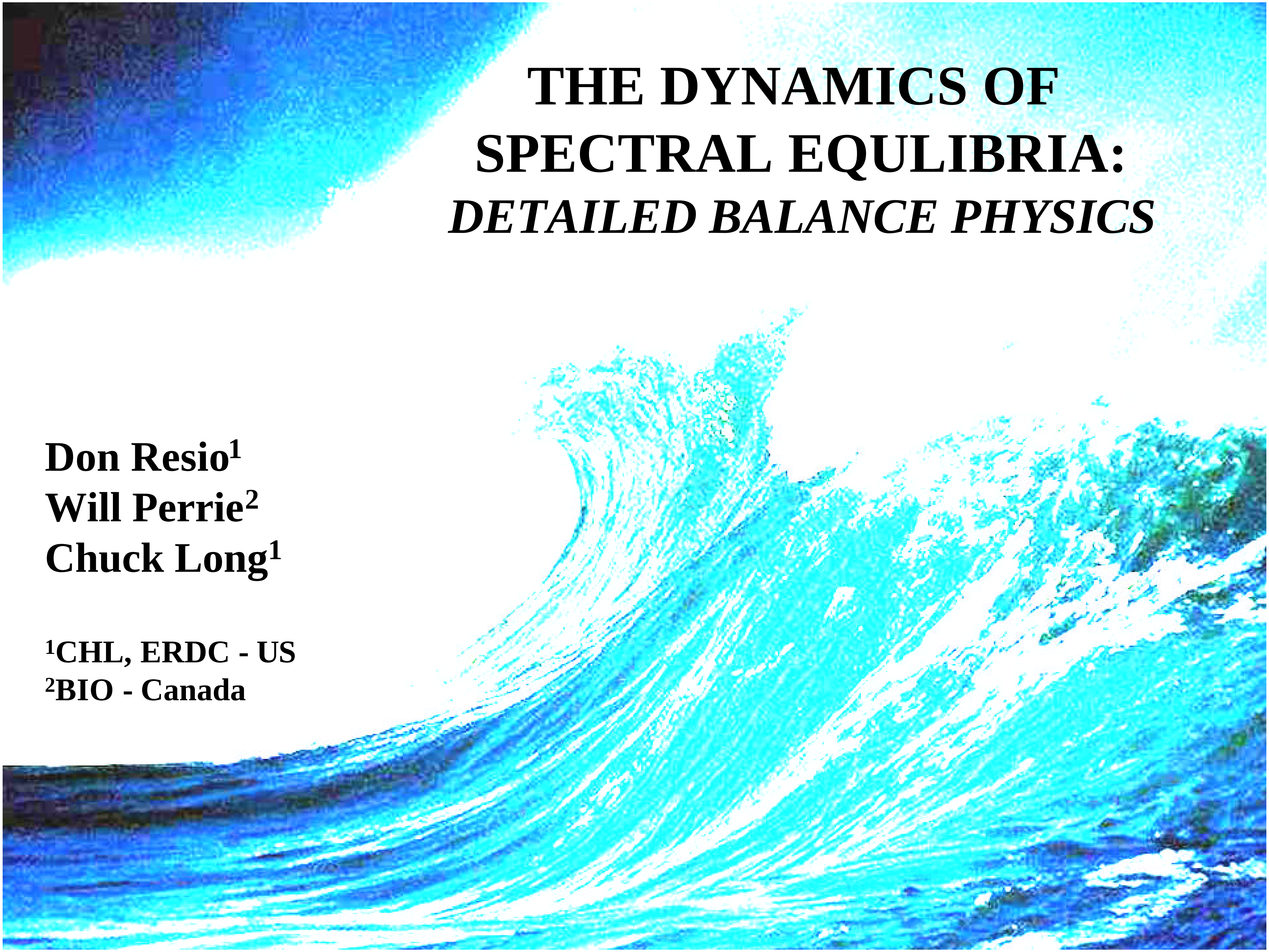

THE DYNAMICS OF SPECTRAL EQULIBRIA: *DETAILED BALANCE PHYSICS*

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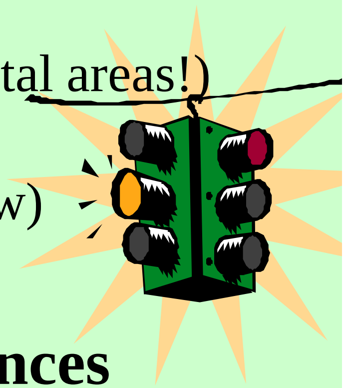
In 1985, The WAM Model Was Published - WAVEWATCH

- 1st detailed balance model – response to Hasselmann parametric attempt?
- 3 source terms included (S_{in}, S_{ds}, S_{nl})
- Initially tuned to fetch laws and fully-developed constraints

$$\hat{E} = m_1 \hat{x}^{q_1}, \quad \hat{f}_p = m_2 \hat{x}^{q_2}$$

$$\text{where } \hat{E} = g^2 E_0 u^{-4}, \quad \hat{f}_p = u f_p u^{-1}, \quad x = gxu^{-2}$$

- Also, tuned to give reasonable results in various operational tests – improving skill! (but 0G, 1G, 2G were good at this, too)
- Parameterization of S_{nl} (DIA) not very accurate even for simple spectra in deep water
- Model accuracies are evaluated using “global” statistics (slowly varying wind systems)
- No significant attention paid to detailed spectral dynamics
- Does not do well in narrow or slanting fetches and shallow water (coastal areas!)
- WAM4 overpredicts wave heights in strong hurricanes (25-30%)
- Models require basin specific tuning (deep) site specific tuning (shallow)



Premise:

Understanding the processes that produce dynamic balances within wind wave spectra can help us attain accurate wave model performance correct without continual “tweaking” via tunable coefficients.

MOTIVATION FOR THIS STUDY IS NEED FOR ACCURATE WAVES IN SHALLOW COASTAL AREAS

Does Detailed Balance Wave Modeling Mean Better Physics? (LCA)

Do detailed-balance source terms in existing 3G models provide a reasonable representation of observed spectral balances in coastal areas?

How can we obtain some useful information on the wave generation process for modeling?

Option 1: Run models and improve their performance via continued calibrations – very site/basin specific

Option 2: Perform careful detailed-balance field studies to define wind input and wave breaking – “micro-scale studies” very difficult and subject to interpretation

Option 3: Examine similarity constraints on wave processes (energy flux balances)
- “macro-scale” studies – The balance within each domain provides unique information on the physics governing the processes.

I guess my option came up “3”.

For many years spectral shapes have been based on the characteristics of their "equilibrium range" Phillips (1958) Pierson-Moskowitz (1964), JONSWAP, Toba (1973), Donelan et al (1985), Resio et al (2004), Smith and Vincent (2004)

shallow water

From observed spectra we can define

b**=<ß>=<?F(k)k**$^{5/2}$**>   or   a**$_4$**=<?E(f)f**^{4}**>**

Where the <> represents an average over the equilibrium range. For dimensional consistency we expect:

$$b = \frac{a_0}{2}(u_a - u_0)g^{-1/2}$$

Where u is a velocity term expected to relate to wind speed

Range of parameters included in Resio et al. (2004) data sets:

$0.39 < u/c_p < 6.65$

$0.7 < k_{eq}h <$ very deep [kh>>1]

$0.0004 < H_{m0}/h < 0.4$

$0.05 < k_p H_{m0} < 0.41$

Based on Zakharov (1999) Stokes Number has some stringent limits on weak interaction theory

$$N = \frac{ka}{(kh)^3} << 1 \quad \{narrow - banded\}$$

$$N(kh)^2 << 1 \quad \{broad - banded\}$$

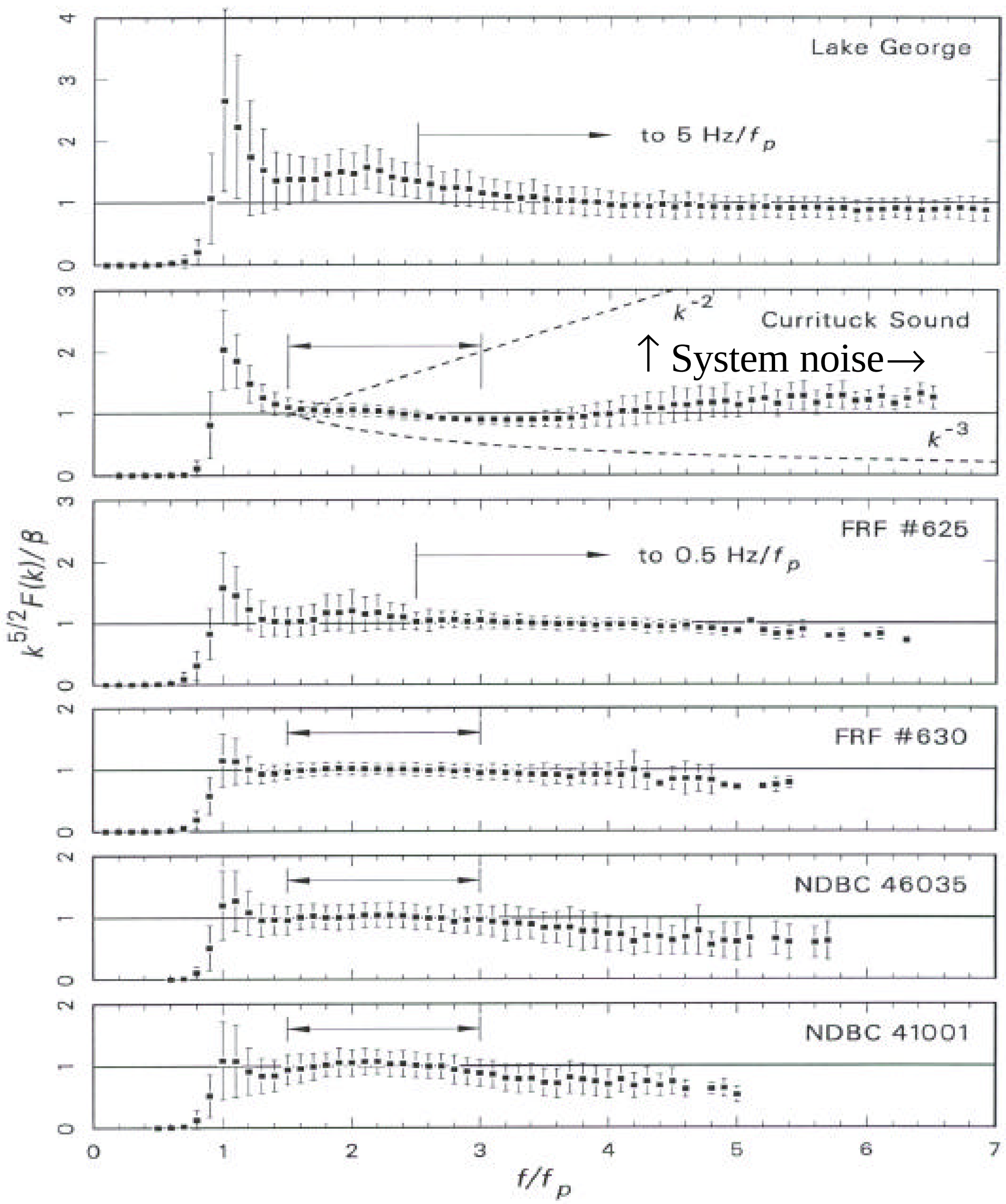
Data are roughly within these limits

Note that these are plotted on a linear scale.
ß is related to Toba's equilibrium constant (velocity)

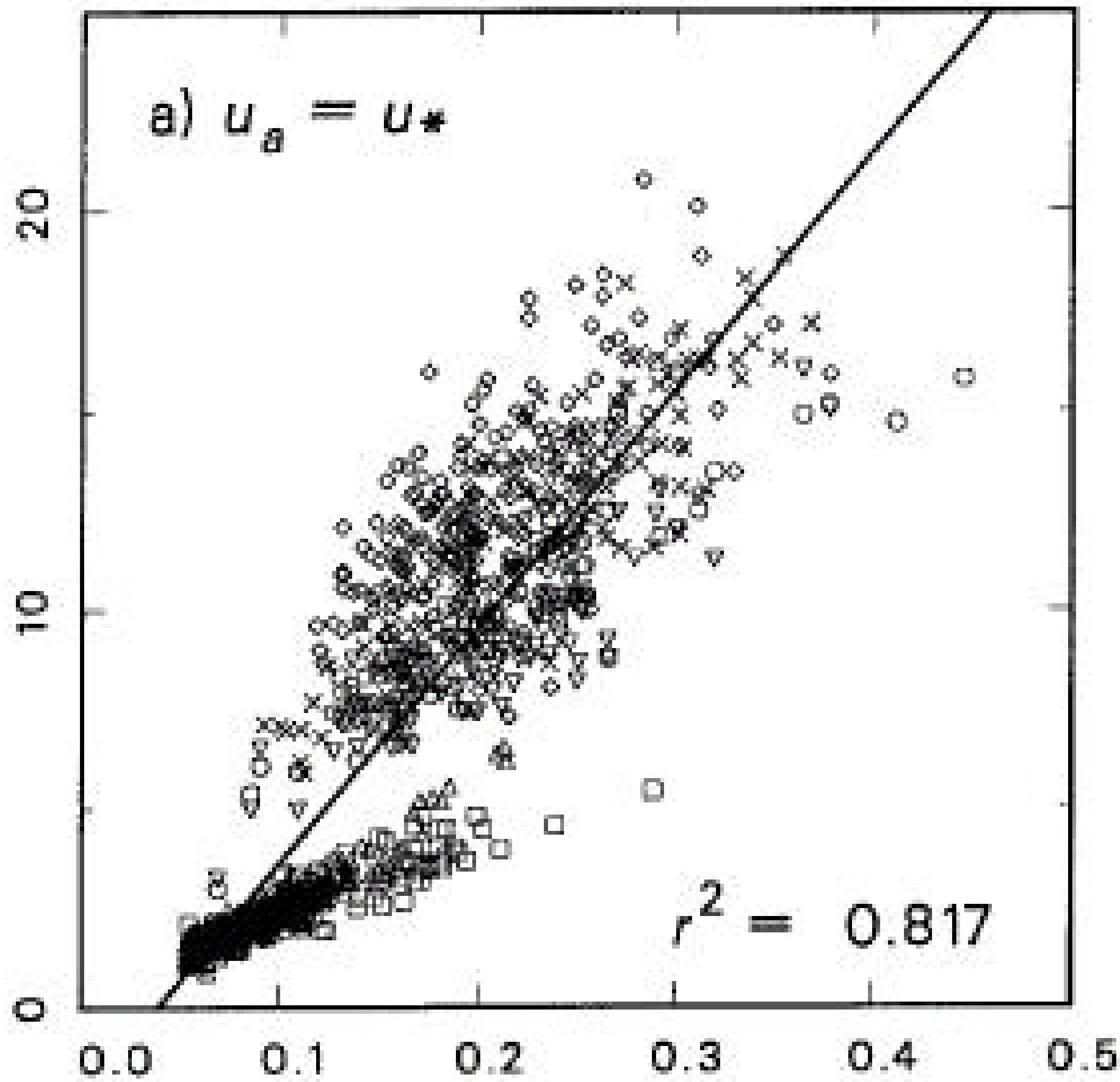
But what do data show that the ß values depend on?

Most assume that ß:wind speed
Since the equilibrium Range gets written as

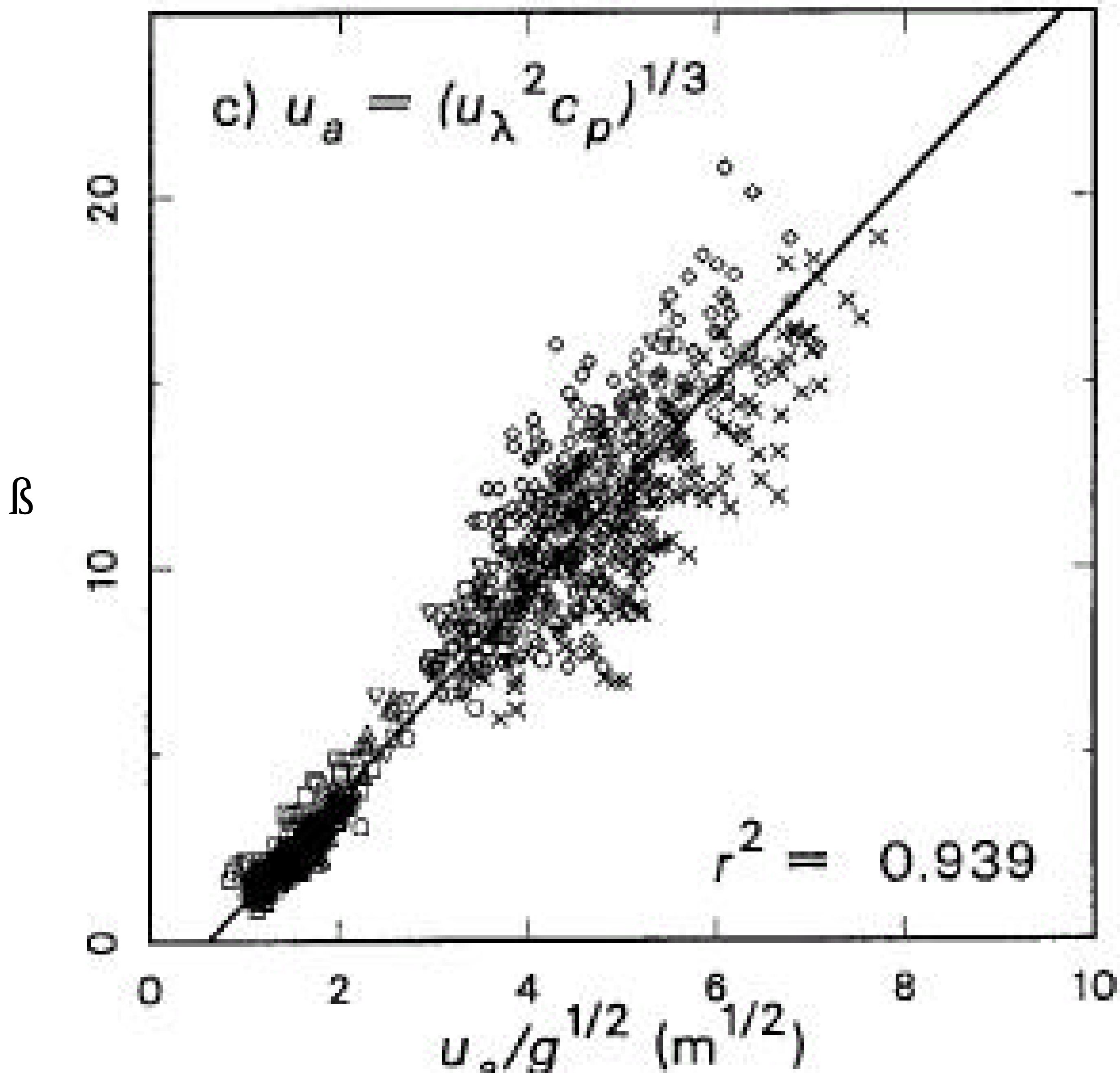
E(f) :ugf^{-4}



ß

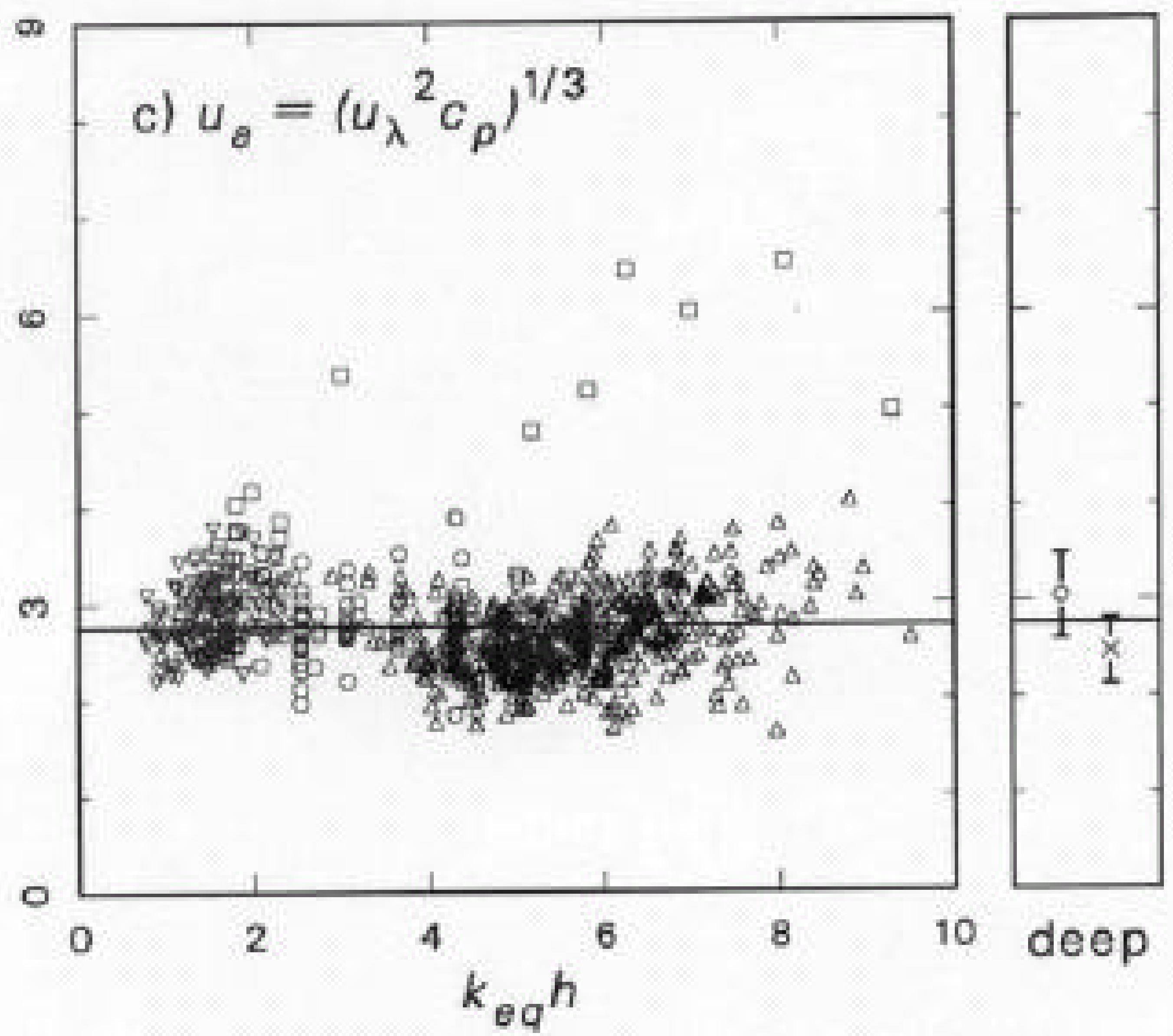


$u_a /g^{1/2}$ ($m^{1/2}$)



c) u_a = (u_λ^2 c_p)^1/3
r^2 = 0.939
ß
u_a/g^1/2 (m^1/2)
0
10
20
0
2
4
6
8
10

$$\frac{b'}{u_a - u_0}$$

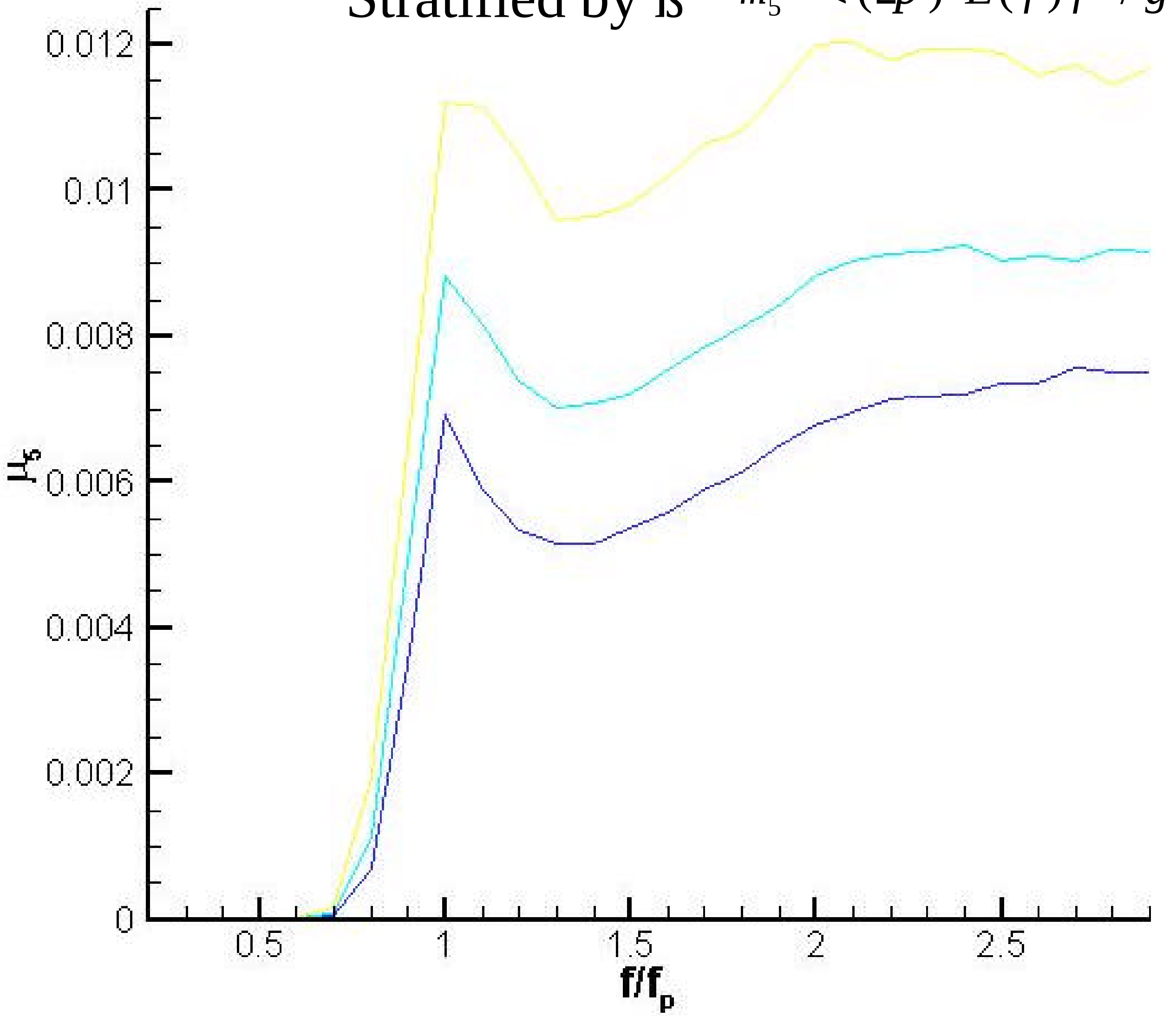


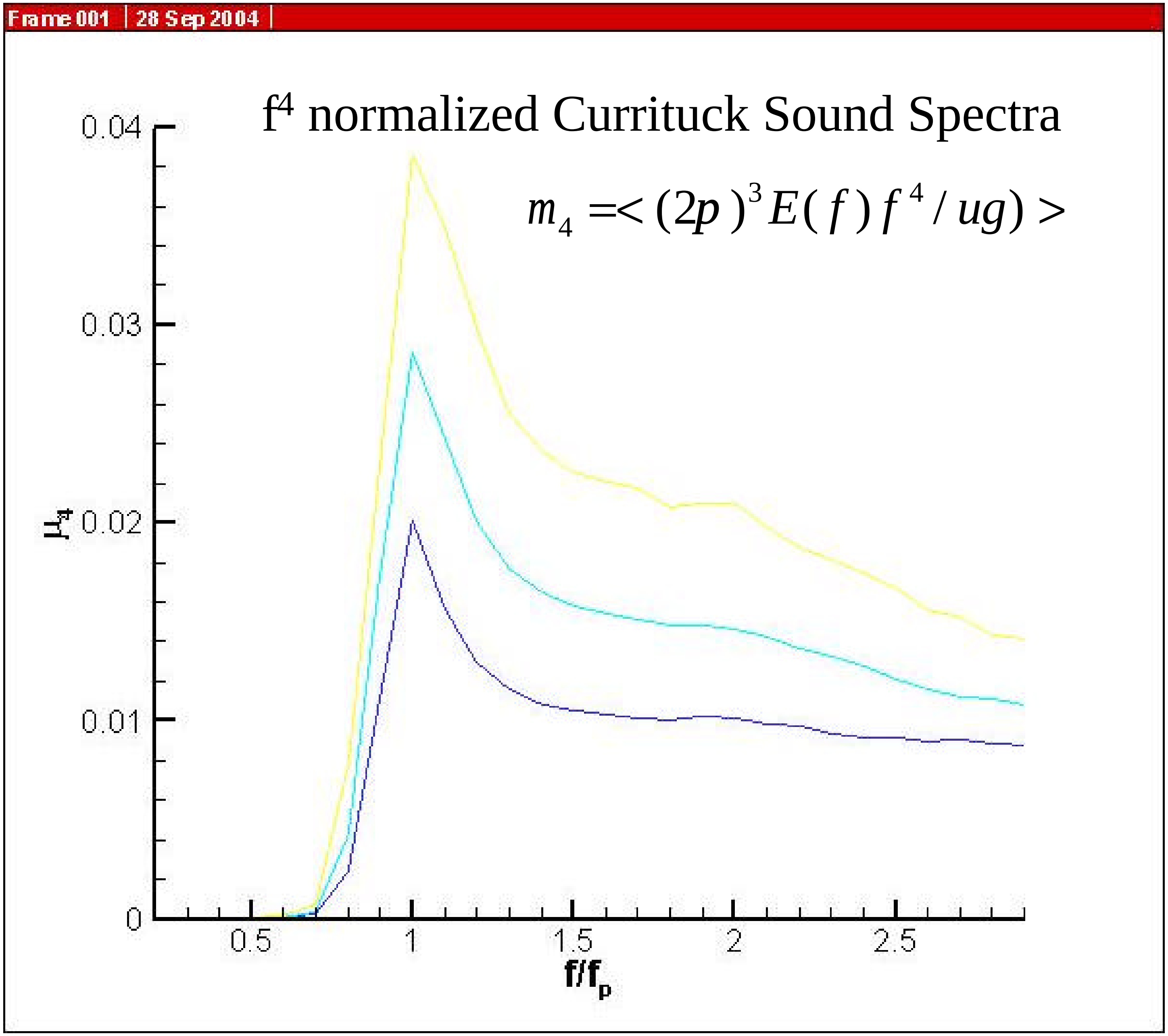
Resio et al results show no dependence of the equilibrium range coefficient on relative depth

Frame 001 | 28 Sep 2004

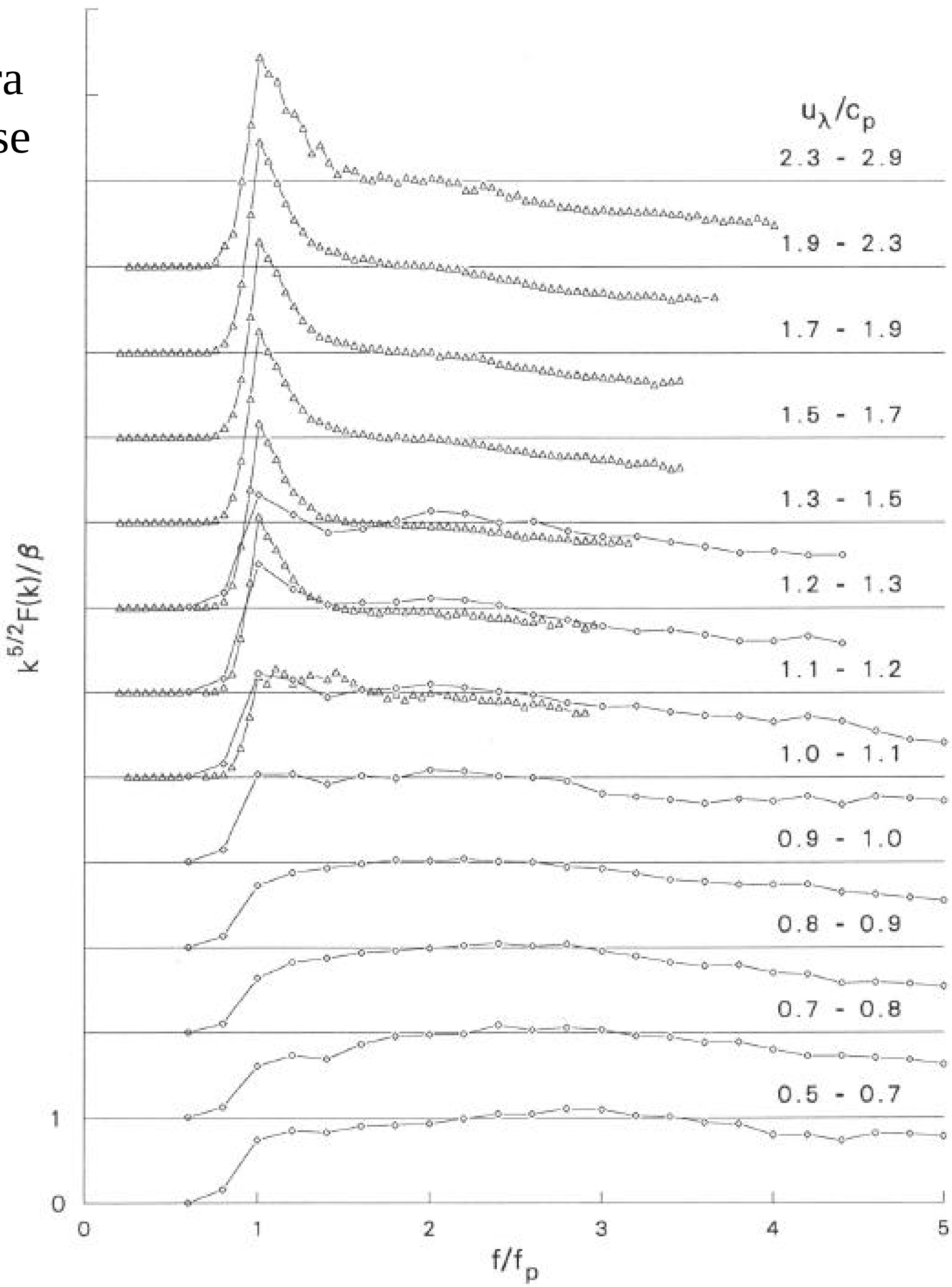
f^5 normalized Currituck Sound spectra

Stratified by ß $m_5 =< (2p)^4 E(f) f^5 / g^2 >$




Frame 001 | 28 Sep 2004
f4 normalized Currituck Sound Spectra
m4 =< (2p)3 E(f) f 4 / ug) >
0.04
0.03
0.02
0.01
0
μ4
0.5
1
1.5
2
2.5
f/fp

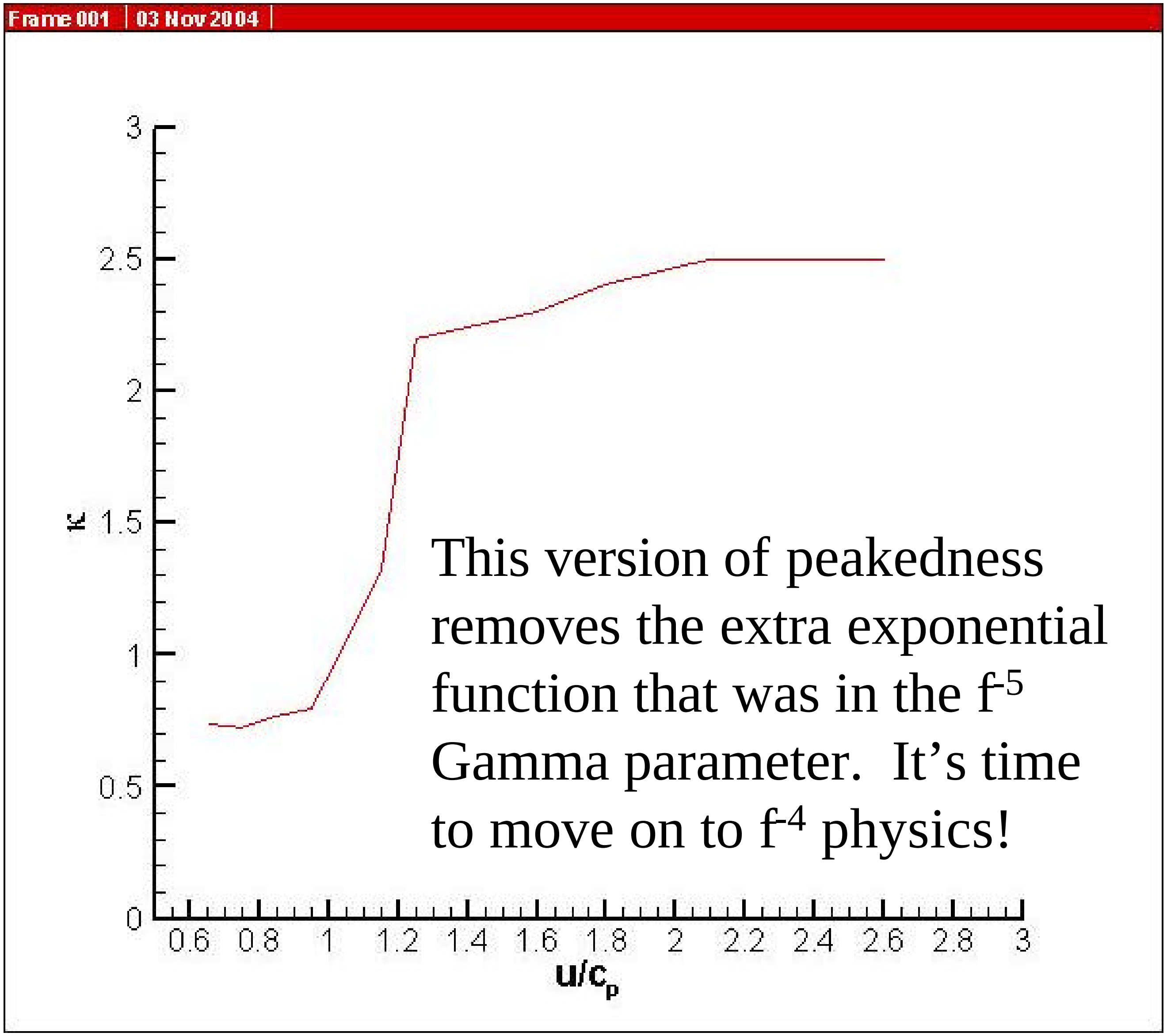
Normalized spectra stratified by inverse wave age based on data from Currituck Sound Bering Sea Atlantic Ocean



Note the "interesting" slope at the front of very old spectra

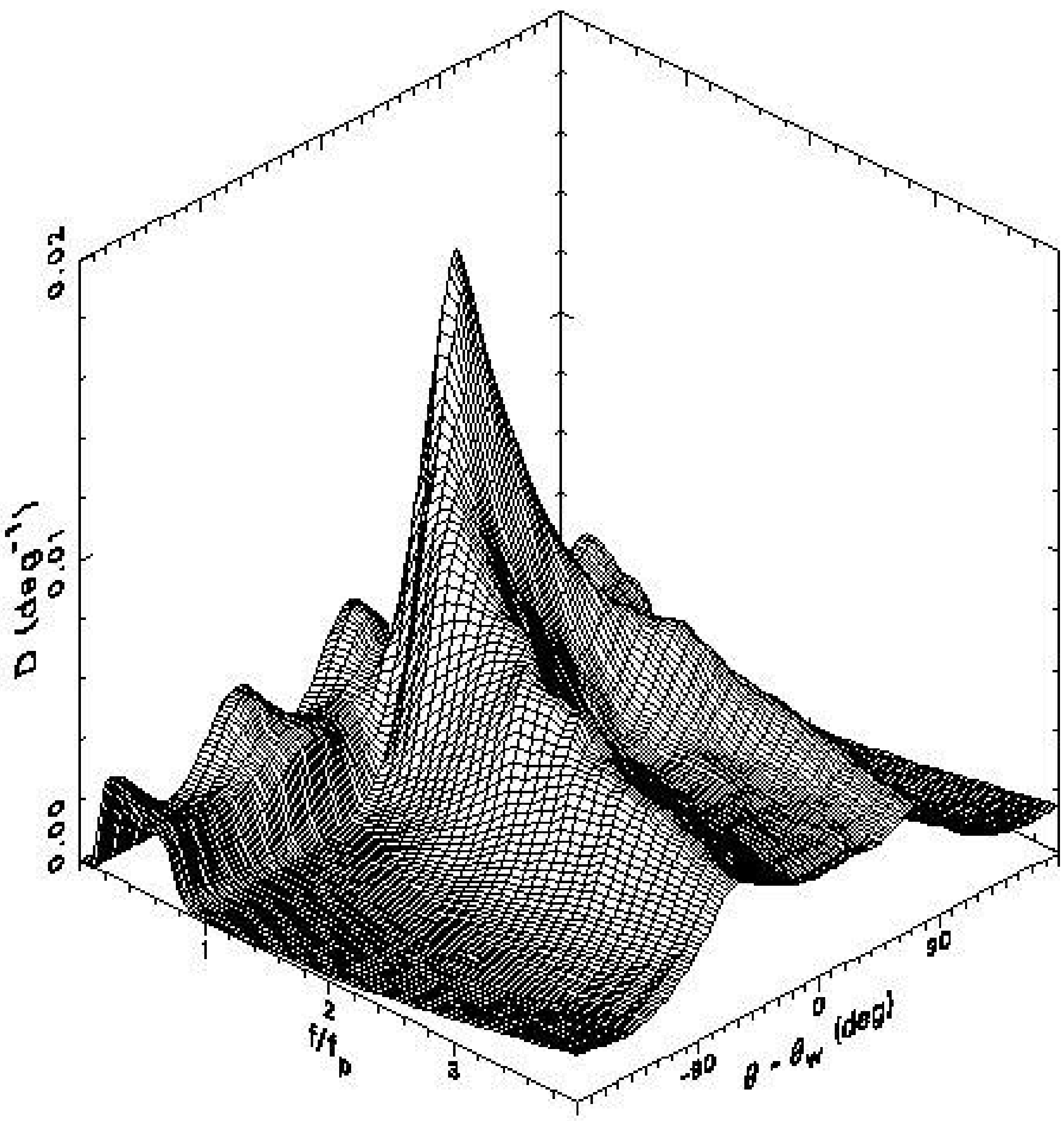
New definition of peakedness
For f^{-4} spectra:

$$k = \frac{E(f_p)}{< a_4 u g (2p)^{-3} >}$$

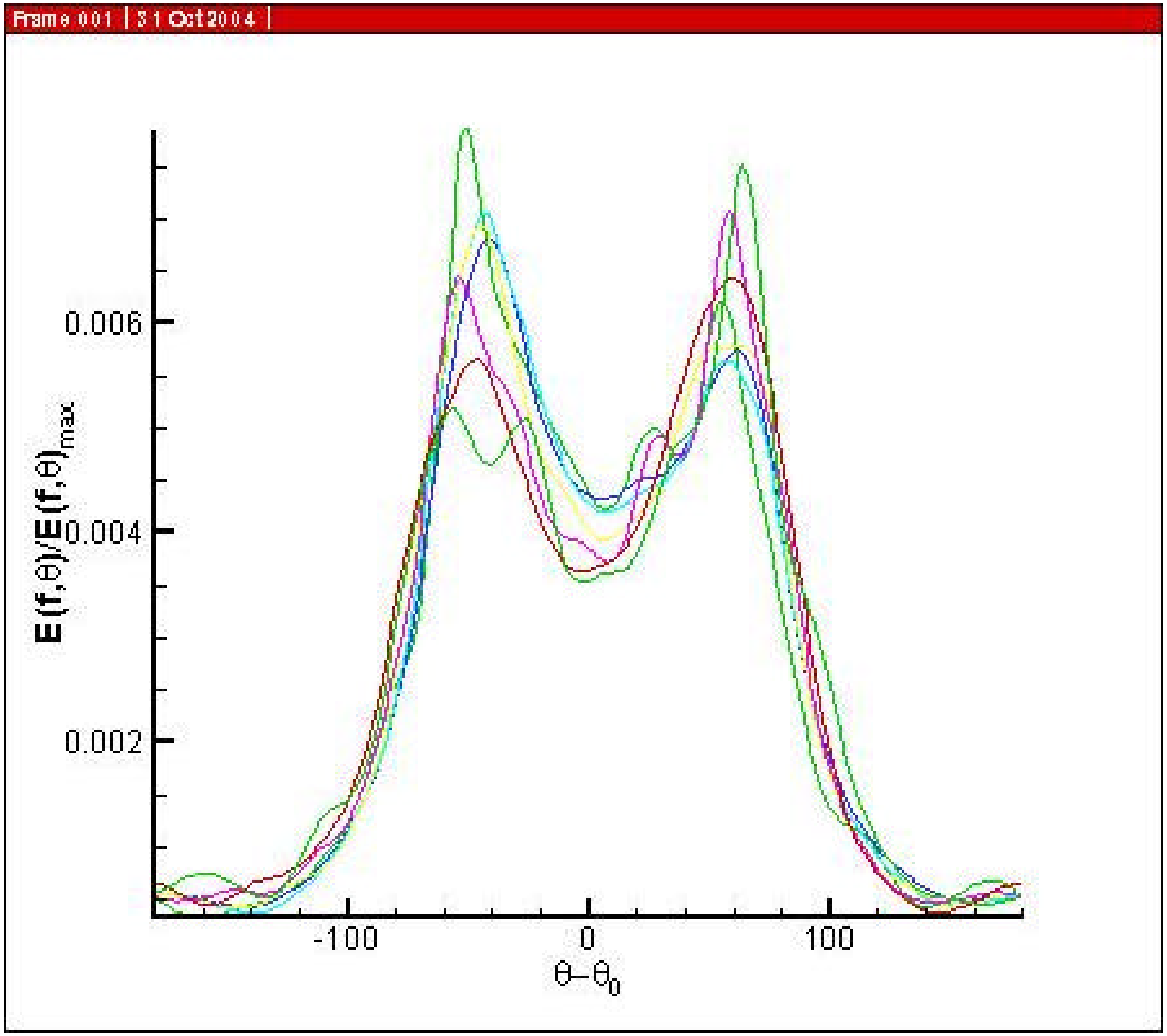


This version of peakedness removes the extra exponential function that was in the f^{-5} Gamma parameter. It's time to move on to f^{-4} physics!

Currituck Sound
Mean Directional Distribution Function
all long-fetch cases (327), $u_{10} > 7$ m/s
centered on wind direction, grouped by f/f_p



Essentially all recent studies have shown that the tail of the spectrum bifurcates at about f/f_p=1.4-2.0 Below we see data from Long and Resio (2004) study in Currituck sound for “slices” at f/f_p>2.



Directional distribution observed by Wang and Hwang (2001)

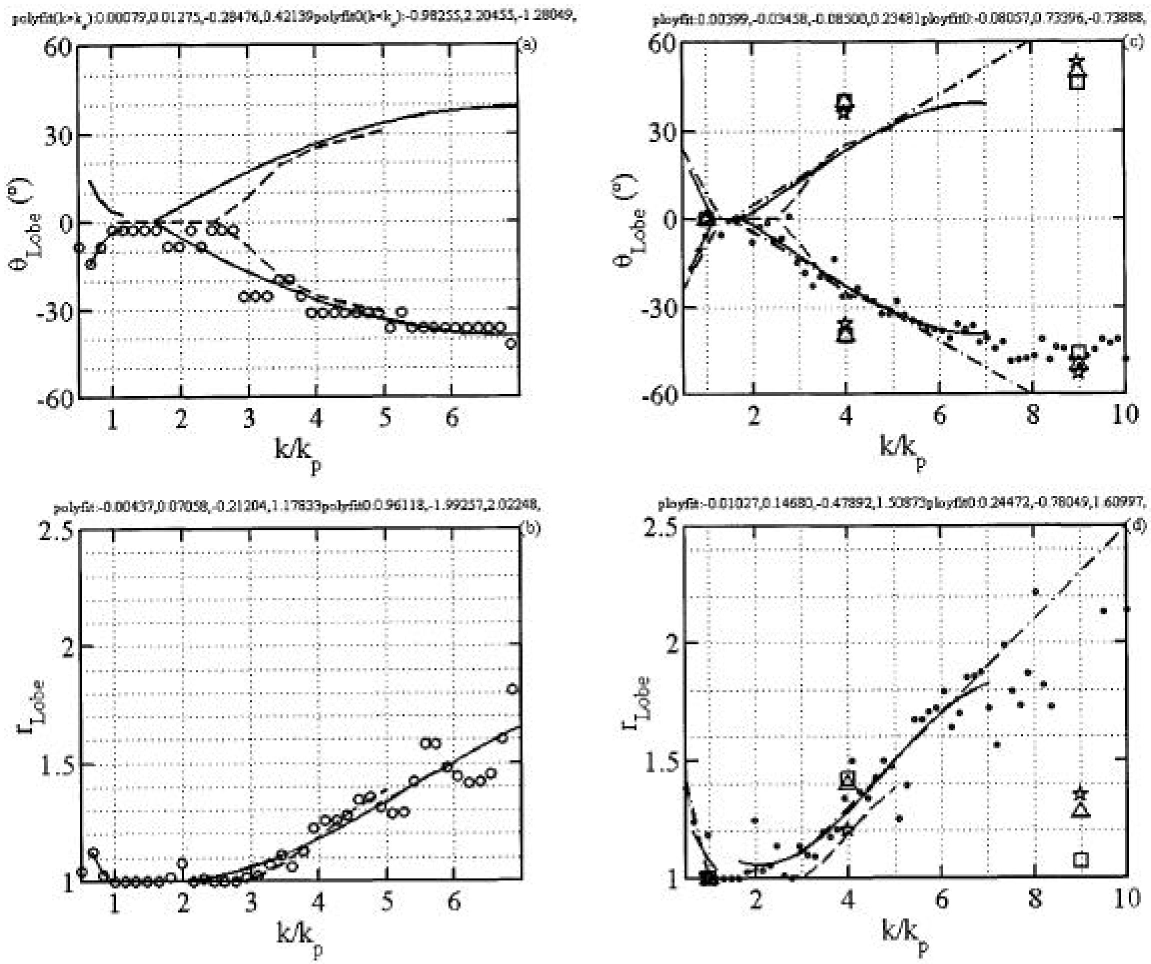


FIG. 10. The lobe angle (a, c) and the lobe ratio (b, d) of the bimodal distribution. In (a) and (b), the directional resolution is degraded to have a uniform resolution as done for the Fourier decomposition procedure. Solid curves: from polynomial fitting (coefficients listed in Table 2), and dashed curves: computed by $D_{kFFTs}(\theta)$. In (c) and (d), the directional resolution is not degraded. The dashed-and-dotted curves are computed from Eqs. (20) and (21). Numerical results of Banner and Young (1994) on the effect of dissipation functions are shown with stars: quadratic, triangles: cubic, and square: quartic frequency dependence. Quasi-steady wave field.

GOOD NEWS/BAD NEWS:

- Lots of data suggests that nature may be fairly well organized in terms of the “self-similar” patterns generated in wave spectra
- These self-similar patterns are not reproduced by existing third-generation wave models, i.e. we need to revisit the physics.

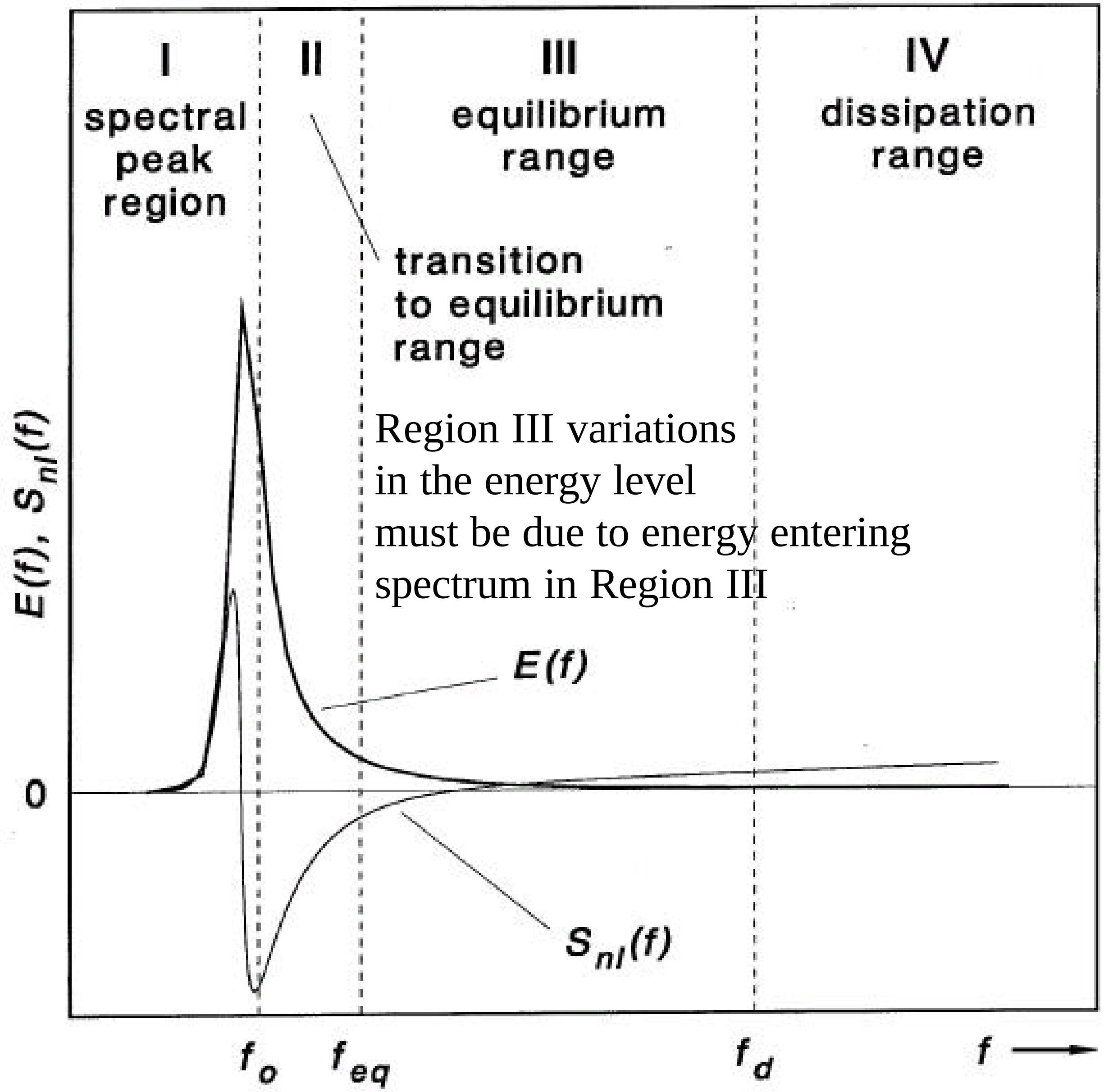
A simple way to understand the parameter f_0 is that

$$\int_0^{f_0} S_{nl}(f)\,df = 0$$

It also is where the net Flux = 0

Region I – all net energy is retained – integral of wind input –dissipation from 0 to f_0

Region II – must provide energy into equilibrium range



Concept of Spectral Regions Based on Source Term Balances

In deep water, with

g = gravity

u=appropriate wind speed scale $(u_l^2 c_p)^{1/3}$

f = frequency

$$E(f) \sim g^a u^b f^{-n}$$

$$E(f) \sim a_5 g^2 f^{-5}$$

$$E(f) \sim a_4 g u f^{-4}$$

Dimensionally consistent forms for the equilibrium range

The radiative transfer equation for wind waves:

$$\frac{\partial E(f,q)}{\partial t} = \vec{c}_g \cdot \vec{\nabla} E(f,q) + \sum_k S_k(f,q)$$

"Accepted" representation for source terms

$$\sum_k S(f,q) = S_{in}(f,q) + S_{ds}(f,q) + S_{nl}(f,q)$$

wind dissipation nonlinear interactions

Method of definition of f_0

$$\int_0^{f_0} \int_0^{2p} S_{nl}(f,q)\, dq\, df = 0$$

Fluxes through the equilibrium range via Hasselmann/Zakharov

Note: These 2 forms have been shown to be exactly equivalent –

There are no tunable coefficients for these interactions

$$\Gamma_{E-net} = \frac{\Lambda b^3}{g}$$

Any net external source term inside the equilibrium range MUST create a divergence in S_{nl}

$$\frac{\partial \Gamma_{E-net}}{\partial f} = \int_0^{2p} S_{in}(f,q) + S_{ds}(f,q)\,dq \approx \frac{\Lambda}{g}\frac{\partial b^3}{\partial f}$$

How certain are we that the slope is "0" for f^{-4}

Using Student's t test we have

$$b = b' \pm t_{a/2}\frac{\sqrt{\dfrac{\sum_1^n [y-\bar{y}(x)]^2}{n-2}}}{\sqrt{x^2-(1/n)(\sum_1^n x)^2}}$$

$$b = 0 \pm 0.14$$ (95% confidence interval)

Indicates that net external source inside equilibrium range is only about 10% of net external source inside Region II

Rate of extraction of energy from the wind is given by [Lighthill, 1962)]

$$\frac{\partial E}{\partial t} = r_a \int_0^\infty \langle uw \rangle \frac{\partial \bar{u}}{\partial t} dz$$

Where u,w are the horizontal
And vertical wind components

This extraction rate can be related to the curvature of the wind profile

$$r_a \frac{\partial \bar{u}}{\partial t} \approx r_a \frac{\partial^2 \bar{u}}{\partial z^2} \langle hw \rangle$$

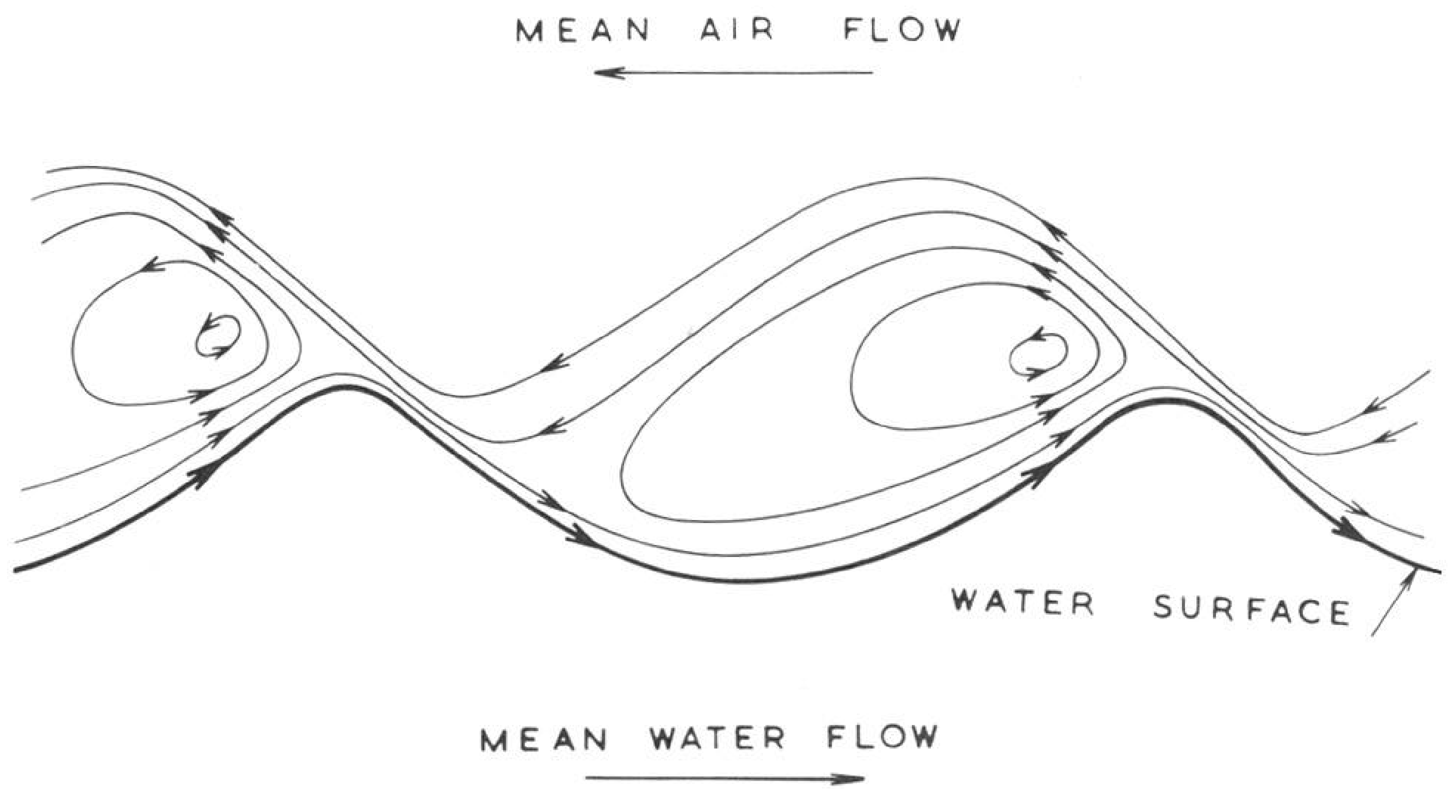
Where h is a small
displacement

And the total rate of energy and momentum loss from the atmosphere
can be written as

$$t_E = ct_M = \frac{1}{4} r_a L \frac{\bar{u}''(z_c)}{\bar{u}'(z_c)} w_0^2$$

This in turn yields the conventional form for the Miles wind input

$$S_{in}(f,q) \square\ b_{in} E(f,q) f \cos(q - q_{wind})$$



Flow field in air passing over waves "visualized" from smoke injected into a laboratory flume. Frame of reference is moving with the phase speed of the spectral peak. Note that the "cats eyes" are shifted with respect to the wave crests.

For infinitesimal waves the Orr-Sommerfield instability theory provides a reasonable approximation for monochromatic waves; but what about the air-flow above wave spectra

In this case we no longer have a vortex force concentrated exactly at a critical matching height, but instead have a quasi-resonant interaction

$$\langle hw \rangle = \lim \frac{w_0^2(z)}{2w} \sin(2wt) \neq 0$$

$$w = \frac{2p[\bar{u}(z) - c_p]}{L}$$

In this form we expect the approach to a delta function to be of the typical exponential form

$$S_{in}(f,q) \square\, b_{in} E(f,q) f_p \frac{e^{-(s_n n_f)^2}}{\sqrt{2p s_n}}$$

Or for frequency-direction input

$$S_{in}(f,q) \square\, b_{in} E(f,q) f_p \frac{e^{-(s_n n)^2}}{\sqrt{2p s_n}} \frac{e^{-(s_{q'} q')^2}}{\sqrt{2p s_{q'}}}$$

In this case, the rate of extraction of momentum will be

$$t_{M-total} = \iint b_{in} E(f,q) f_p \frac{e^{-(s_n n)^2}}{\sqrt{2p s_n}} \frac{e^{-(s_q q')^2}}{\sqrt{2p s_{q'}}} df dq$$

If we choose the power of u/c in S_{in} to be

$$b_{in} \square \left(\frac{u}{c_p} \right)^{4/3}$$

We have for the total rate of momentum transfer

$$t_{M_{in}-total} = l \frac{u^2 c_p}{g}$$

Which is consistent with the power law in the relationship between Dimensionless energy and dimensionless fetch being approximately 1

For the case of wave breaking, if there is little or no wind input into the equilibrium range, and if we no longer assume that we must Force wind input and wave breaking to balance at "full-development" we can allow S_{ds} to operate <u>primarily</u> at high frequencies- where it Must provide an energy sink that balances the flux of energy past f_d

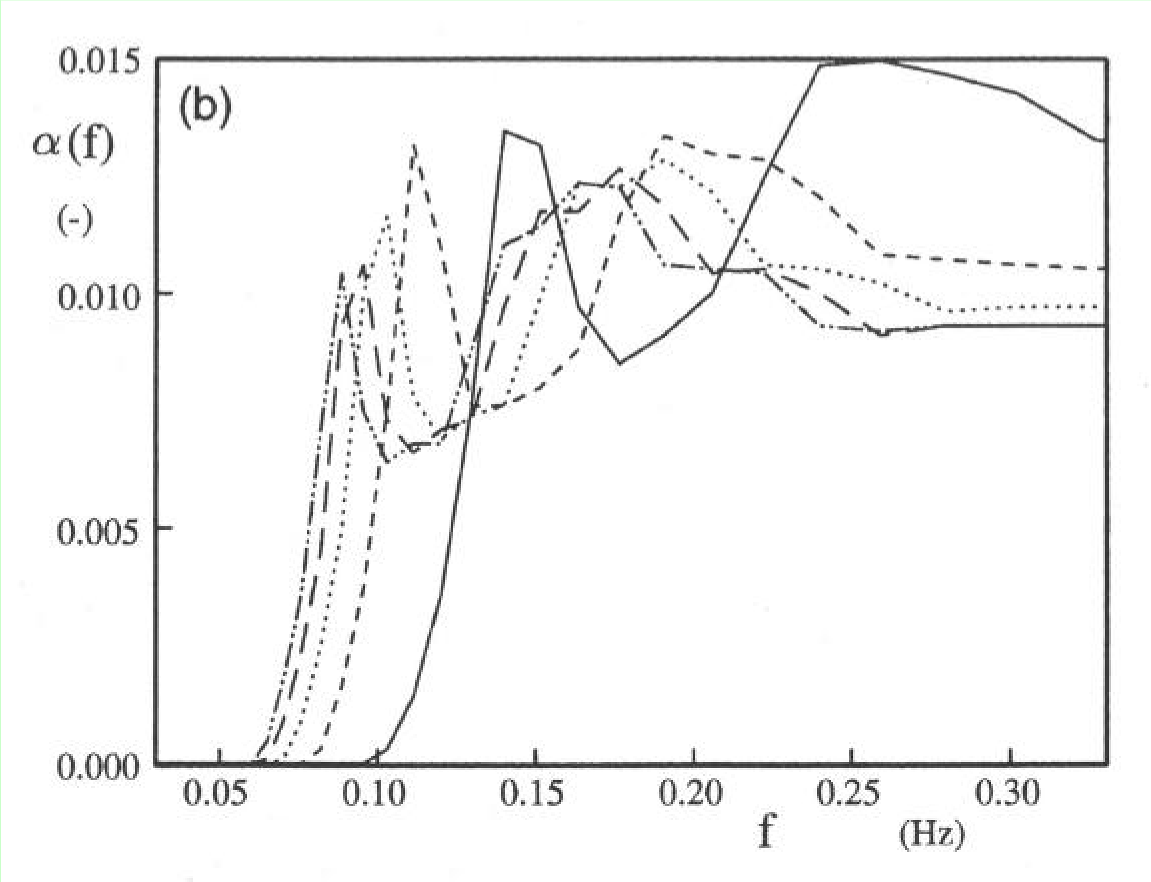
Would this create a change spectral shape – f^{-5} , k^{-3}?

$$\int_{f_d}^{\infty}\int_{0}^{2p} S_{ds}(f,q)\,df\,dq = \Gamma_{E-net}\,\Big|_{f_d}$$

For a simple approximation within a model we can use a relaxation Toward a parametric (dynamic) "saturation" limit

$$S_{ds}(f,q) \square -\left(1-f\left(\frac{E_s(f)}{E(f)}\right)\right)E(f,q)$$

Important note for shallow water – existing breaking concepts do not maintain a $k^{-5/2}$ spectral balance in shallow water.



The application of a f^{-5} “tail” in WAVEWATCH III

But what about the need for wave breaking to “exactly” balance the sum of wind input and nonlinear interactions in the spectral peak region at full development?

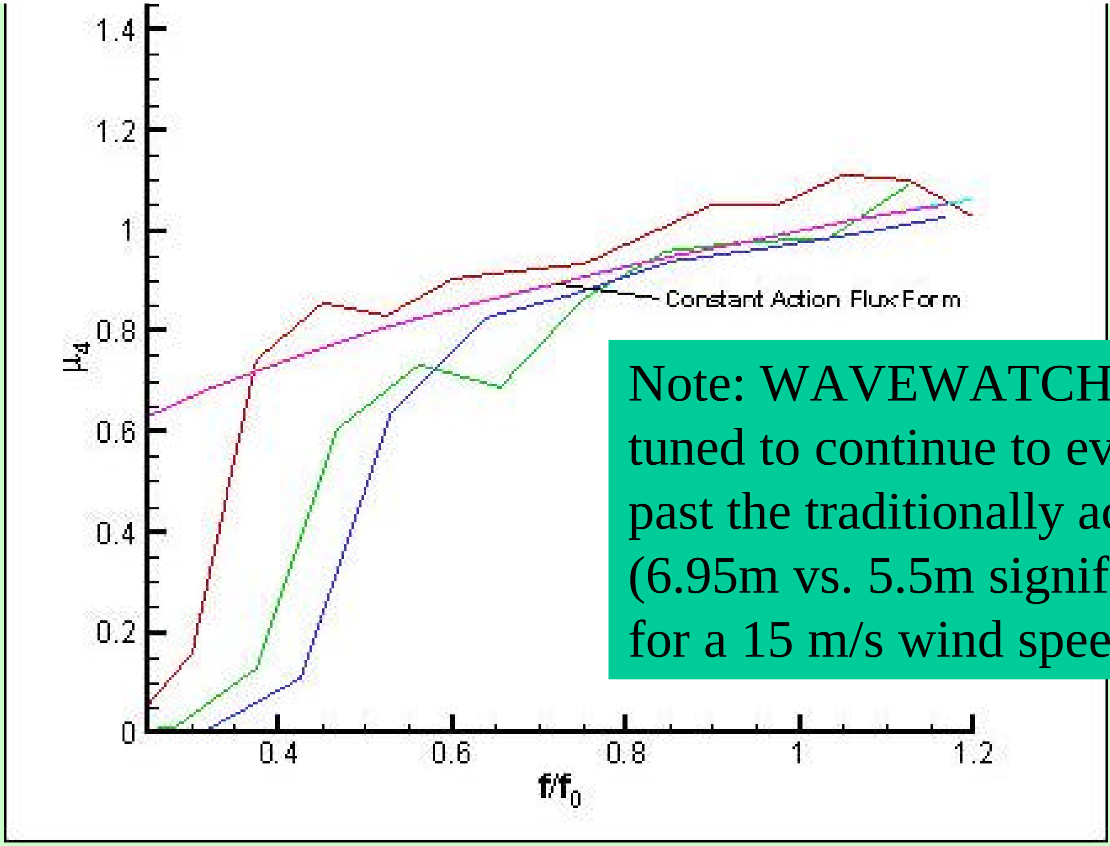
The concept of full development has been a mainstay of the wave generation paradigm for over 50 years?

Does this mean it’s real?

Some data suggests that although the rate of energy growth slows down, the peak frequency continues to propagate into lower values, but at a decreasing rate.

If spectra continue to evolve, what should they look like? What are the model consequences?

Comparison of front portion of wave spectra (frequencies below The traditional P-M cutoff) to the constant action (inverse) flux form [the $f^{-11/3}$ form first shown by Zakharov (1966, 1982)]

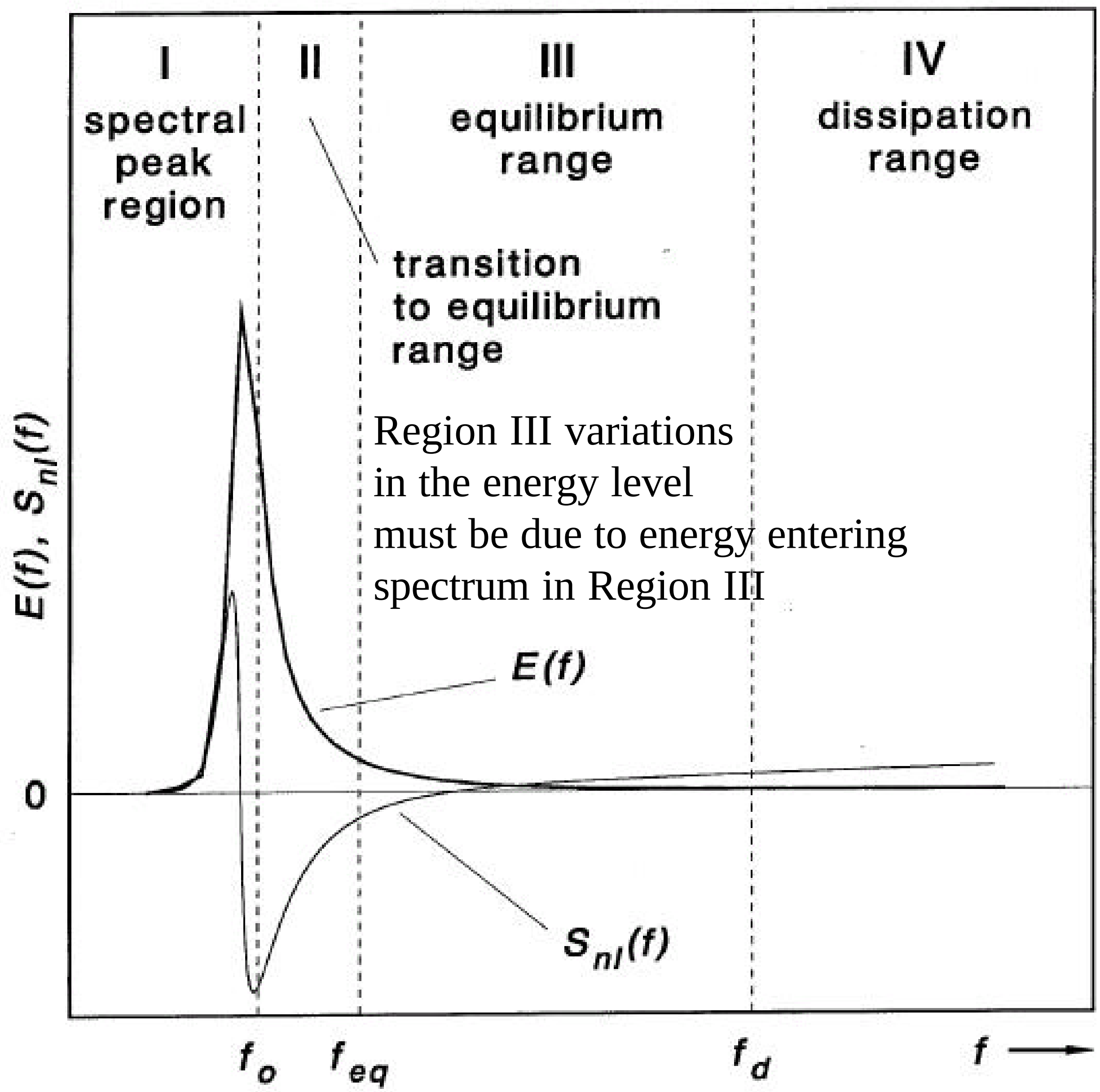


Note: WAVEWATCH III has been tuned to continue to evolve far past the traditionally accepted PM limit (6.95m vs. 5.5m significant wave height) for a 15 m/s wind speed

Note: This is a "work in progress" and will be the focus of a subsequent paper

We may need a new spectral region added to this figure.

As c_p approaches the limit of effect feedback from the Miles' input, the form/rate of wave generation will be drastically effected – but will continue to develop asymptotically slower creating an $f^{-11/3}$ form (Zakharov range) at frequencies less than the PM limit. H_s will continue to grow much along the lines of what theWW3 model has been tuned to do.



Concept of Spectral Regions Based on Source Term Balances

Scaling for nonlinear interaction in f^{-5} spectra

$$S_{nl}(f) \propto a_5^3 f_p^{-4} G_1(f/f_p)$$

Scaling for nonlinear interaction in f^{-4} spectra

$$S_{nl}(f) \propto b^3 f_p^{-1} G_2(f/f_p)$$

Scaling for nonlinear interaction in f^{-4} spectra with Resio-Long energy level coefficient

$$S_{nl}(f) \propto (uc_p)^2 G_2(f/f_p)$$

Total momentum transfer rate into spectrum and fluxed Through equilibrium range

$$t_{M_{nl}} \propto \frac{u^2 c_p}{g}$$

Same form as wind input so self similar shape is a real Possibility – even in shallow water – since shape of eq range is controlled by Snl.

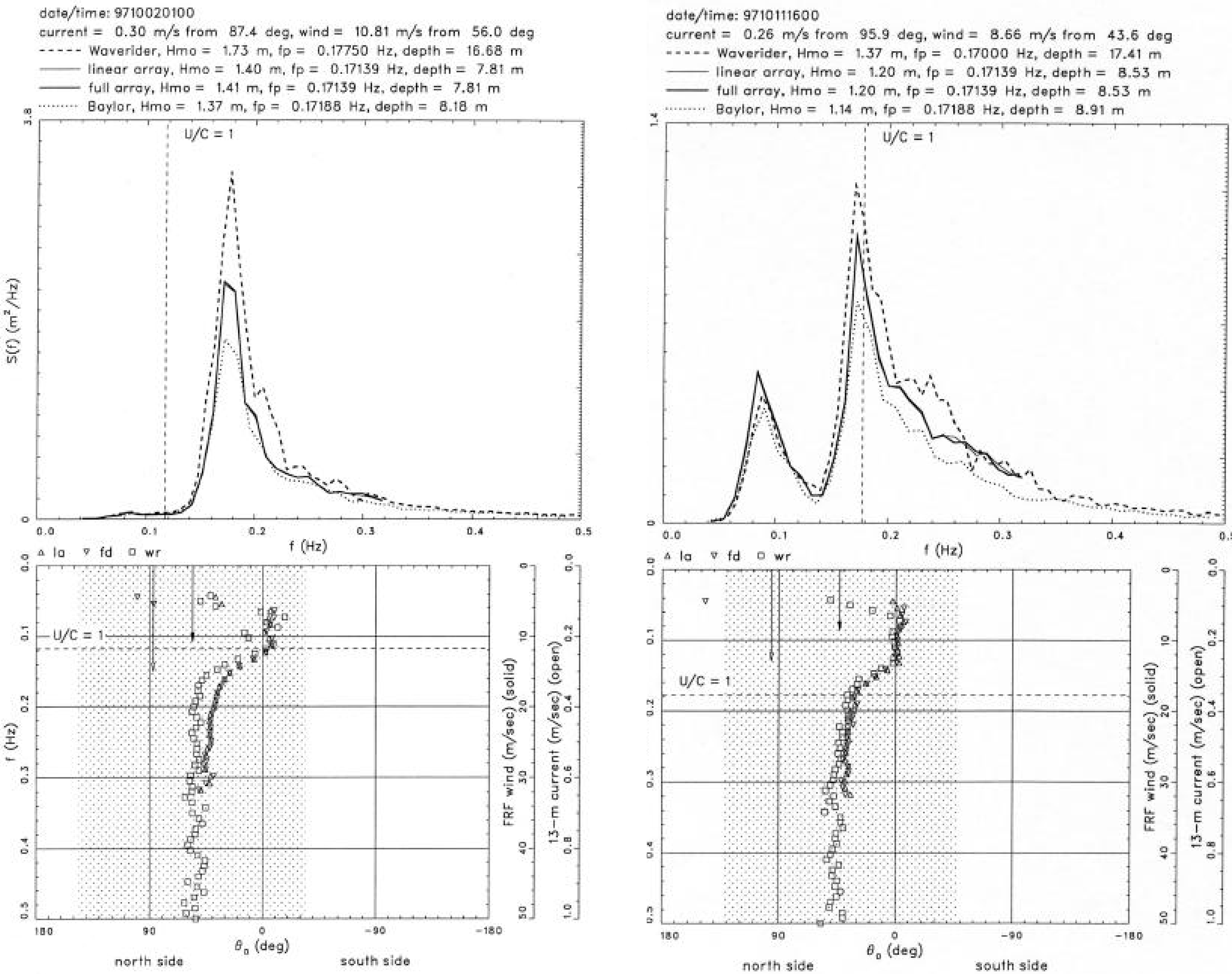
Summary of Problems in DIA

Sampling is inadequate to represent S_{nl} in even simple spectra

In spectra with angular shear, the representation is very bad

Herterich and Hasselmann scaling in SWAN type models is not applicable to shallow coastal areas

Does this mean SWAN type models cannot be tuned to work OK – no, but is does mean that SWAN does not have "better physics"



Coastal wave spectra are very complex!

How well is S_{nl} approximated?
For simple spectra – not well
For complex spectra – very badly
For shallow water – even worse

Shallow-water approximation
Modified DIA

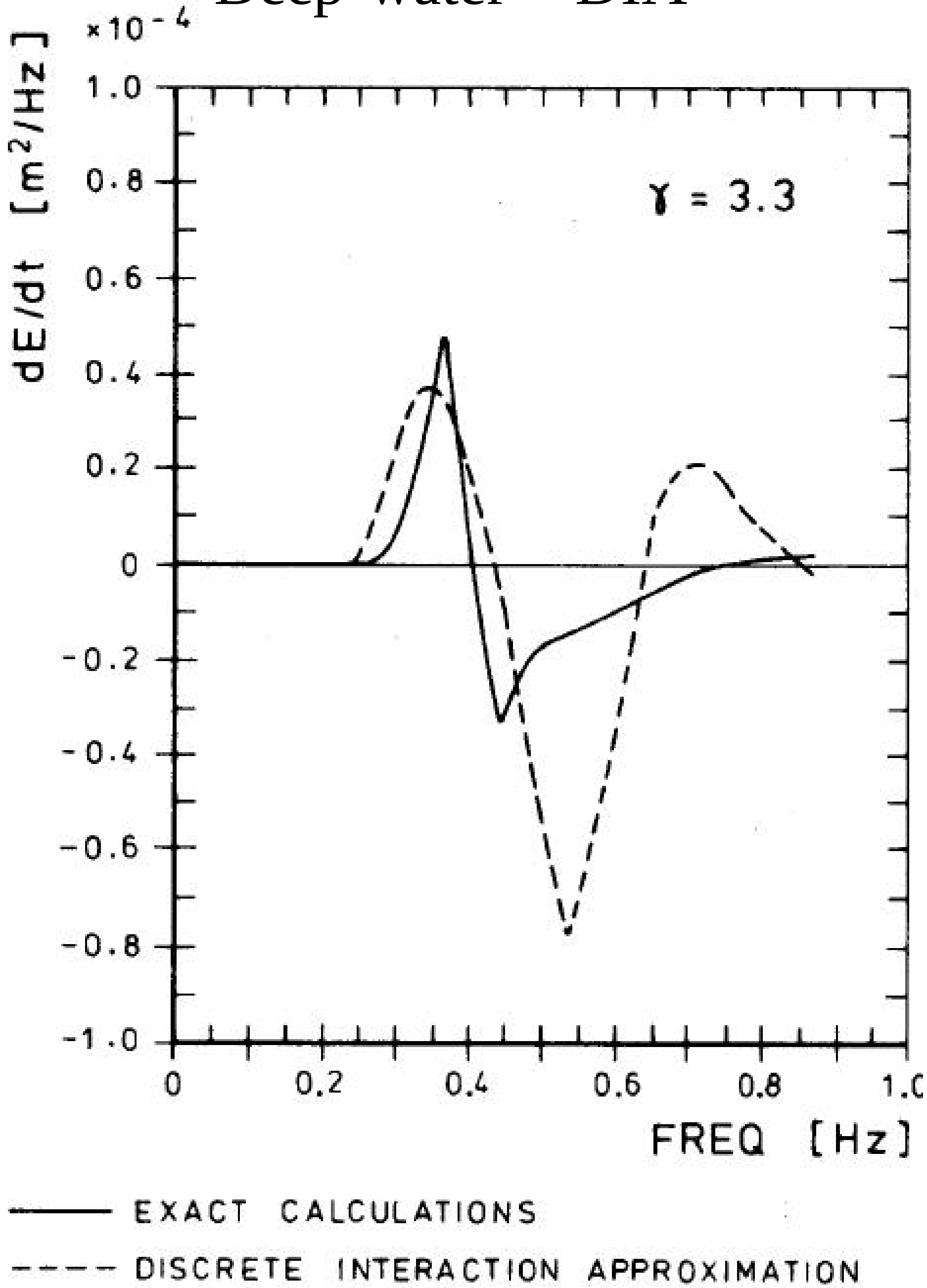
$$S_{nl}(f,q) = R(\bar{k}h) S_{\infty}(f,q)$$

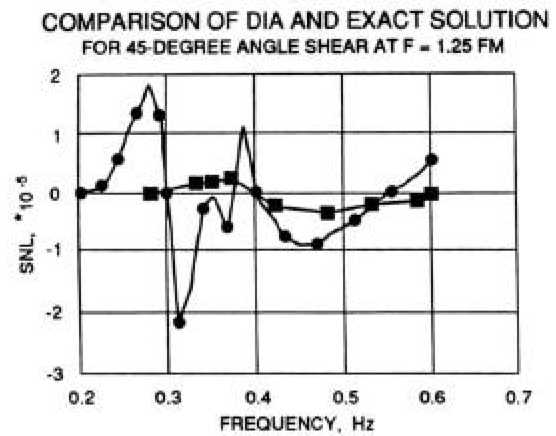
$$R(\bar{k}h) = 1 + \frac{5.5}{x}\left(1 - \frac{5x}{6}\right)\exp(-1.25x)$$

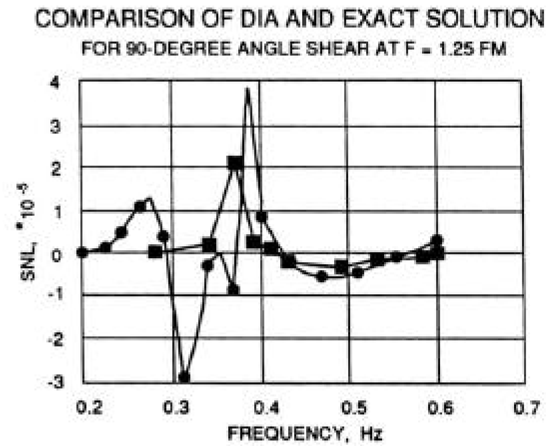
$$x = \frac{3\bar{k}h}{4}$$

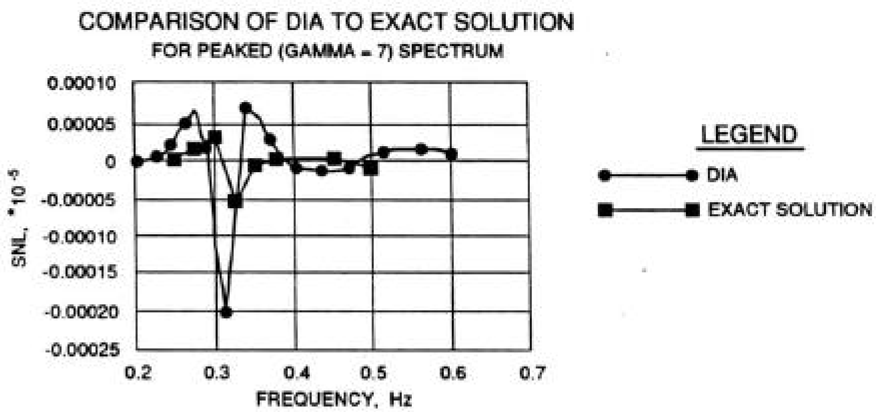
Problem: 4 to 20 sample points are used to represent integral over 3D volume requiring O(10^3) points for accurate integral (R&P 1991) - Selection of “dominant” points depends on shape of spectrum.

Deep-water = DIA

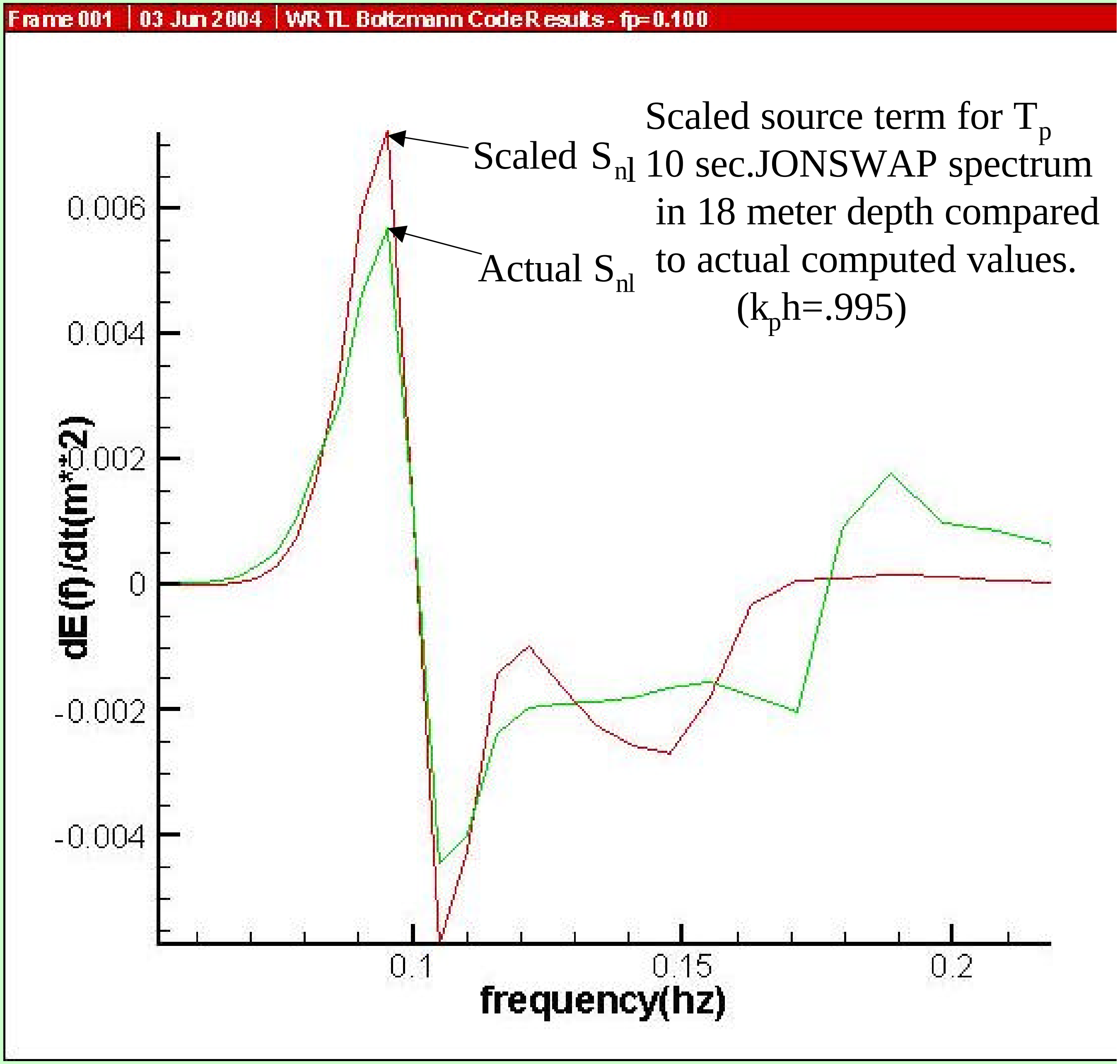


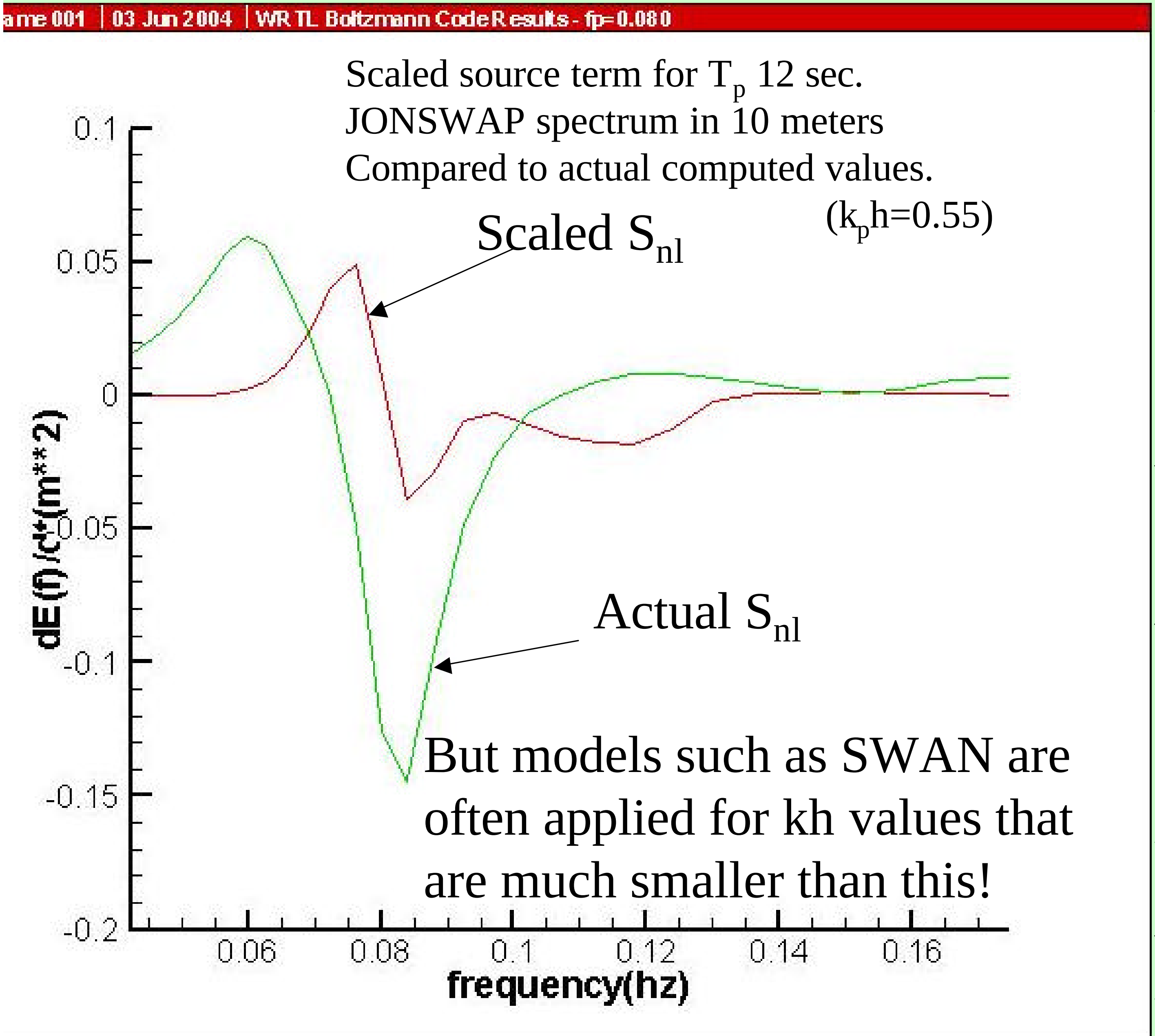

COMPARISON OF DIA AND EXACT SOLUTION
FOR 45-DEGREE ANGLE SHEAR AT F = 1.25 FM
SNL, *10^-5
FREQUENCY, Hz


COMPARISON OF DIA AND EXACT SOLUTION
FOR 90-DEGREE ANGLE SHEAR AT F = 1.25 FM
SNL, *10^-5
FREQUENCY, Hz


COMPARISON OF DIA TO EXACT SOLUTION
FOR PEAKED (GAMMA = 7) SPECTRUM
SNL, *10^-5
FREQUENCY, Hz
LEGEND
DIA
EXACT SOLUTION

Not a comparison Of DIA – but of deep full solution versus scaled full solution.



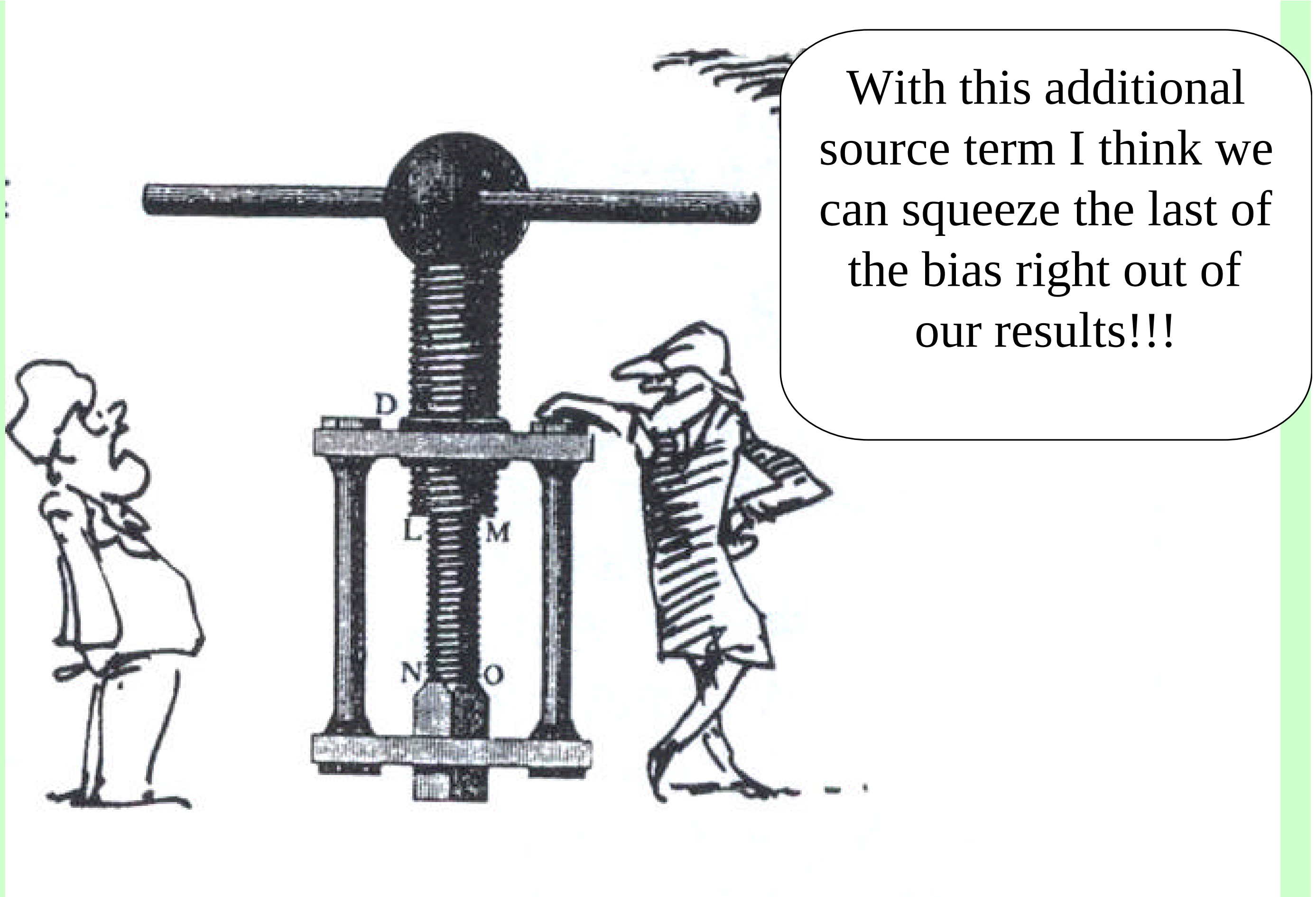


The form of Snl in SWAN cannot provide the self-similar evolution in spectral shape observed in nature.

This is a key aspect of the total energy balance in waves approaching a beach

Without this term correctly specified, empirical tuning of spurious source terms is required to achieve the energy balance actually due to Snl (TMA effects)

But can we get this new concept to work without undue tweaking?



Test:

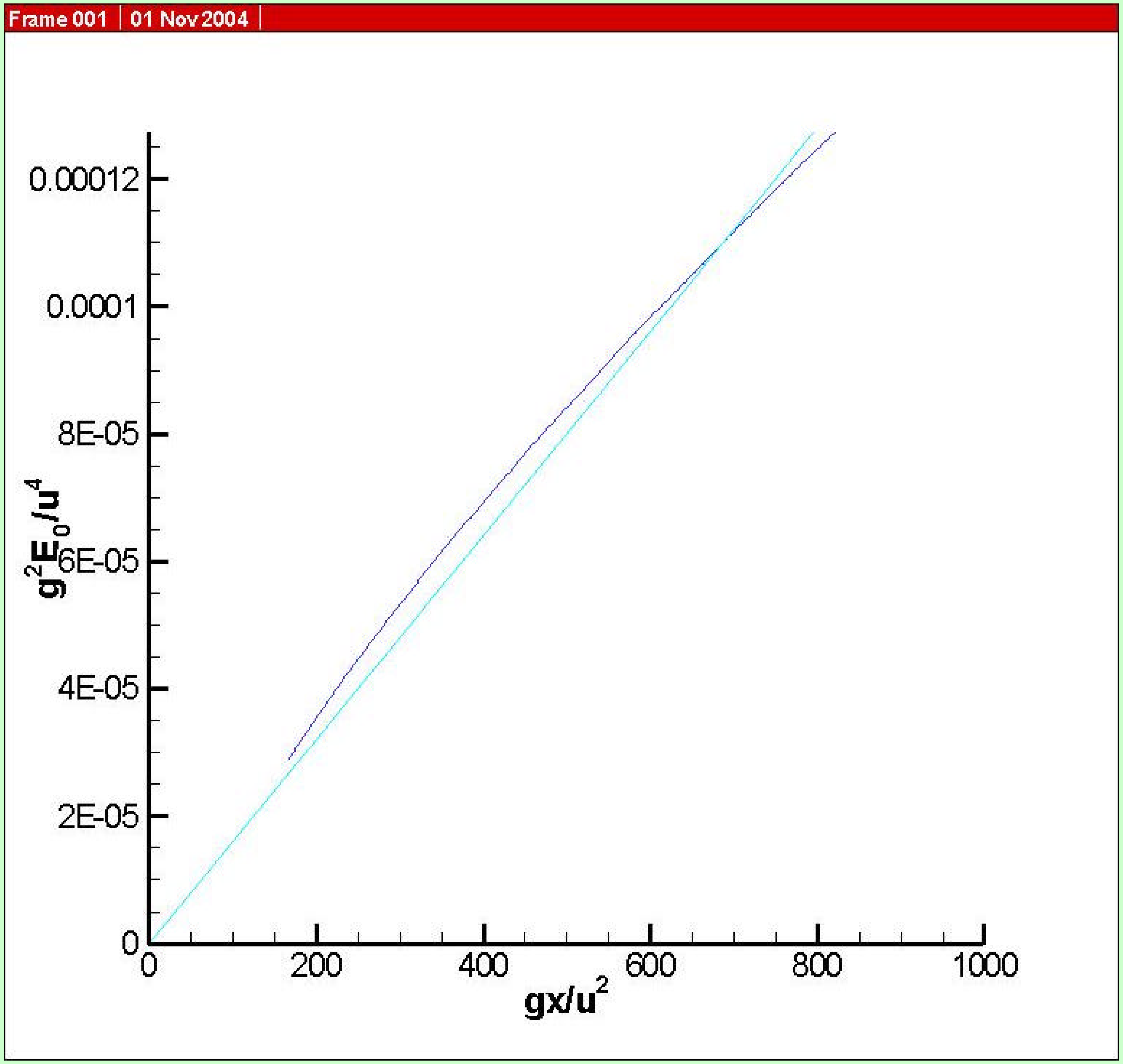
$f_d >> f_p$,

proportion of momentum into waves (RI and RII)= 10%,

frequency rms=0.25, and

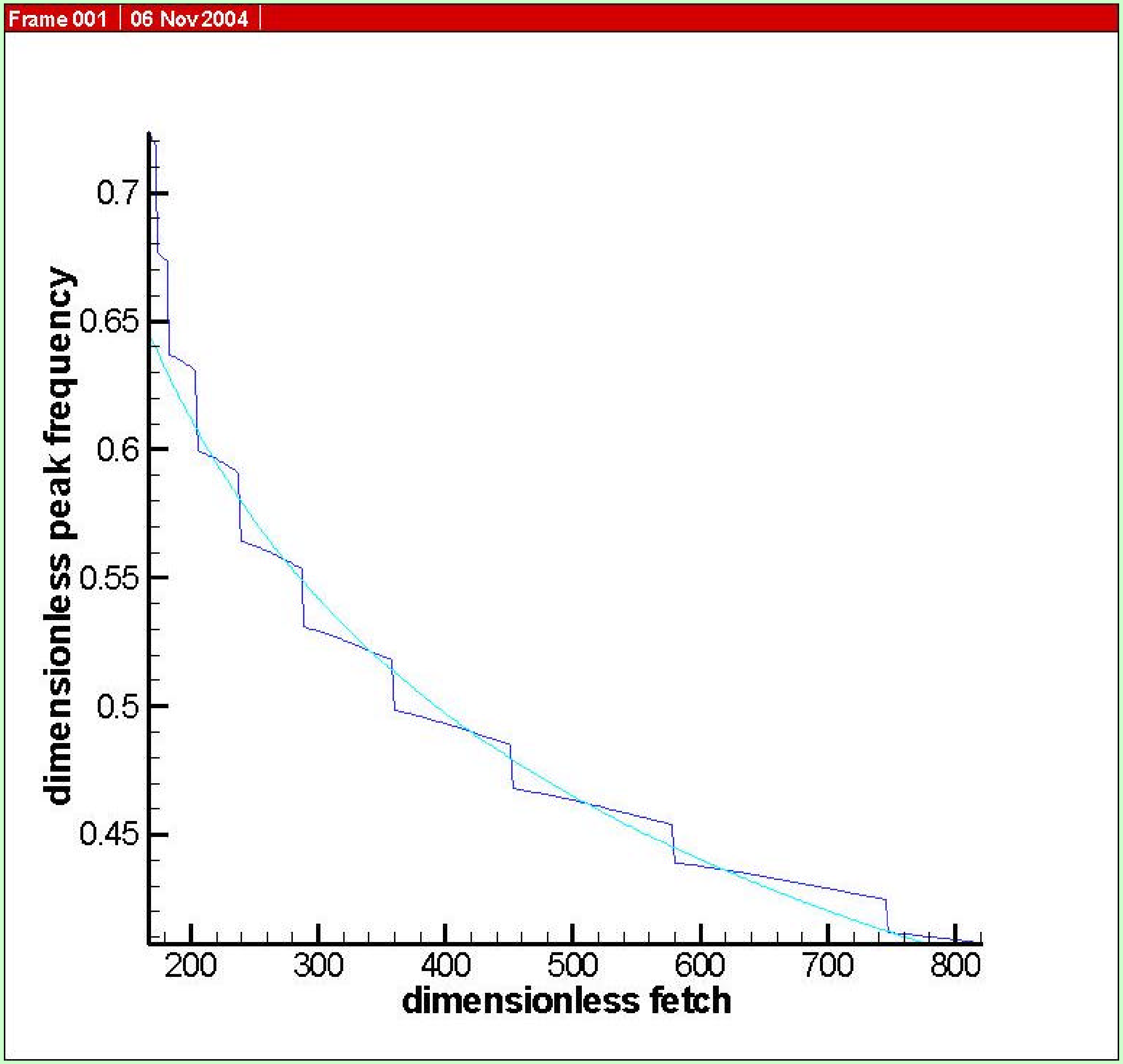
angle rms=20^o

Simulated relationship between dimensionless energy and Dimensionless fetch compared to JONSWAP equation



Note: This is on a linear-linear scale

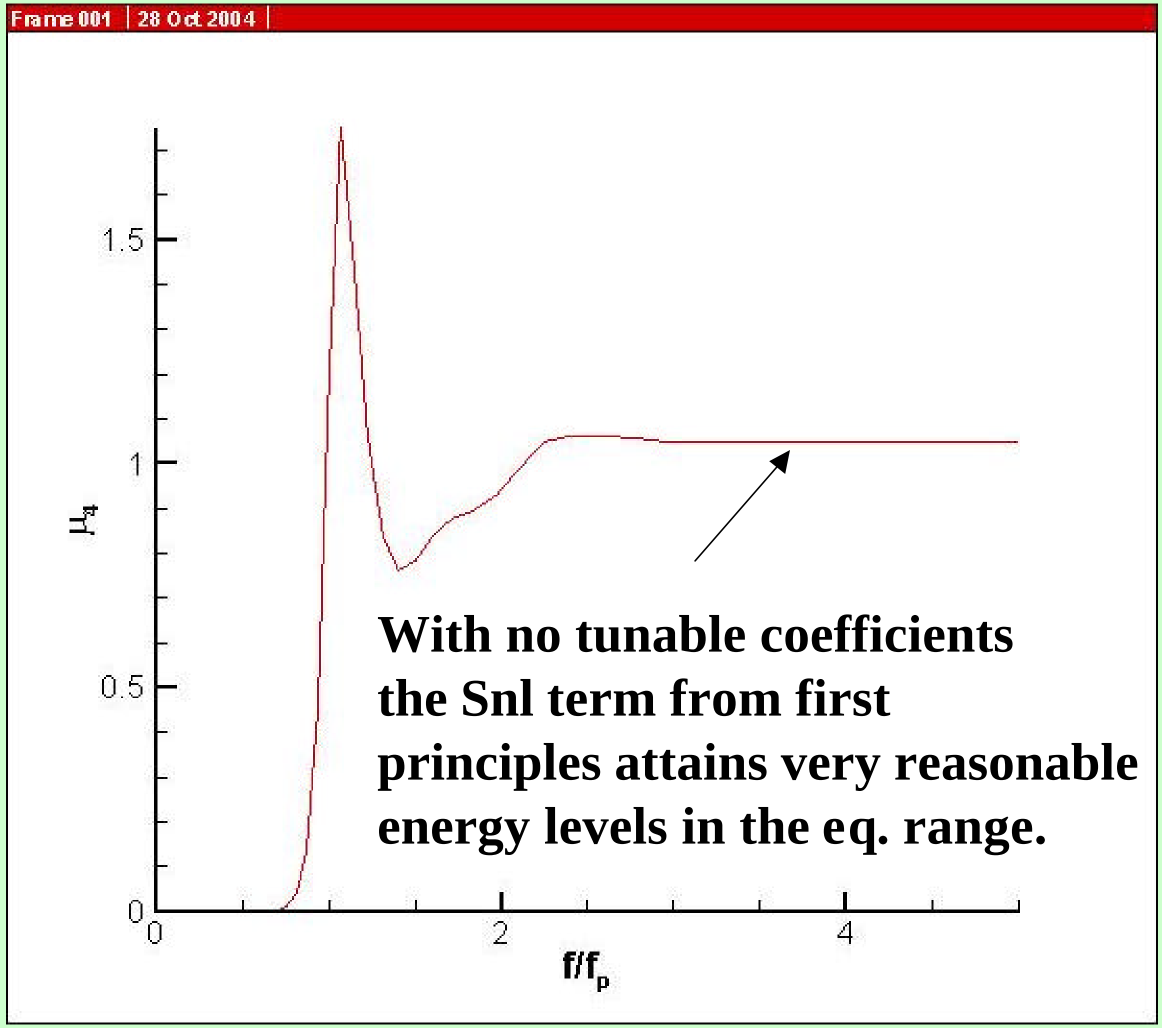
Simulated relationship between dimensionless peak frequency and dimensionless fetch compared to JONSWAP equation



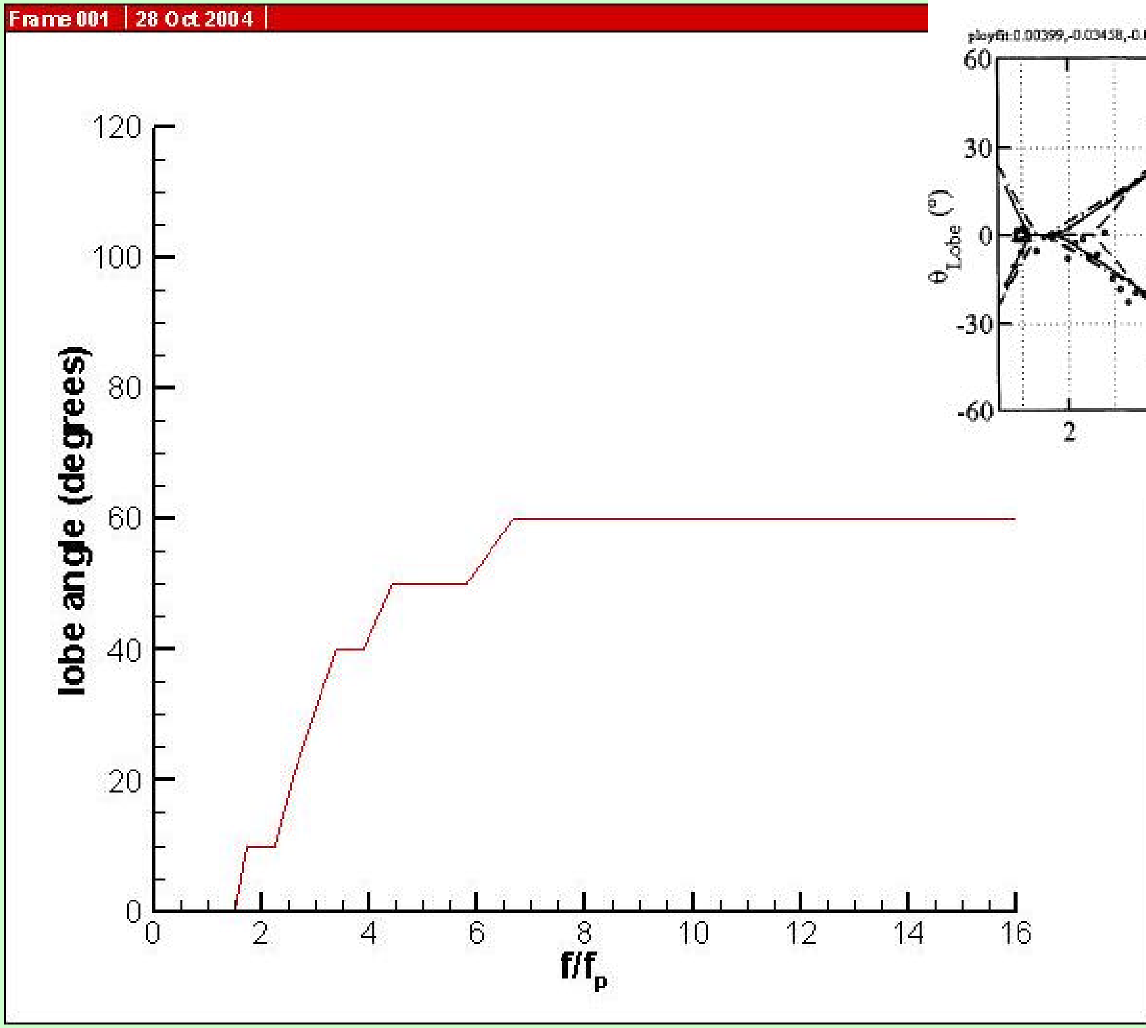
Note: This is on a linear-linear scale

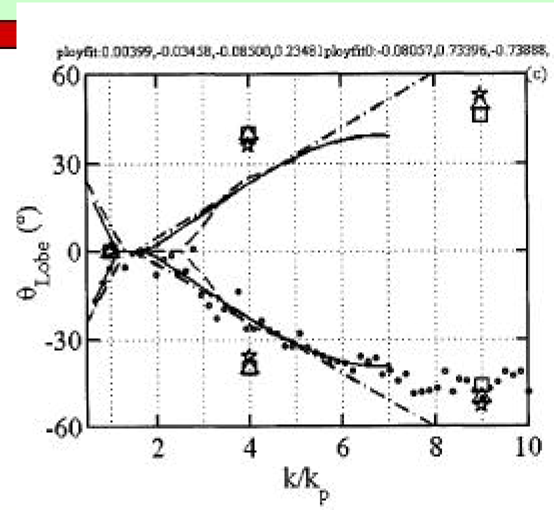
Simulated normalized spectral (f^{-4}) shape using new source terms

Note: This spectral balance will retain a $k^{-5/2}$ form in shallow water.

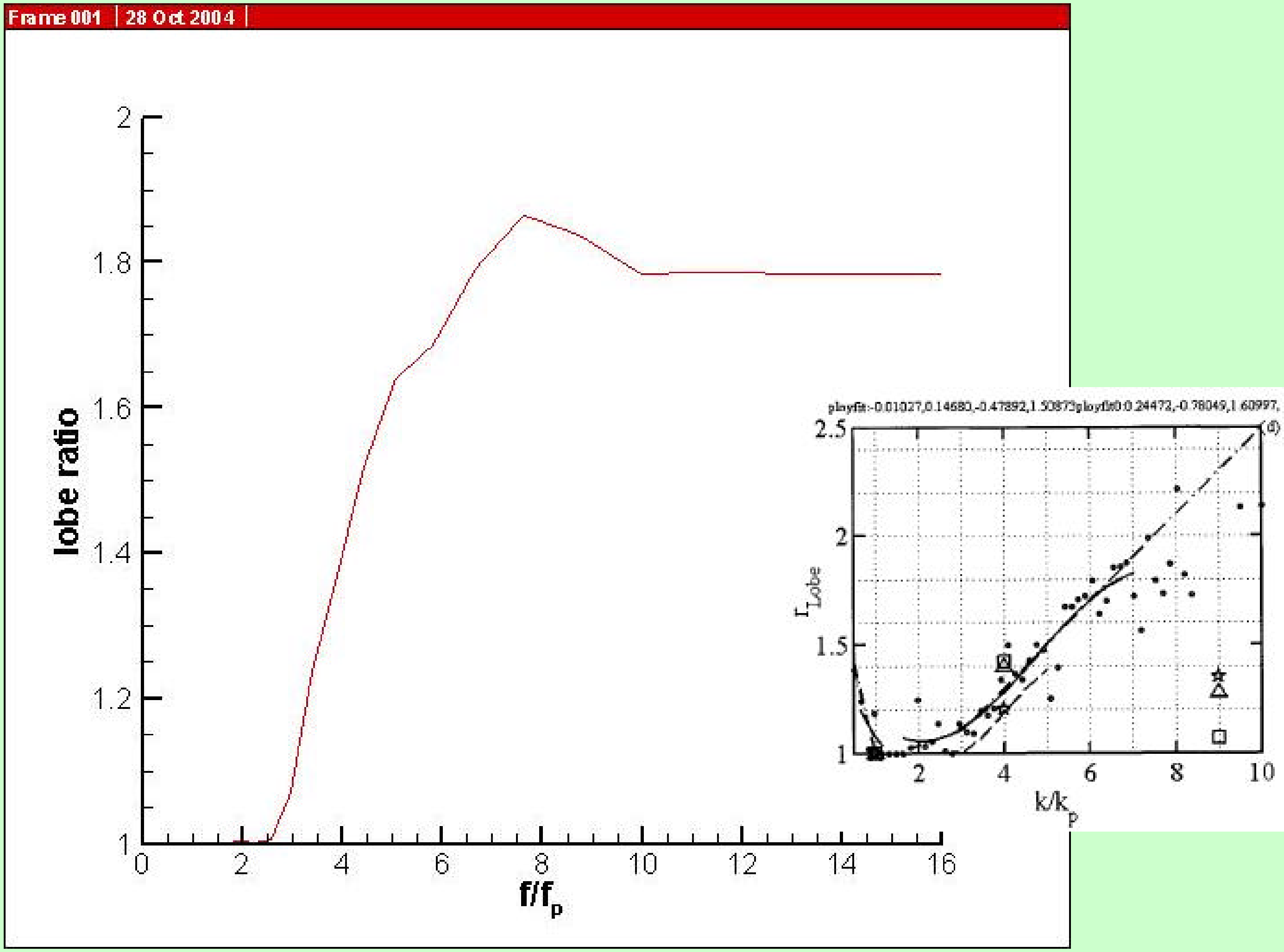


Simulated location of lobes relative to mean direction using new source terms



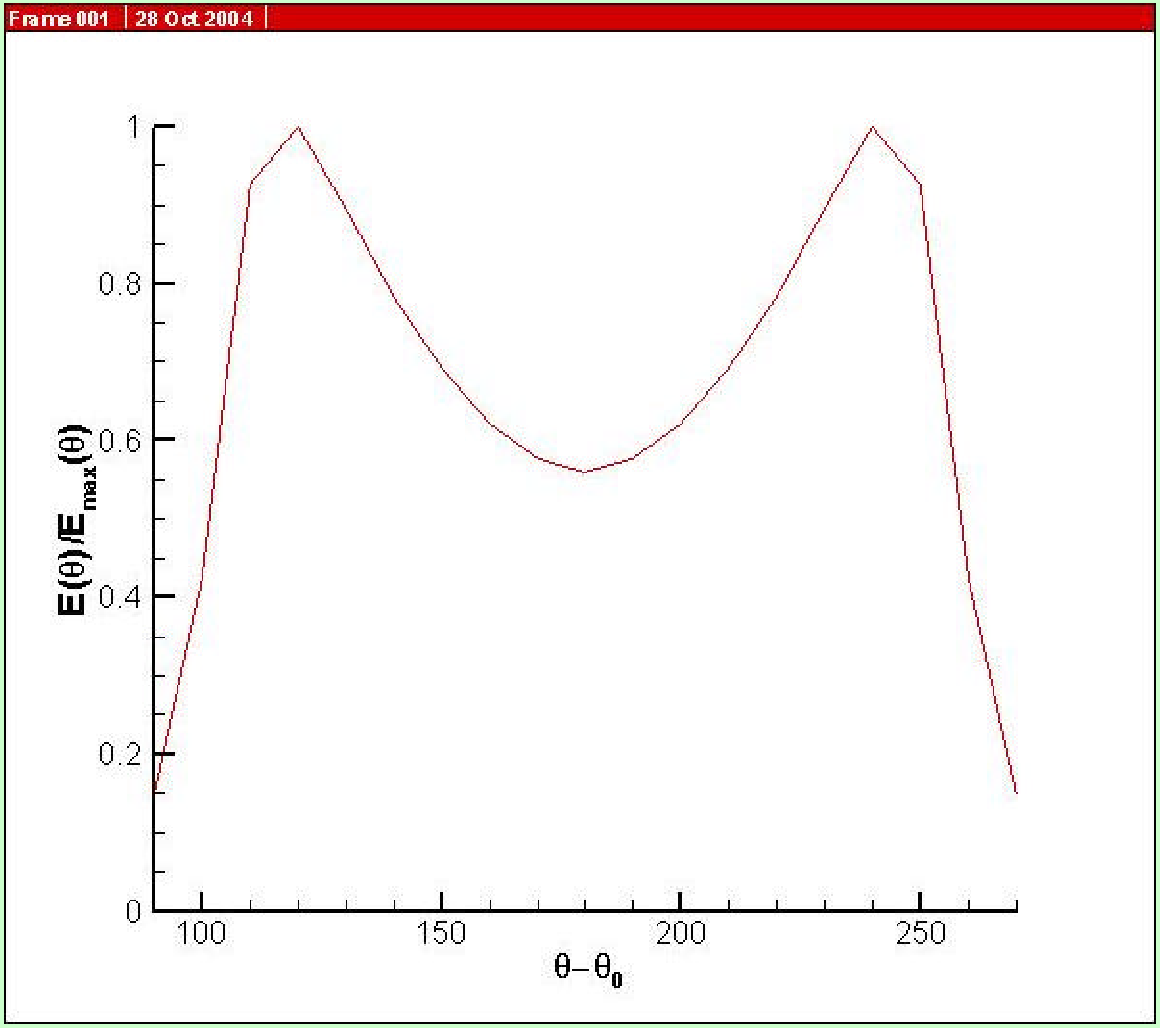


Simulated lobe ratio as a function of relative frequency based on new source terms

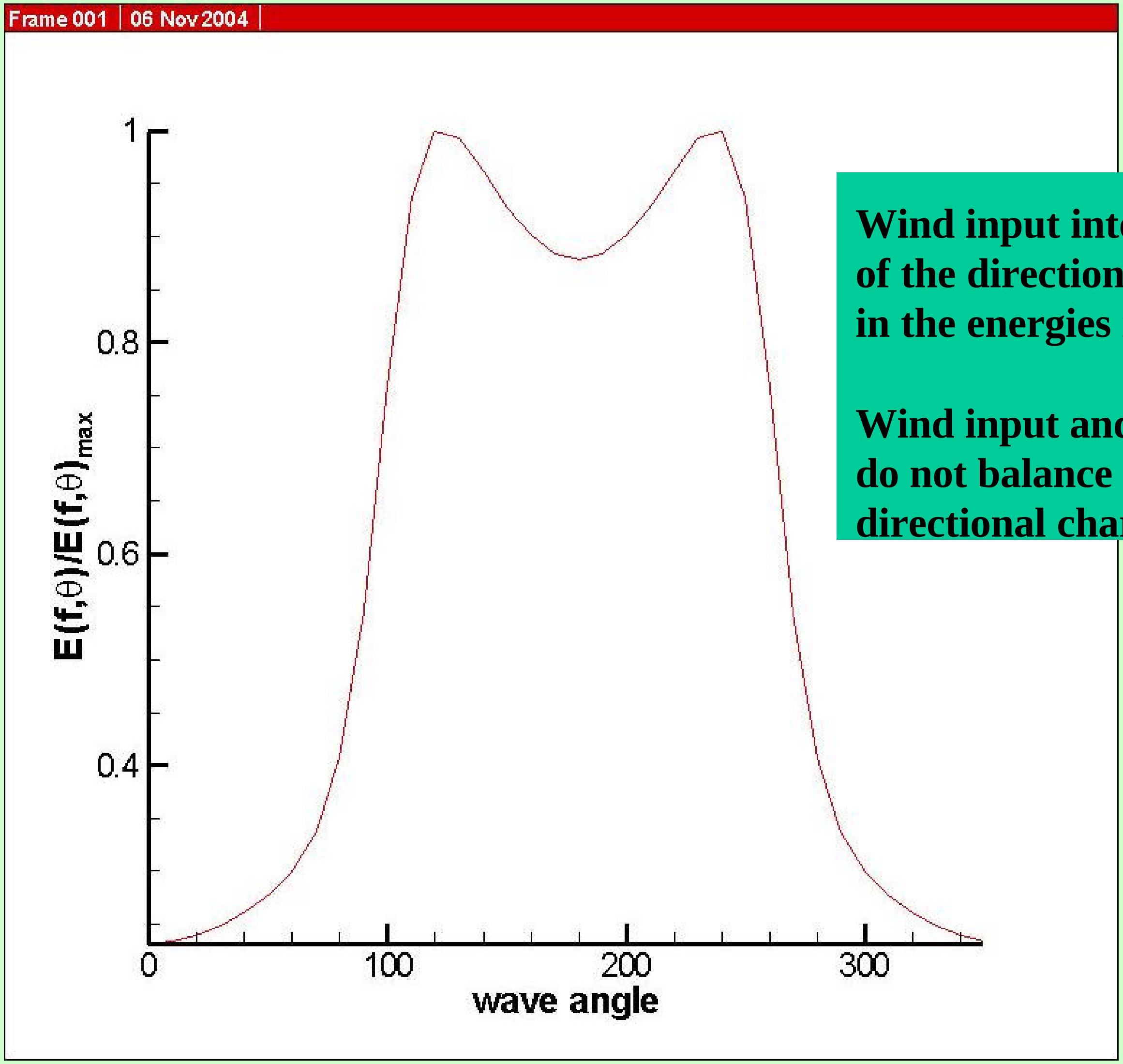


Alves-Banner source terms could only achieve lobe ratio of about 1.4

Simulated lobe ratio at $4f_p$ based on new source terms



Simulated lobe ratio at $4f_p$ based on WAM4 source terms

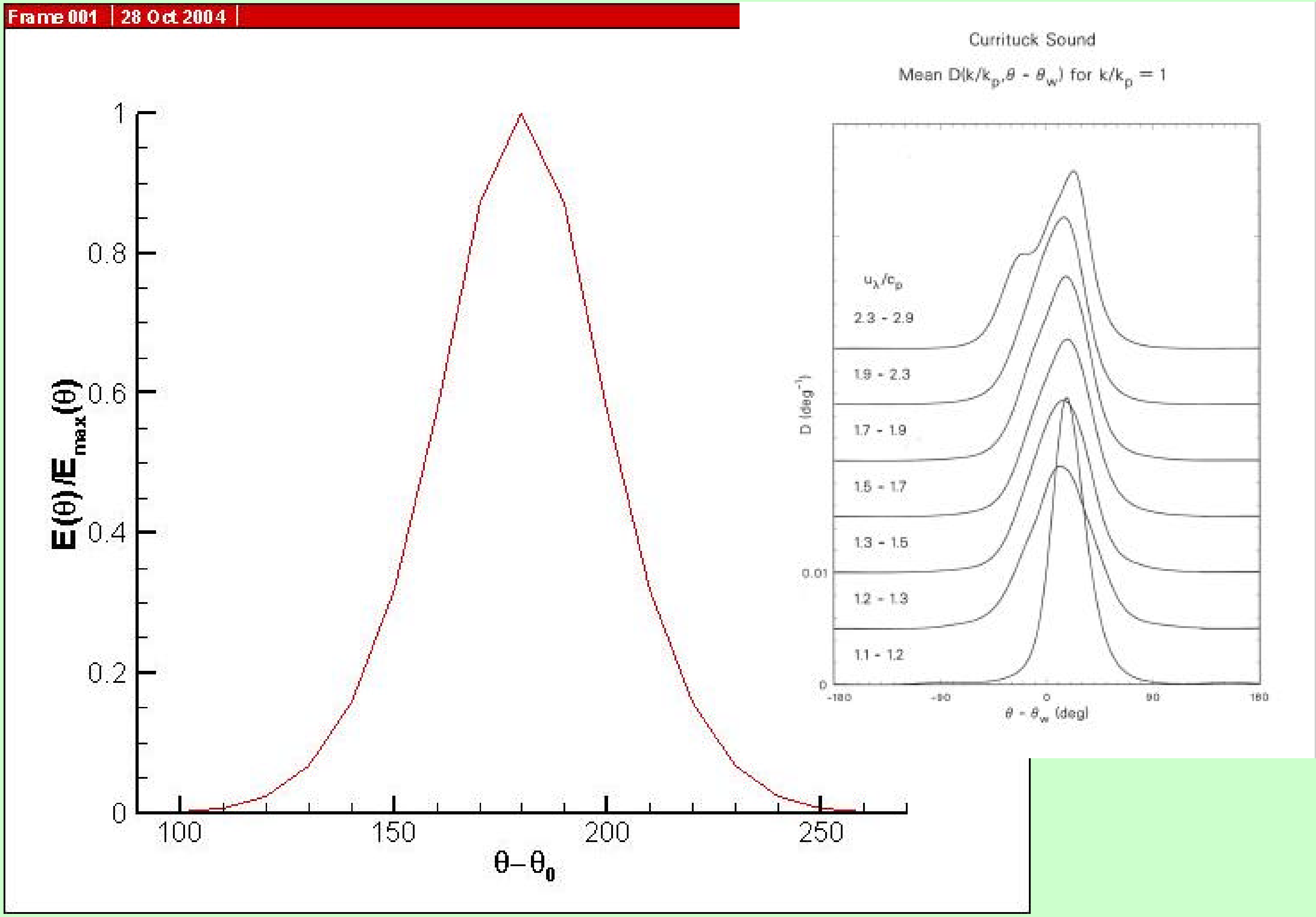


Wind input into the central portion of the directional distribution fills in the energies in this region.

Wind input and wave breaking do not balance with respect to directional characteristics

Lobe ratio is only about 1.12

Simulated directional distribution of energy at spectral peak using new source terms



Is is time to move to a new paradigm for wave generation?

- Concepts of fully developed seas in 3G models require breaking in the spectral peak region – Recent evidence suggests that actual spectra may continue to evolve significantly past the PM limit.

- Concepts of distributed wind and wave input do not duplicate the frequency-direction characteristics in observed spectra.

- Snl in existing 3G models cannot replicate nearshore wave physics due both to a lack of capability to represent complex spectra and the incorrect scaling of Snl in shallow water

- Detailed-balance concepts in contemporary models need to be re-thought for coastal applications

CONCLUSIONS

- Detailed balance characteristics of spectral shape should be used to rigorously test model performance
- Detailed-balance is not sufficient to argue better physics.
- A somewhat different set of physics than that embodied in existing 3G models appears to be more consistent with observed shapes and is being developed into a new model. Three remaining challenges:
    1. New Snl in arbitrary depth
    2. Asymptotic development past PM limit
    3. Replace source terms in SWAN-type model


QUESTIONS