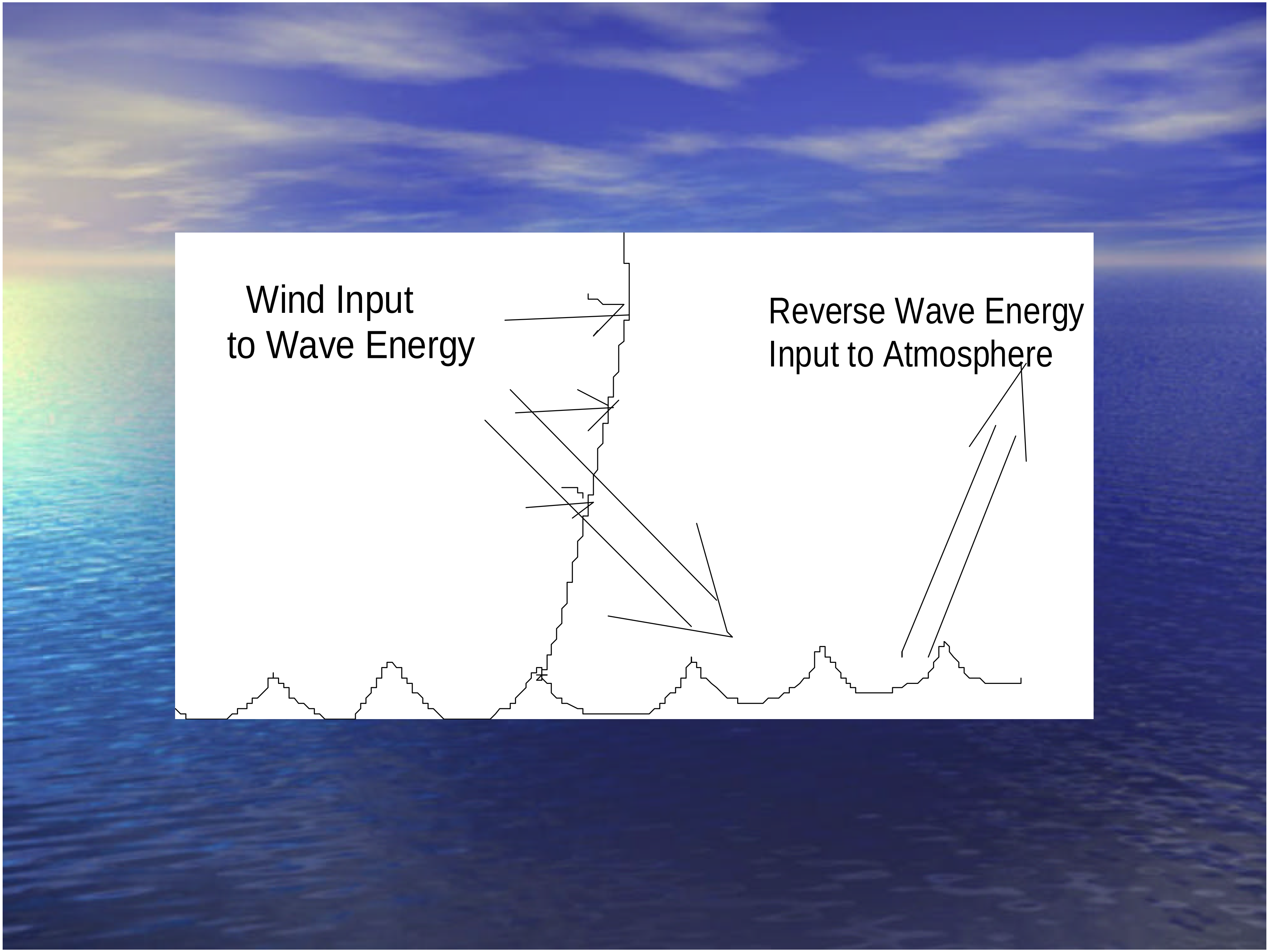
Evolution of momentum fluxes in time

Wave Evolution in fetch: $C_g \cos(b) \frac{\partial}{\partial x} N = G$





Limited source function with:
Non-linear energy transfer
Energy input and Dissipation

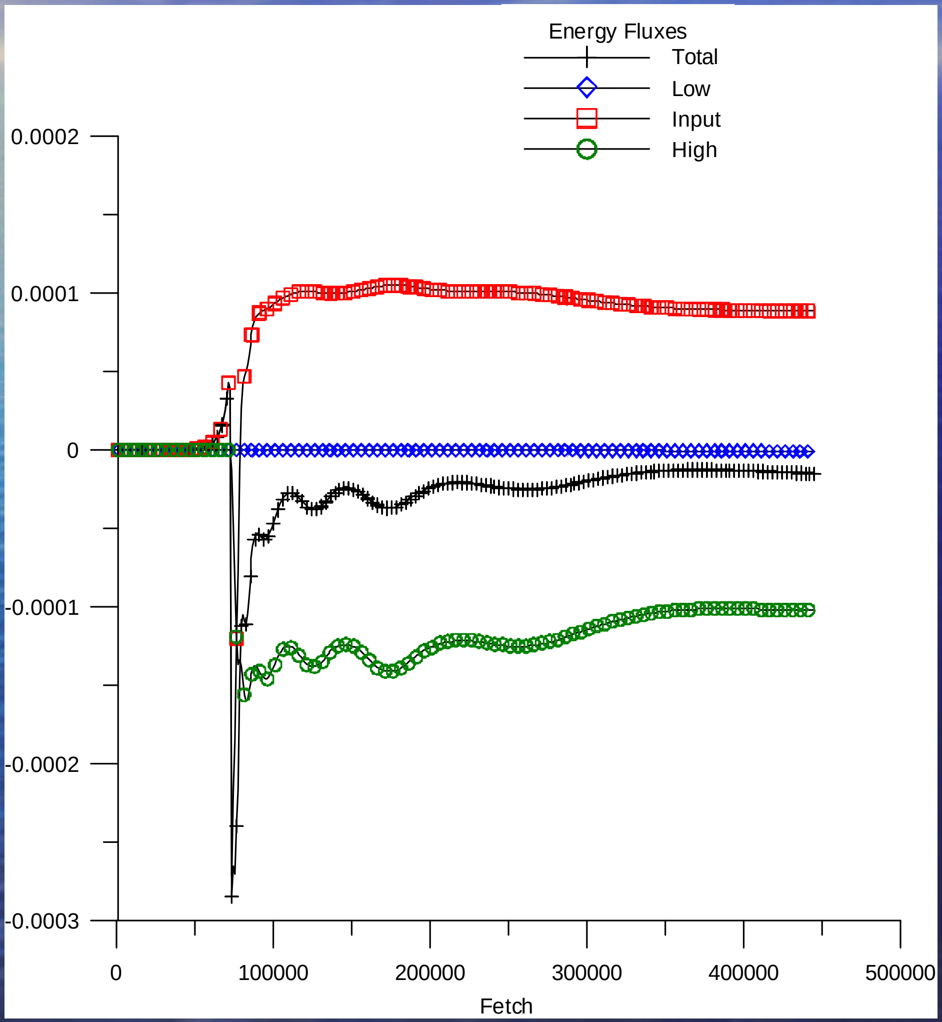


Wind Input
to Wave Energy
Reverse Wave Energy
Input to Atmosphere

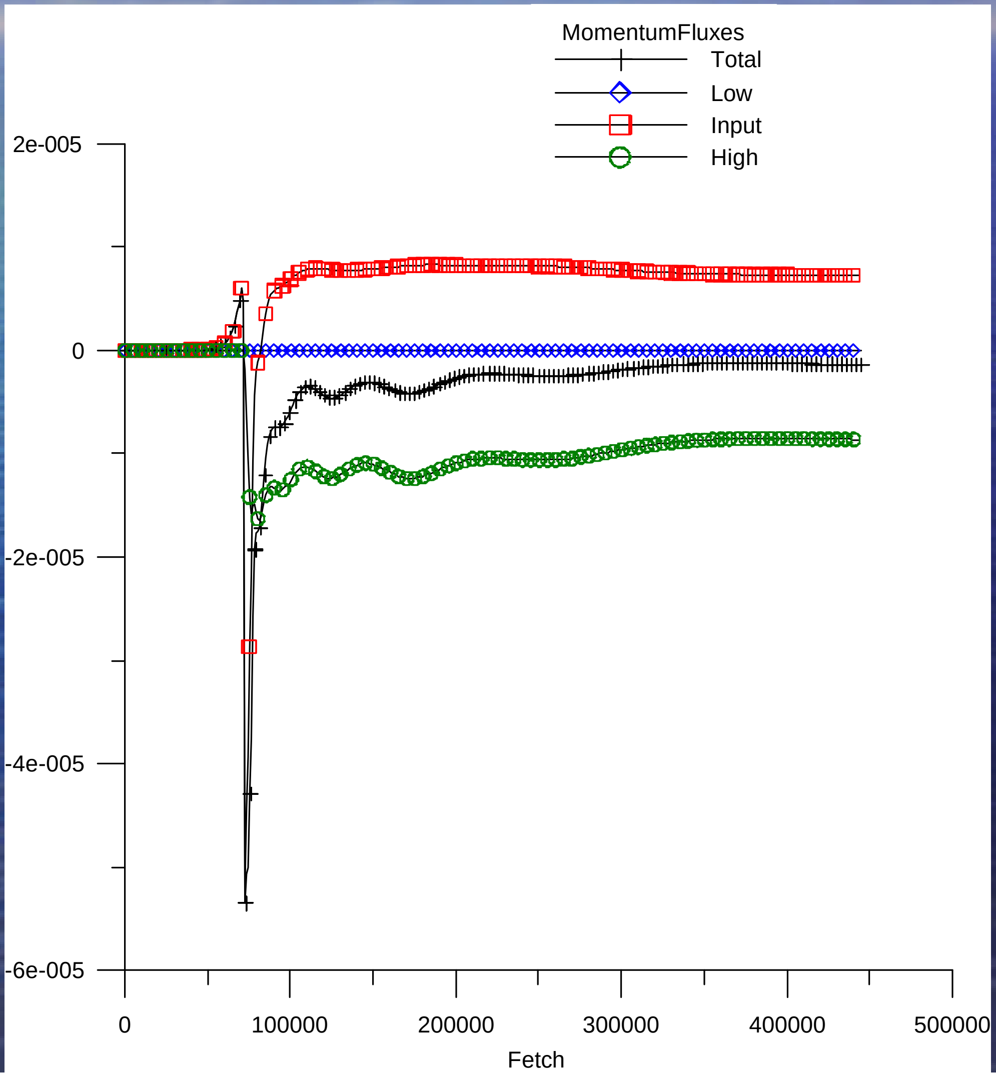
Estimation of relative limited fluxes of the energy, wave action and momentum into high and low frequency ranges In the case of fetch evolution (Makin-Kudriavtcev's input)

Wind U (m/s)	Energy flux to low frequency (%)	Energy flux to high frequency (%)	Wave action flux to low frequency (%)	Wave action flux to high frequency (%)	Momentum flux to low frequency (%)	Momentum flux to high frequency (%)
20	24.4	75.6	67.0	33.0	3.7	96.3
10	23.4	76.6	66.4	33.6	2.8	97.2

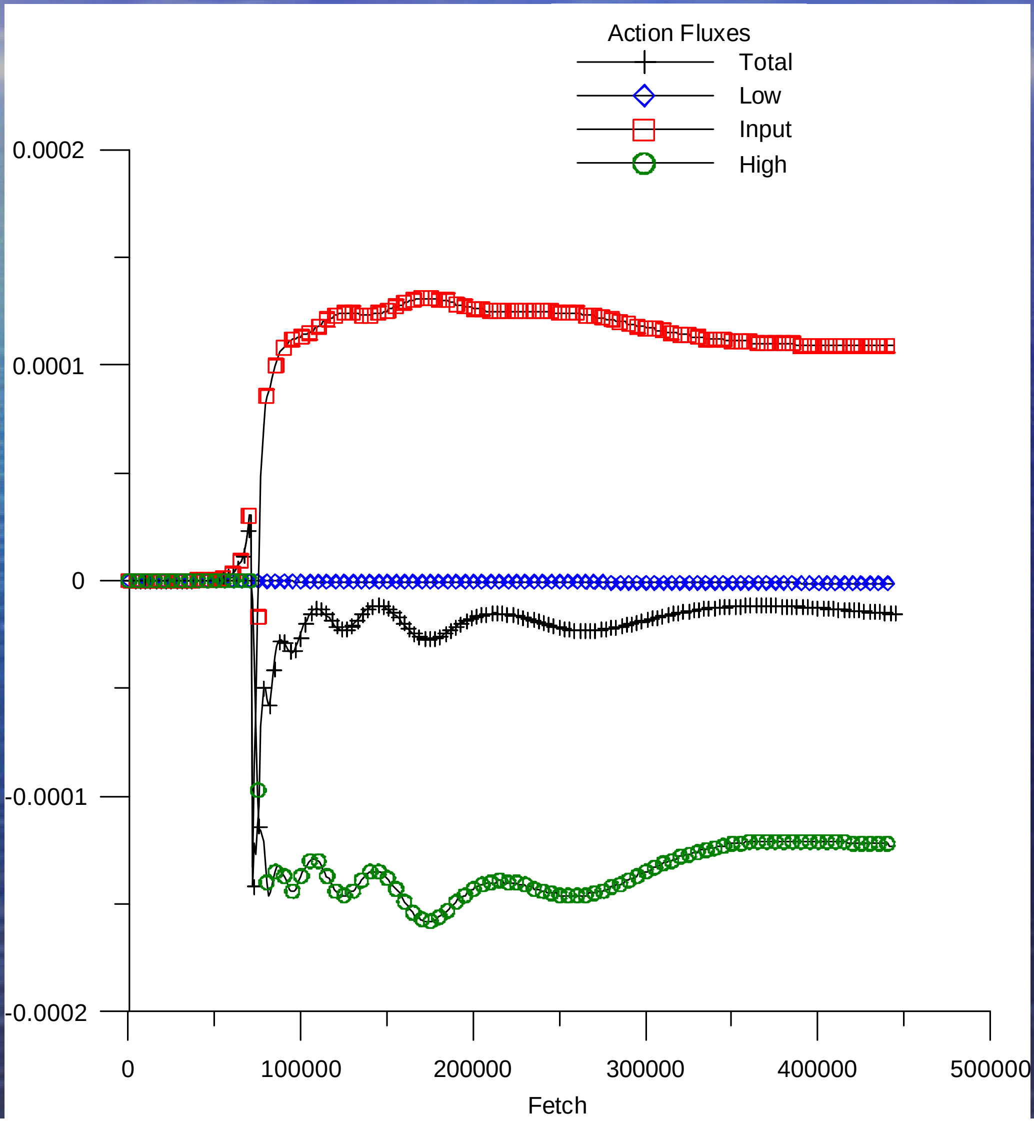
Energy fluxes for WAM+ dissipation



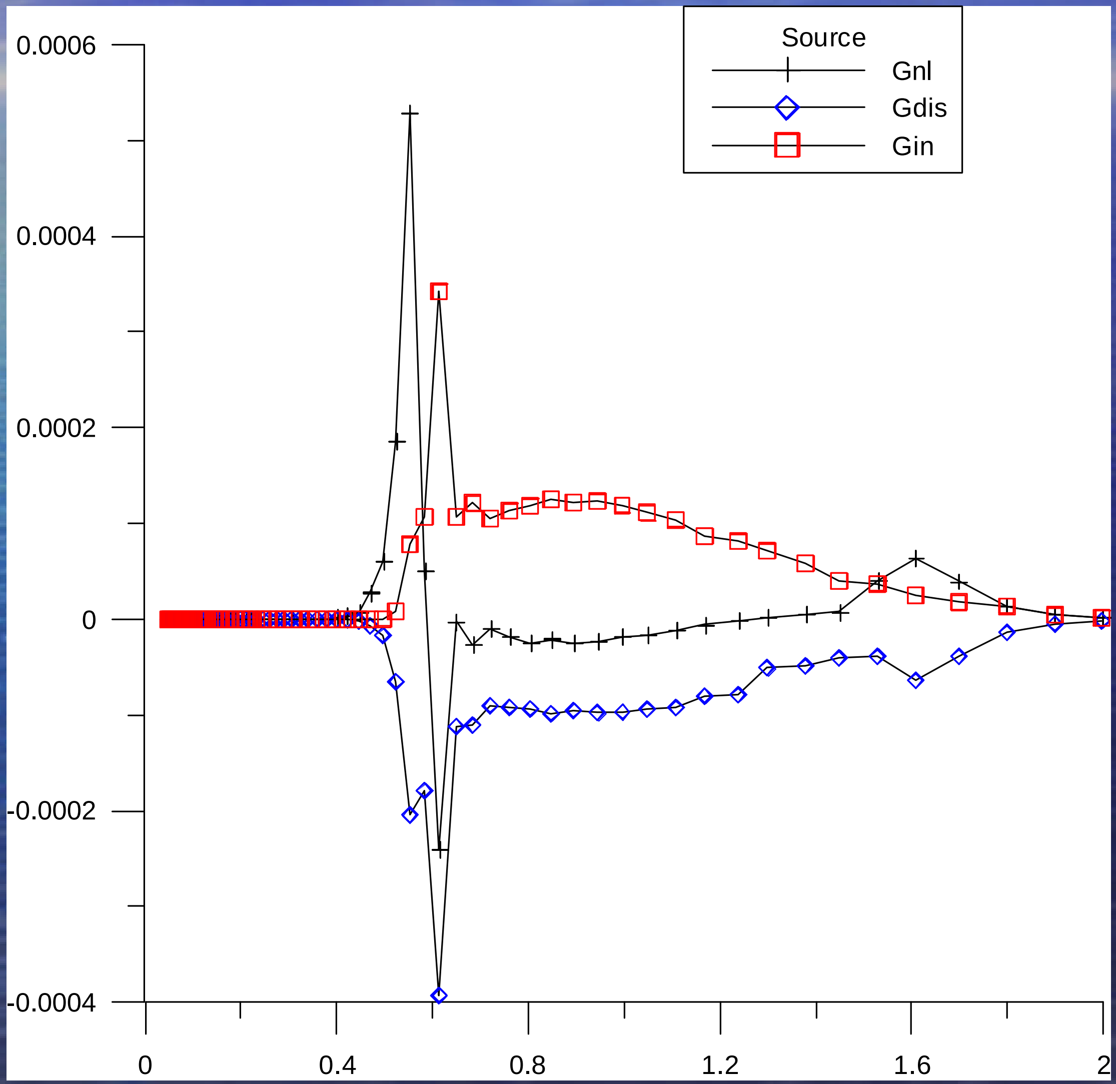
Momentum fluxes for WAM+ dissipation



Action fluxes
for WAM+ dissipation



Source functions balance



Estimation of relative limited fluxes of the energy, wave action and momentum into high and low frequency ranges

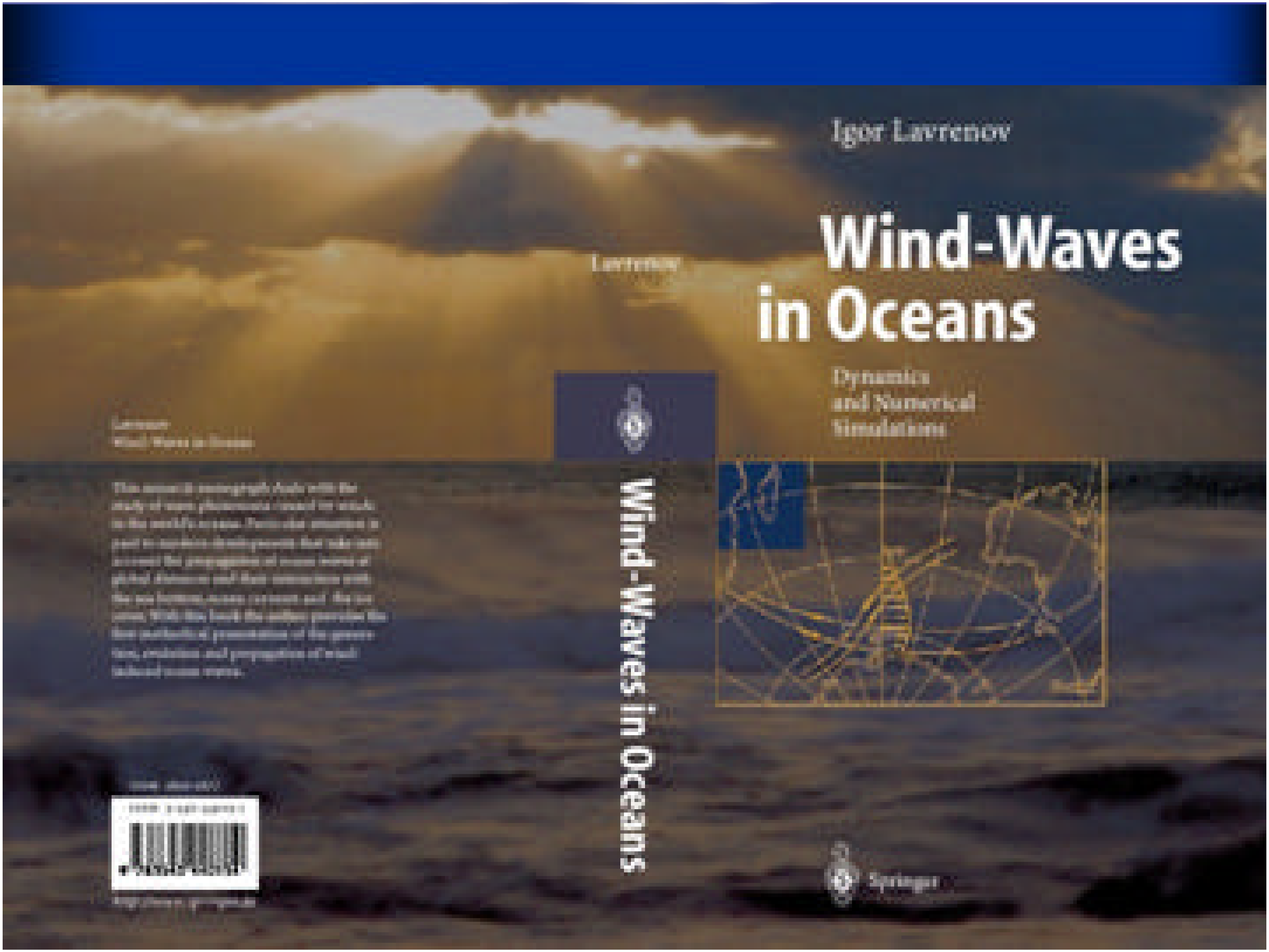
In the case of fetch evolution

(Makin-Kudriavtcev's input + WAM dissipation)

U (m/s)	Energy flux low (%)	Energy flux high (%)	Wave action flux low (%)	Wave action flux high (%)	Momentum flux low (%)	Momentum flux High (%)
20	**0.07**	**99.93**	**0.3**	**99.7**	**1.0**	**99.0**

CONCLUSIONS

- The results of numerical simulations show that there are the main differences between two types of whitecapping dissipation approximations: the WAM dissipation, covering almost the whole frequency range and Zakharov dissipation located in the high frequency range. The values of energy, action and momentum fluxes directed to the high and low frequency ranges differ significantly in these cases.
- Numerical simulation of wind wave development with the Makin-Kudriavtsev wind energy input shows that almost 1/4 part of energy flux can be transferred back to atmosphere boundary layer in the low frequency range for a fully developed spectrum. At the same time implementation of the WAM model dissipation reduces almost up to zero all fluxes of energy, momentum and action to low frequency range. There is no energy flux transferred back to atmospheric boundary layer. Thus, a principal difference is shown for physics of wind wave spectrum development and interaction between ocean and atmosphere depending on wave dissipation function.
- Until recently it was assumed that the whole energy coming from wind to waves was spent on wave spectrum development and dissipation. However, our results show that it is not quite so. The results obtained with the help of Zakharov energy dissipation contrary to WAM dissipation provide the existence of backward energy flux from waves to atmospheric boundary layer. It may reach considerable values presenting a additional source of energy into atmosphere. The fact should be taken into consideration in global ocean-atmosphere interaction models.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Thank you!
With gratitude to the Workshop’s organizers!