Generalized Wave-Ray Approach for Propagation on a Sphere and Its Application to Swell Prediction

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Outline of Talk

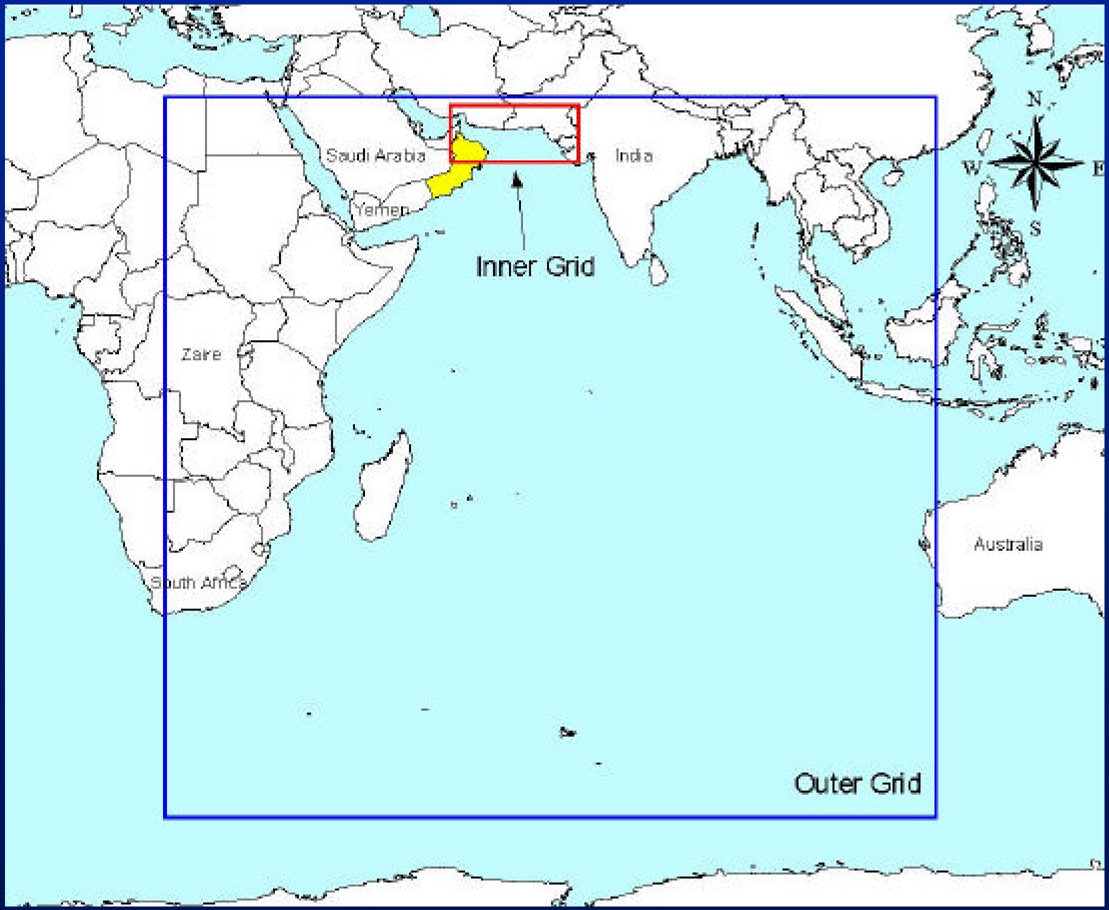
- Introduction
- Wave-Ray Approach
- Line Source for Swell Propagation
- Model Comparisons
  - *Simplified Test Cases*
  - *Pacific Ocean Sea States*
- Conclusions
- A Diversion: Model Source Terms

Motivations for the Work

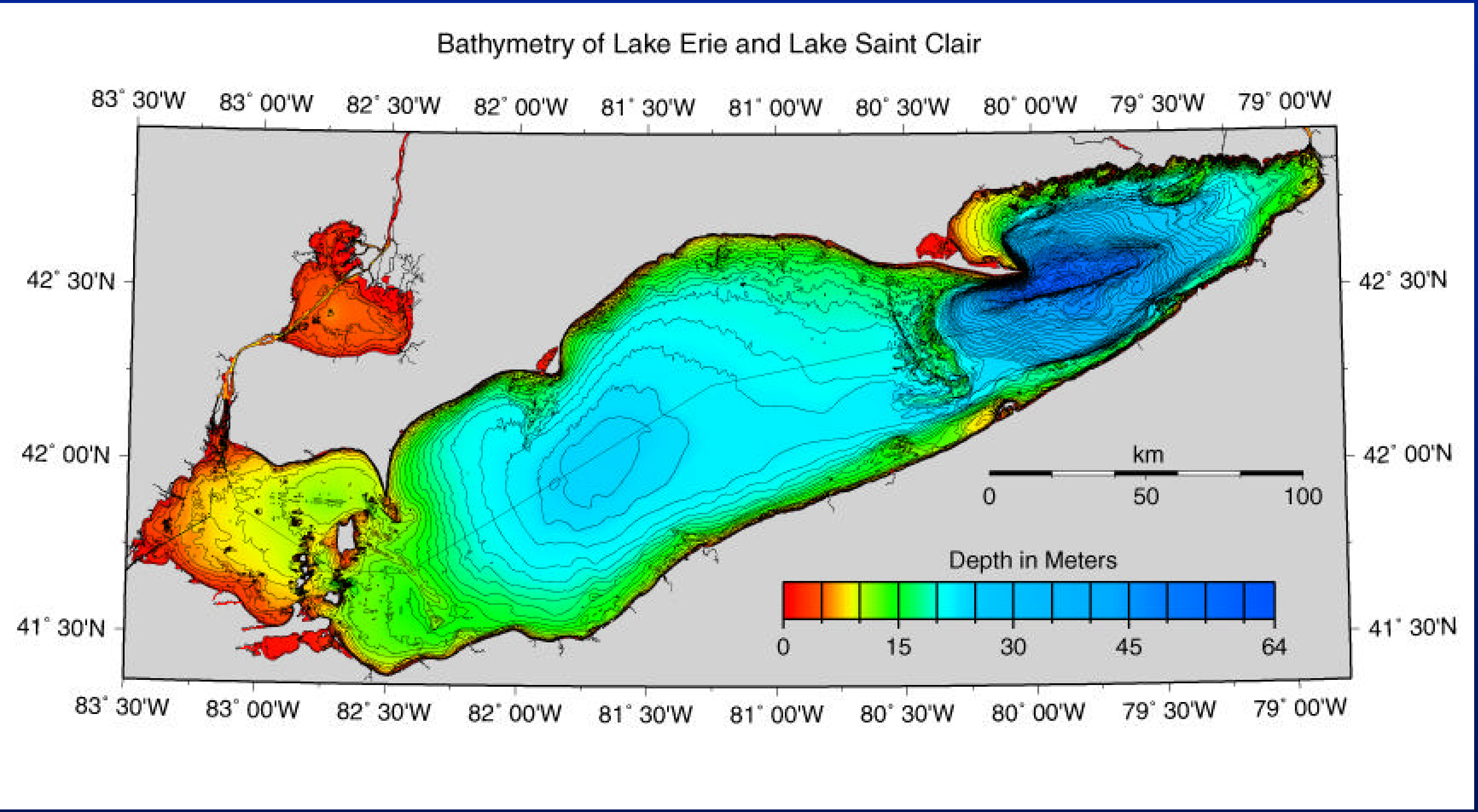
- Improved form for incorporating depth refraction in wave models
- Investigations of swell propagation and swell decay

Refraction Processes

- Not generally important at oceanic scale, but a consideration with nested sub-grids



Lake Hindcasts



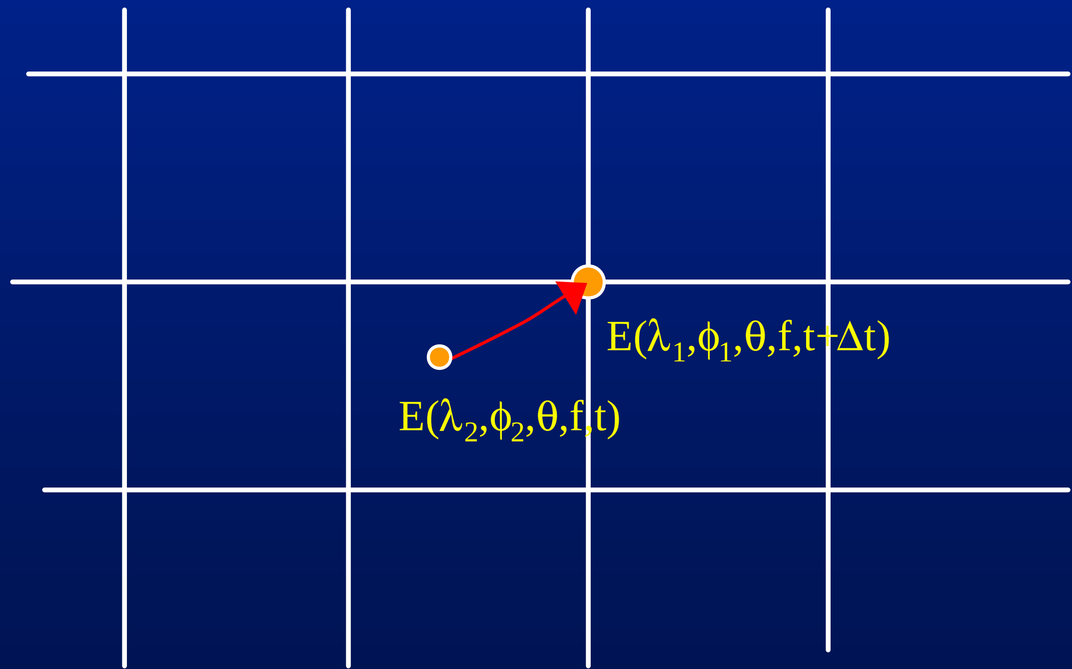
Source: US Geophysical Data Center, NOAA

Model Development

- Context of 2G models where wave energy propagation consumes at least 50% of the computation time – less important for a 3G model
- Appeal of a 2G Model:
  - *Computational speed (25x to 40x factor)*
  - *Scalability (any size waterbody)*

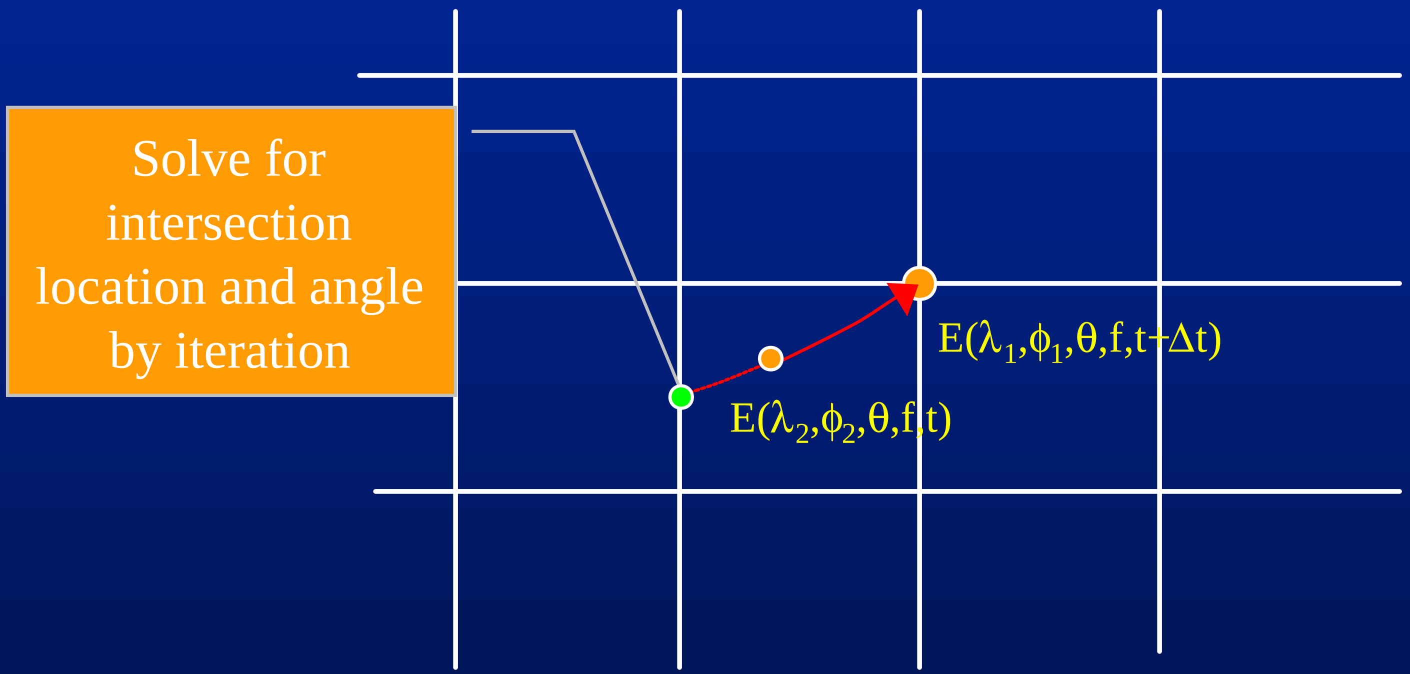
Basic Development

- Method of characteristics approach following great circle routes and depth-induced curvature (ray back-tracking)



Interpolation Approach?

Basic Development



$$\hat{E}(f,q)_{\text{intersection}} = l_{11}\hat{E}_{i\pm n,j\pm m}(f,q_1) + l_{12}\hat{E}_{i\pm n',j\pm m'}(f,q_1) + l_{21}\hat{E}_{i\pm n,j\pm m}(f,q_2) + l_{22}\hat{E}_{i\pm n',j\pm m'}(f,q_2)$$

$$l_t = \frac{c_g \Delta t}{D}$$

$$\hat{E}_{i,j}(f,q)^{(n)} = (1 - l_t)\hat{E}_{i,j}(f,q)^{(n-1)} + l_t \hat{E}(f,q)_{\text{intersection}}$$

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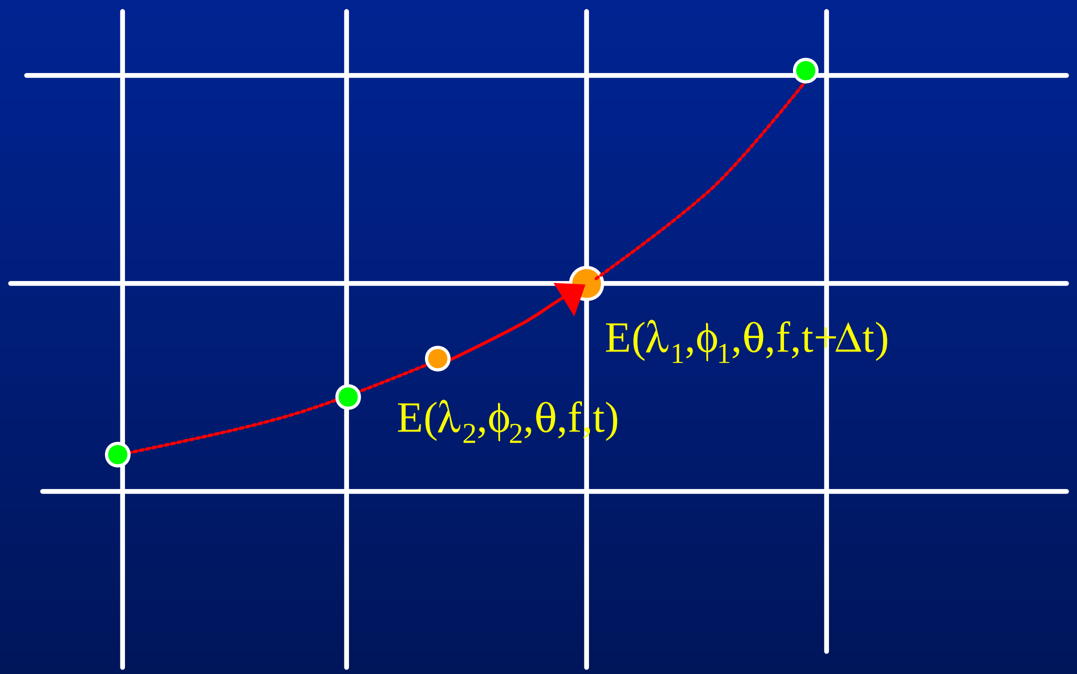
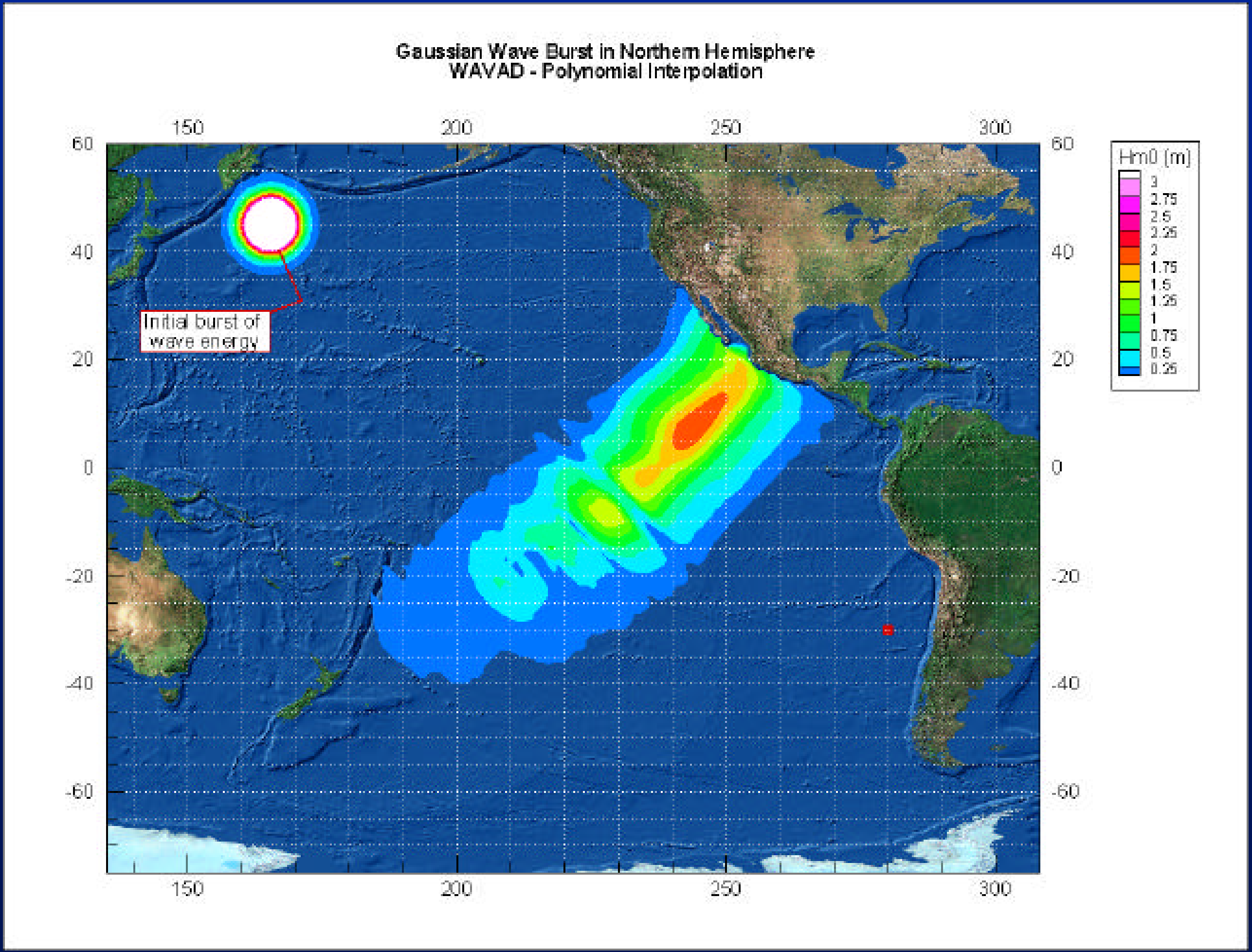
Extension to Higher Order

- Higher order interpolation – cubic spline, polynomial, ….
- In the end we utilized a relatively simple procedure of determining the great circle arc intersections with adjacent grid cells, and using cubic polynomial interpolation

Higher Order – Cubic Polynomial Interpolation



Gaussian Wave Burst



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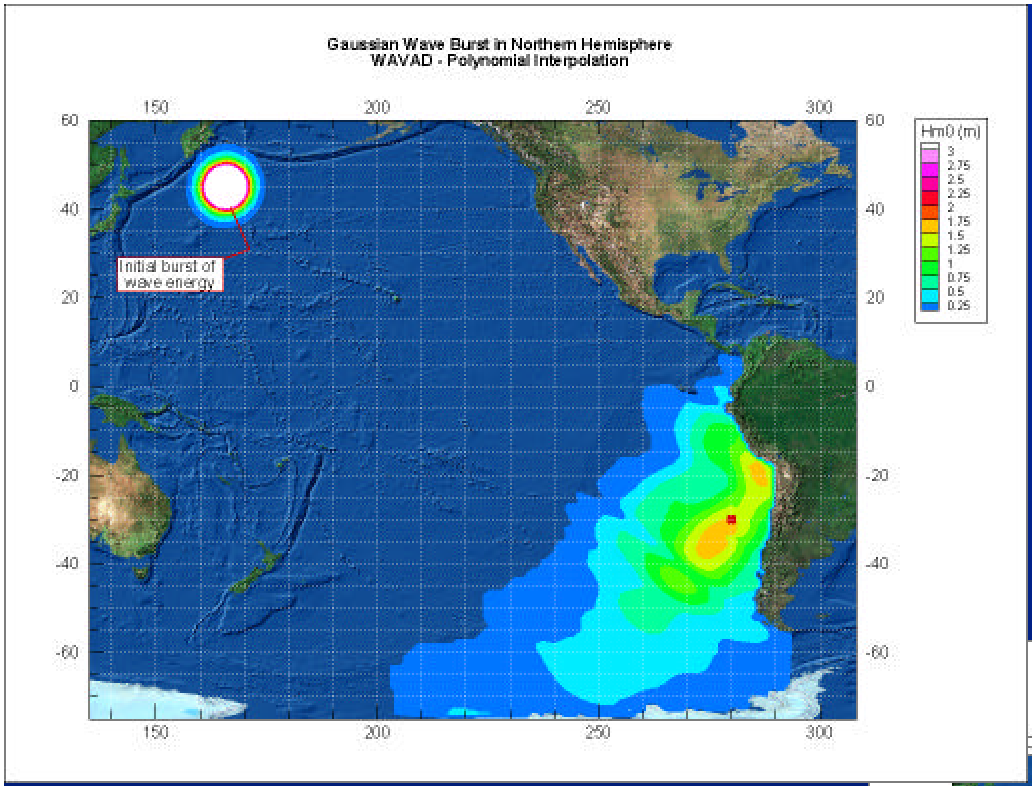
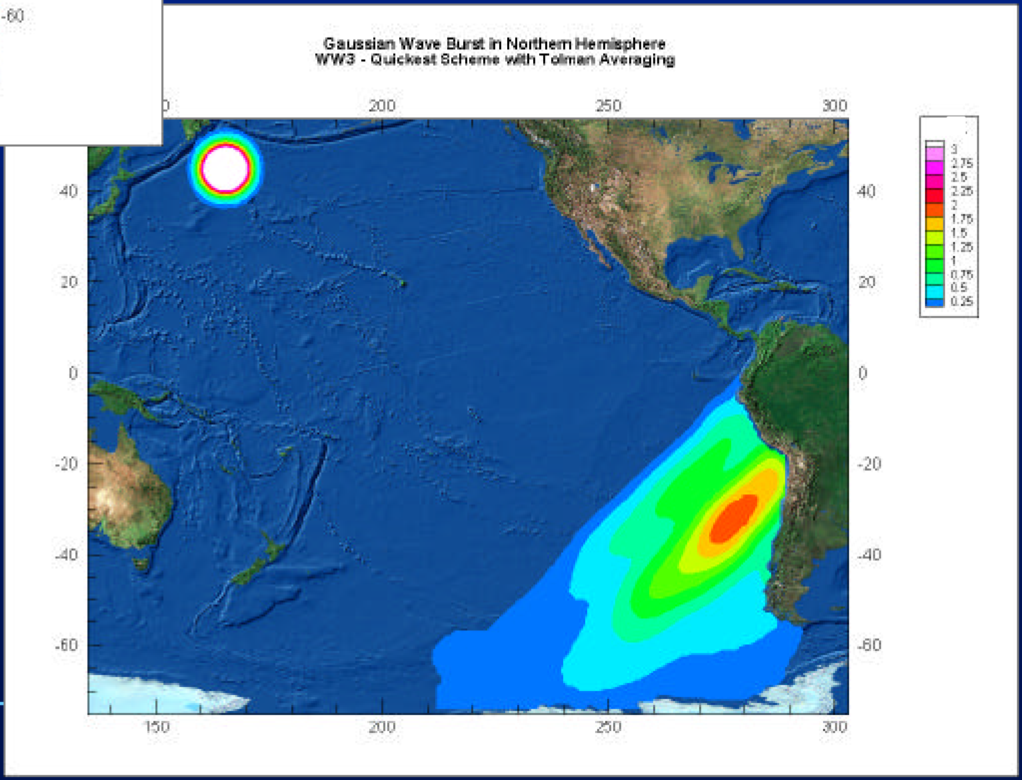

Gaussian Wave Burst in Northern Hemisphere
WAVAD - Polynomial Interpolation
Initial burst of wave energy
Hm0 (m)
3
2.75
2.5
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25


Gaussian Wave Burst in Northern Hemisphere
WW3 - Quickest Scheme with Tolman Averaging
3
2.75
2.5
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25

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Extension to Simulation of Swell Propagation

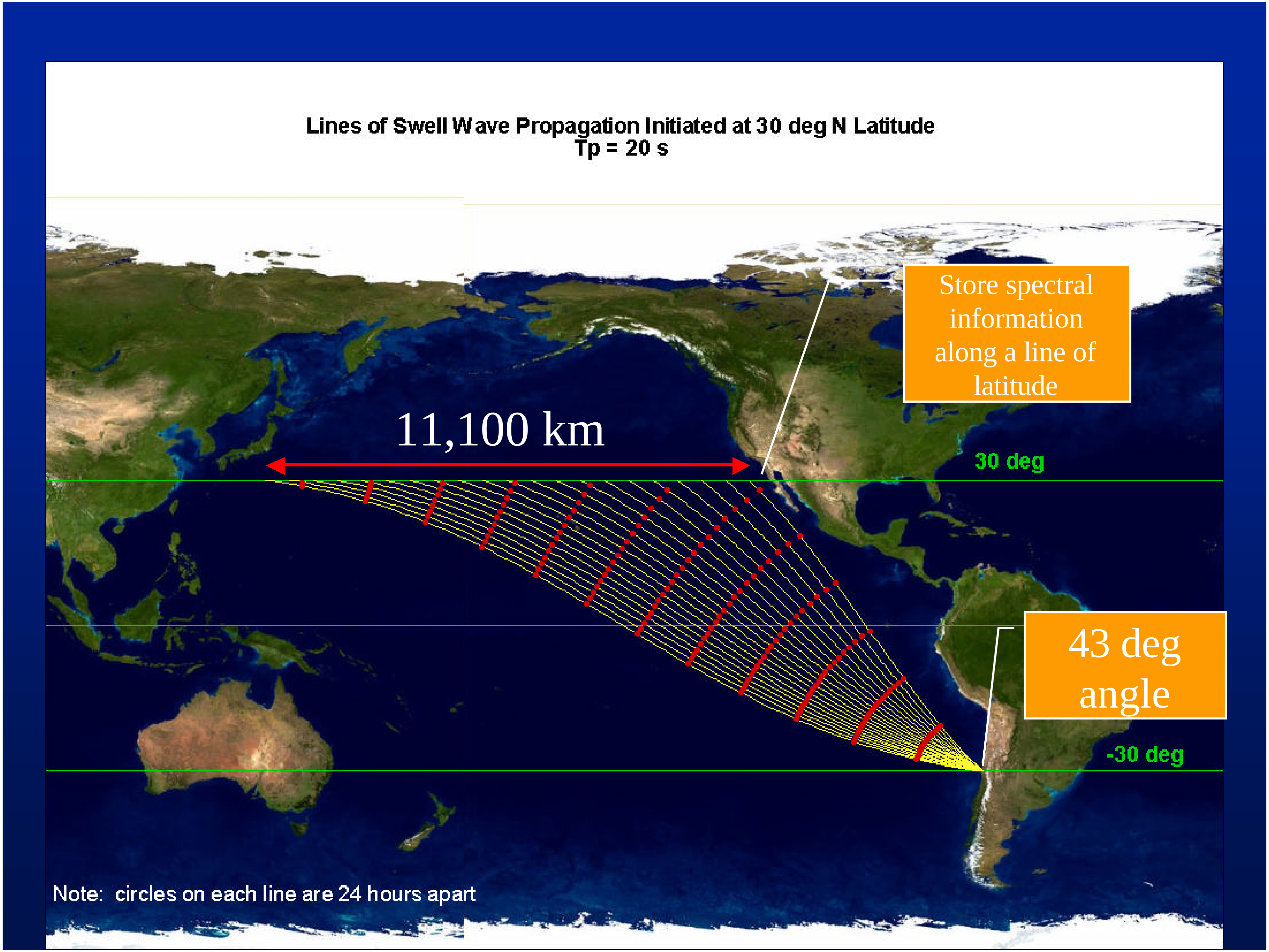
- Relate waves arriving at any point on the earth to waves passing a fixed, non-intersecting line (a "line source")

$$E(x_0, y_0, f, q, t) = E(x, y, f, q(x, y, x_0, y_0), t - D / c_g)$$

- Fundamental assumption: source terms negligible over the propagation distance

Line Source

- Integration of wave information along a given line
- Low-frequency (< 0.1 Hz) spectral information stored at regular intervals & replaces the computed wave energy densities in relevant directional and frequency bins


Lines of Swell Wave Propagation Initiated at 30 deg N Latitude
Tp = 20 s
Store spectral information along a line of latitude
11,100 km
30 deg
43 deg angle
-30 deg
Note: circles on each line are 24 hours apart

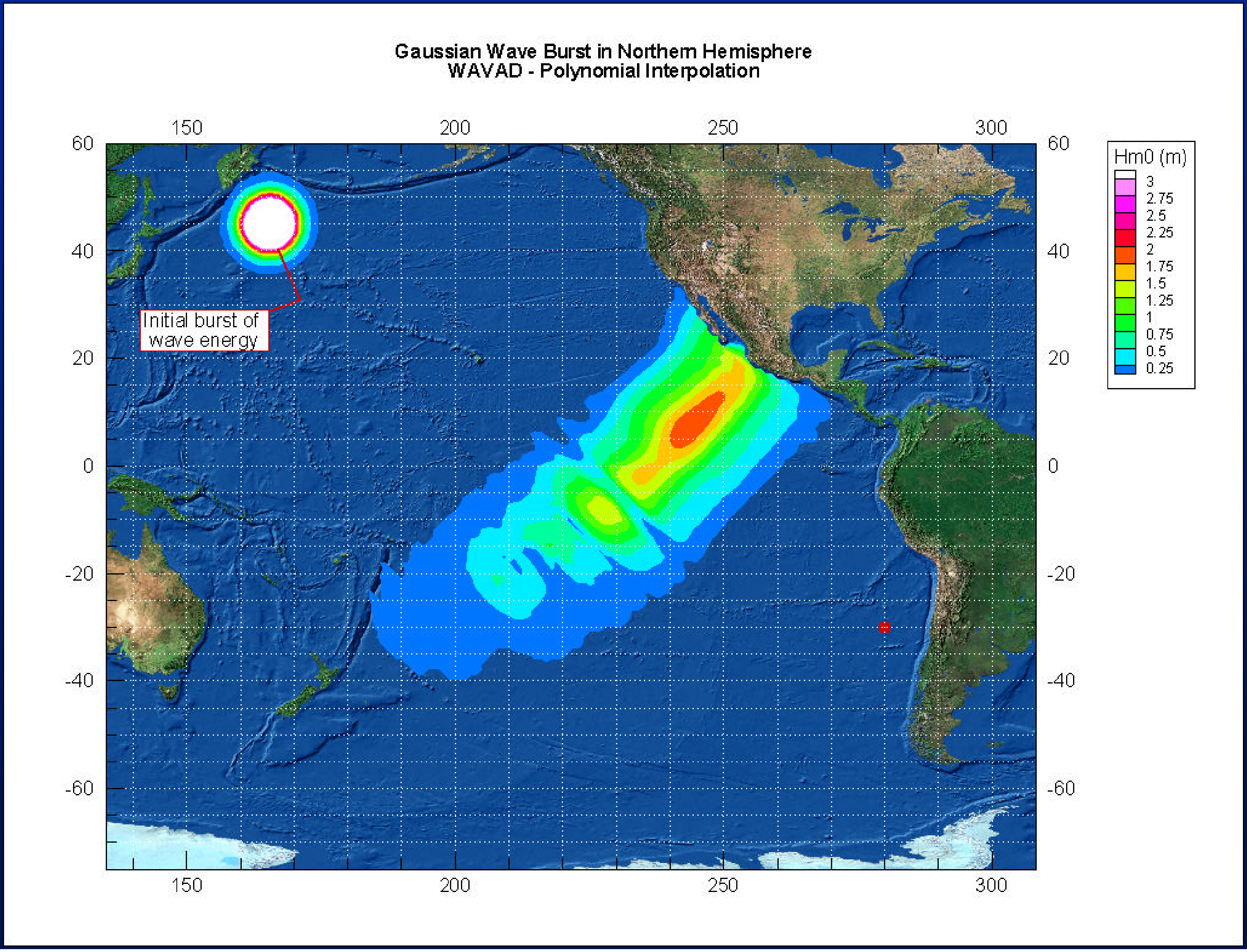
Line Source

- Low-frequency energy propagated without diffusion
- Effects of islands and sub-grid blocking can be readily incorporated

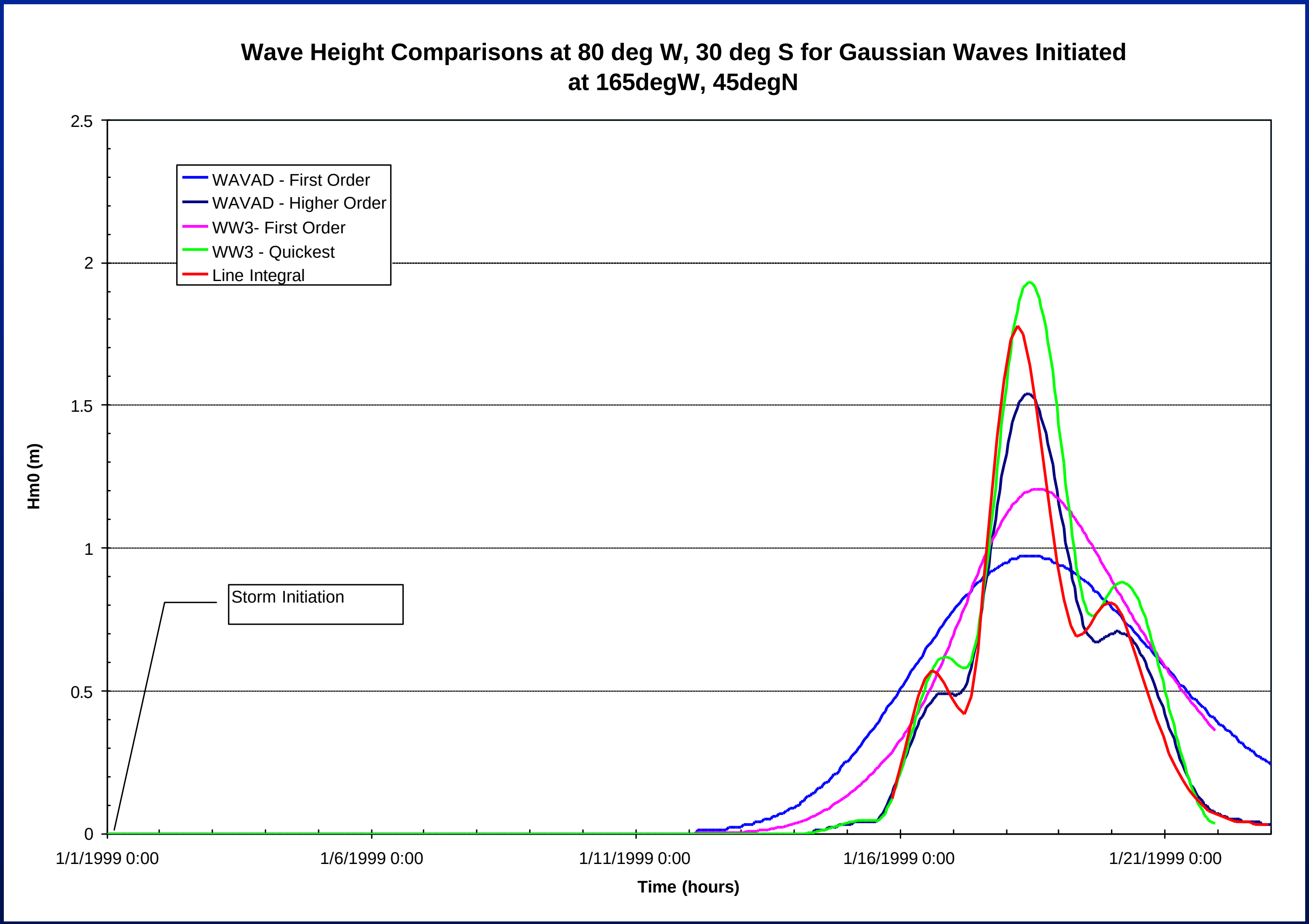
The Wave Model

- Propagation schemes implemented into a 2^{nd} Generation wave model WAVAD
- Some comparisons performed with Wavewatch III v2.22
  - *Settings as per NCEP Operational settings (Tolman & Chalikov, Quickest/Ultimate & Tolman Averaging)*
- Model grid 1°x 1°; 24 directional bins; 24 frequencies; sub-grid blocking represented

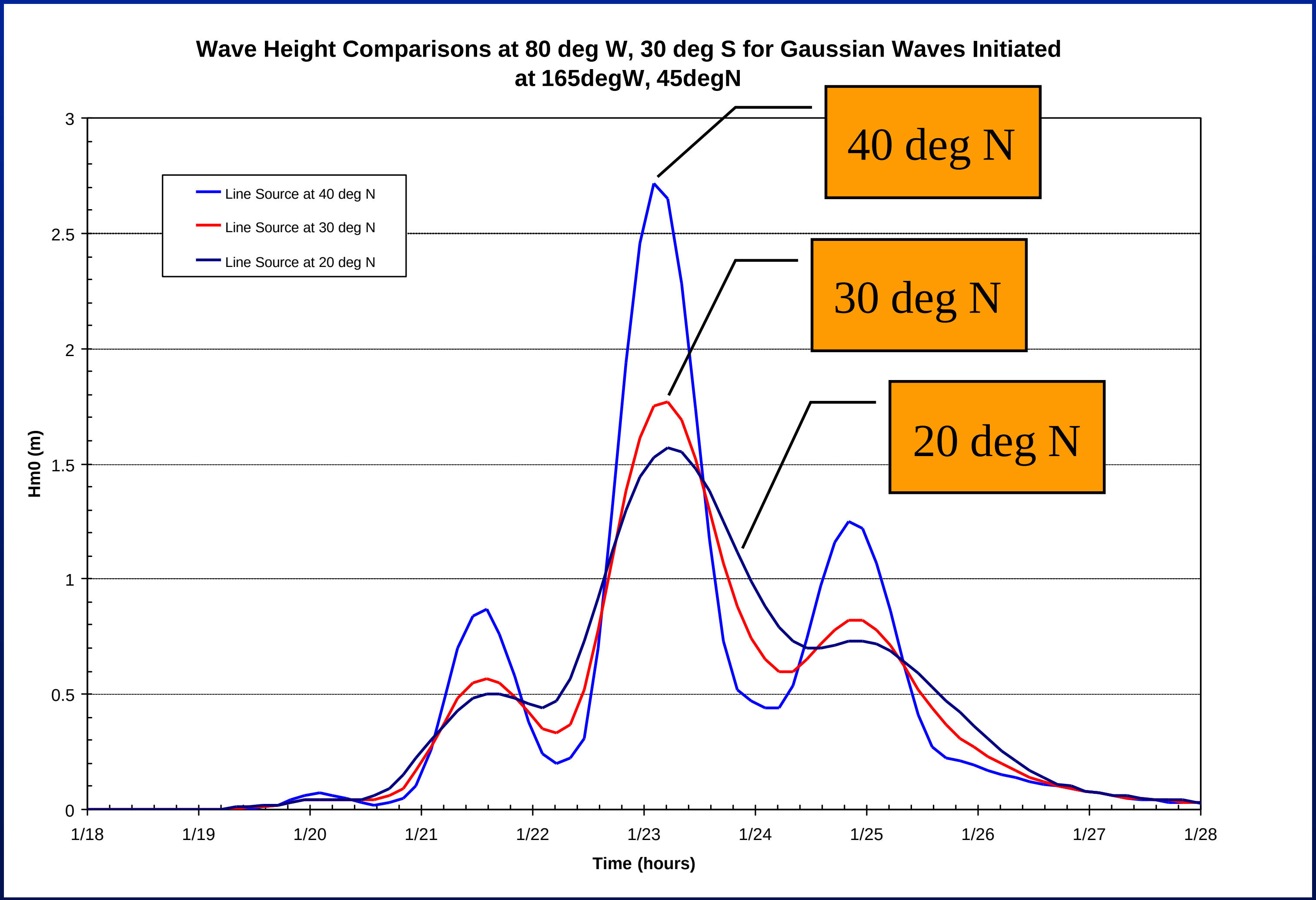
Gaussian Burst of Wave Energy (No Source)



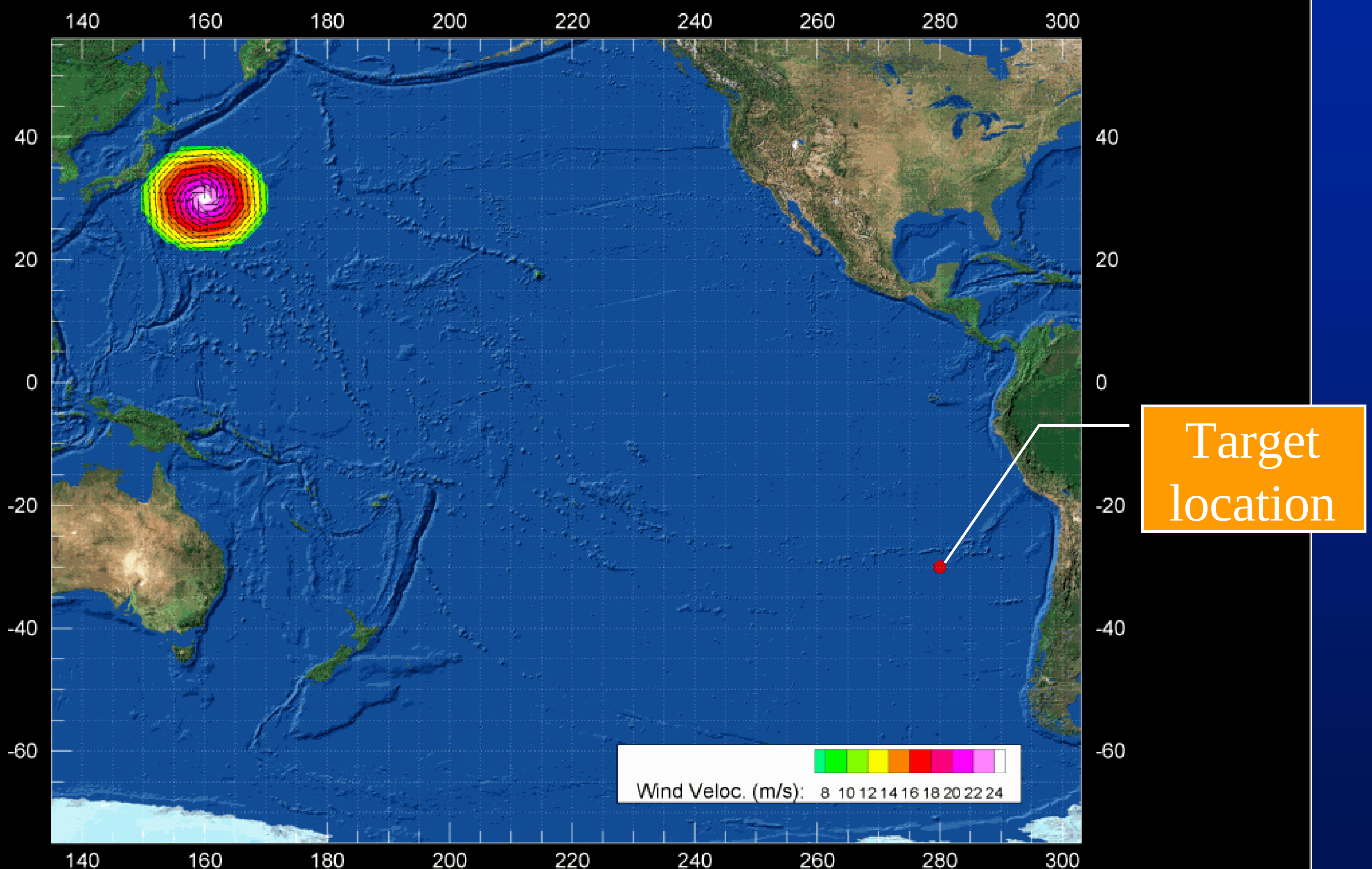
Comparative Results at 80°W, 30°S



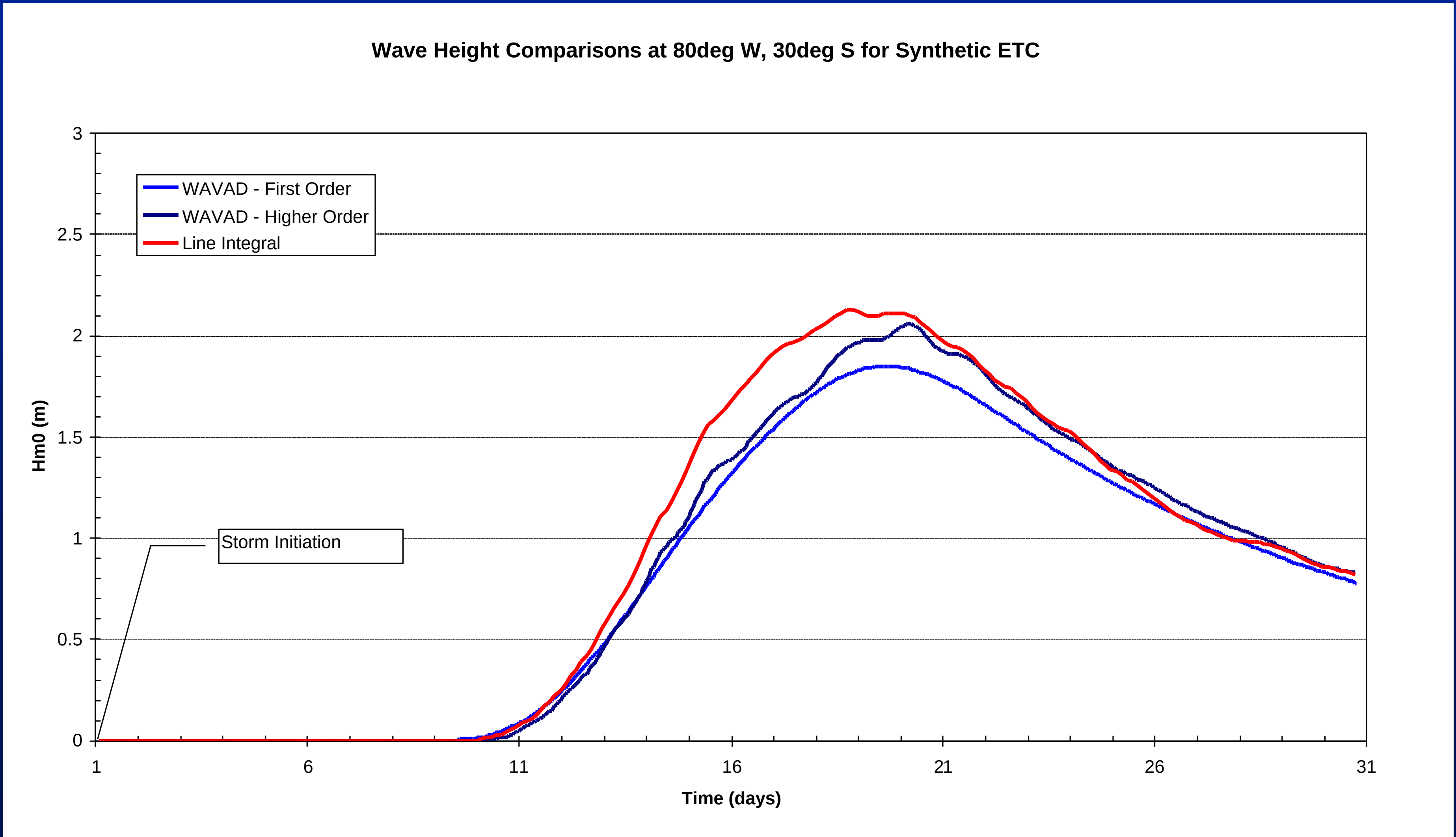
Selection of Different Line Source Locations for Gaussian Burst at 45 deg N



Theoretical Moving Cyclonic Storm

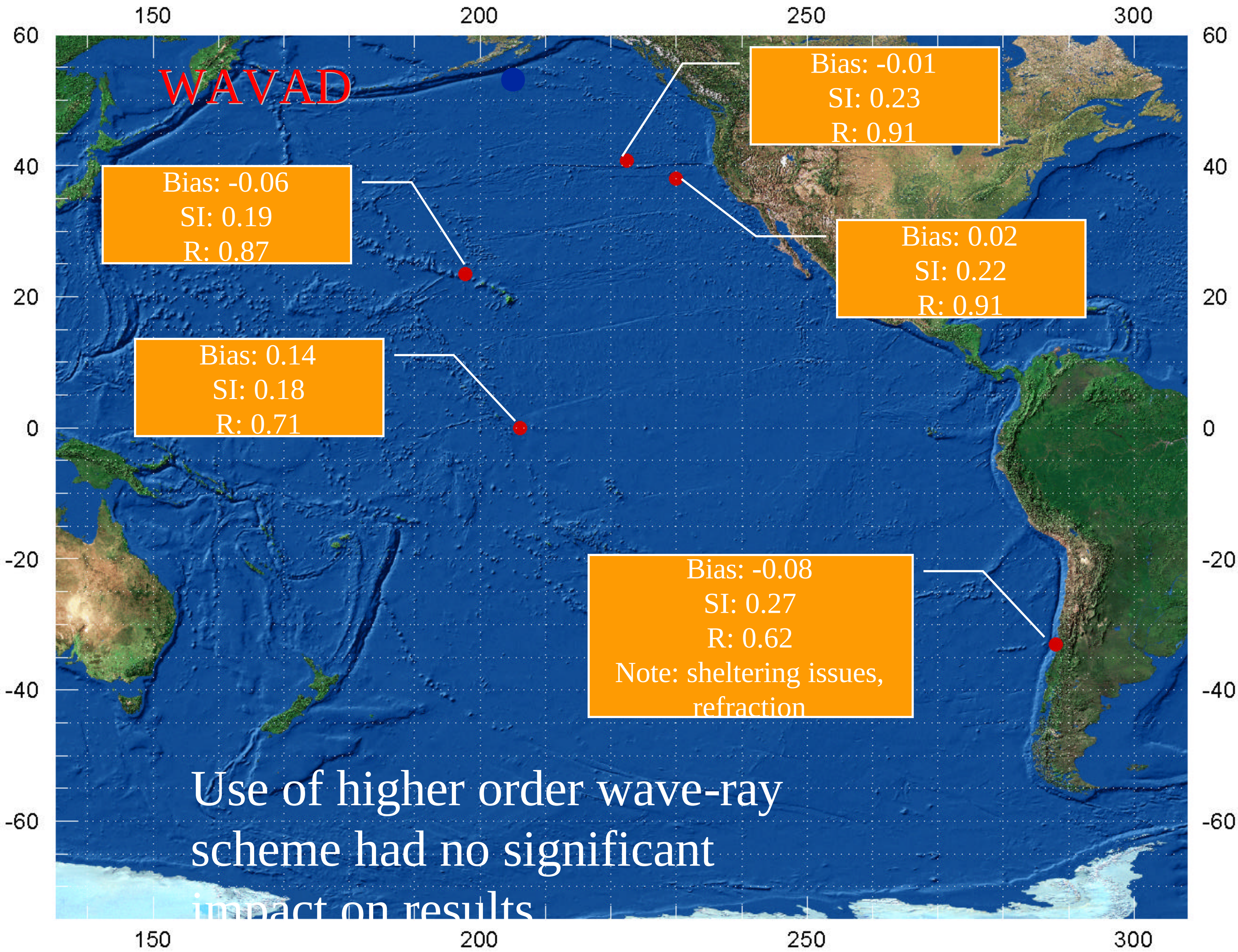


Comparative Results at 80°W, 30°S

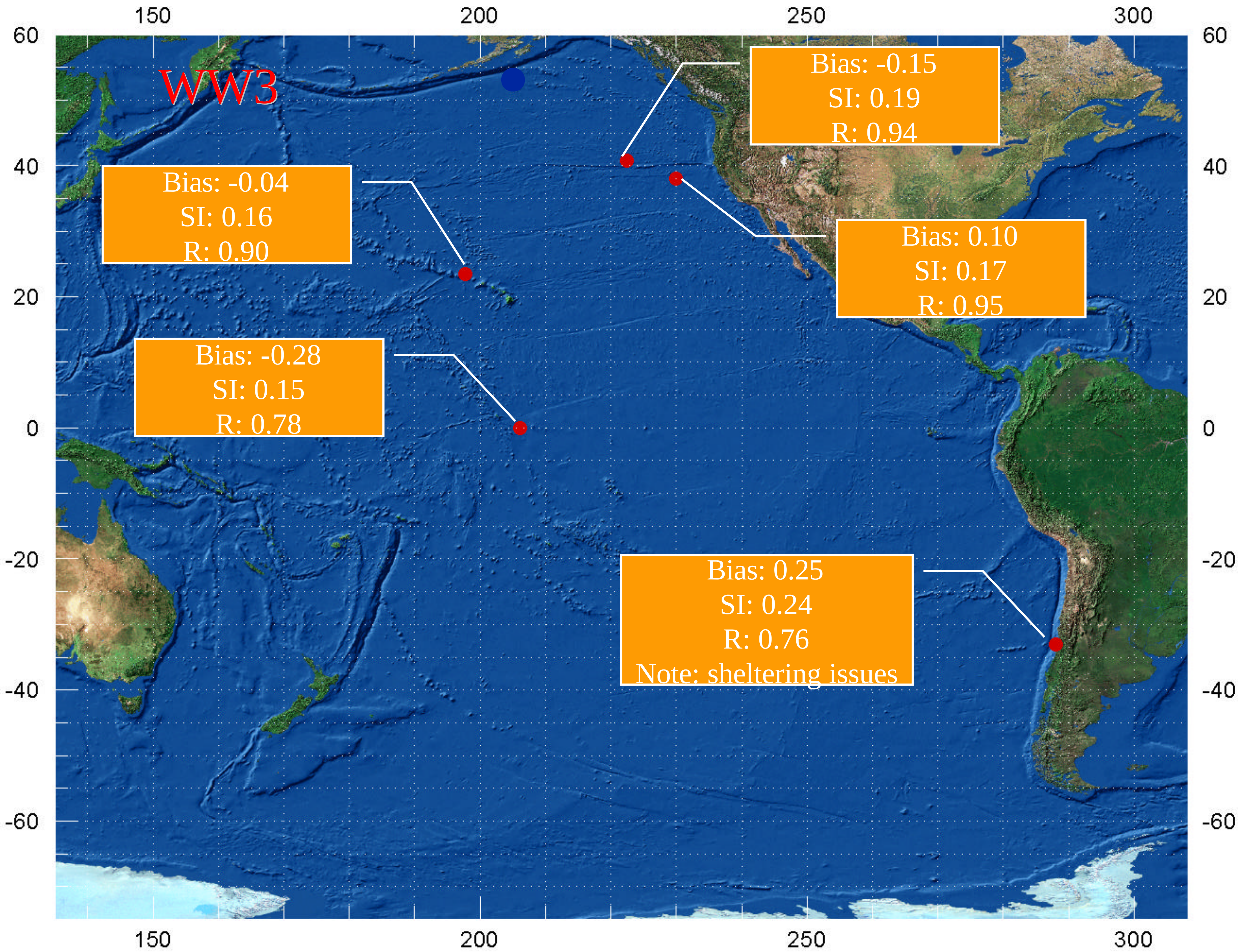


Simulation of More Realistic Sea States for Comparative Purposes

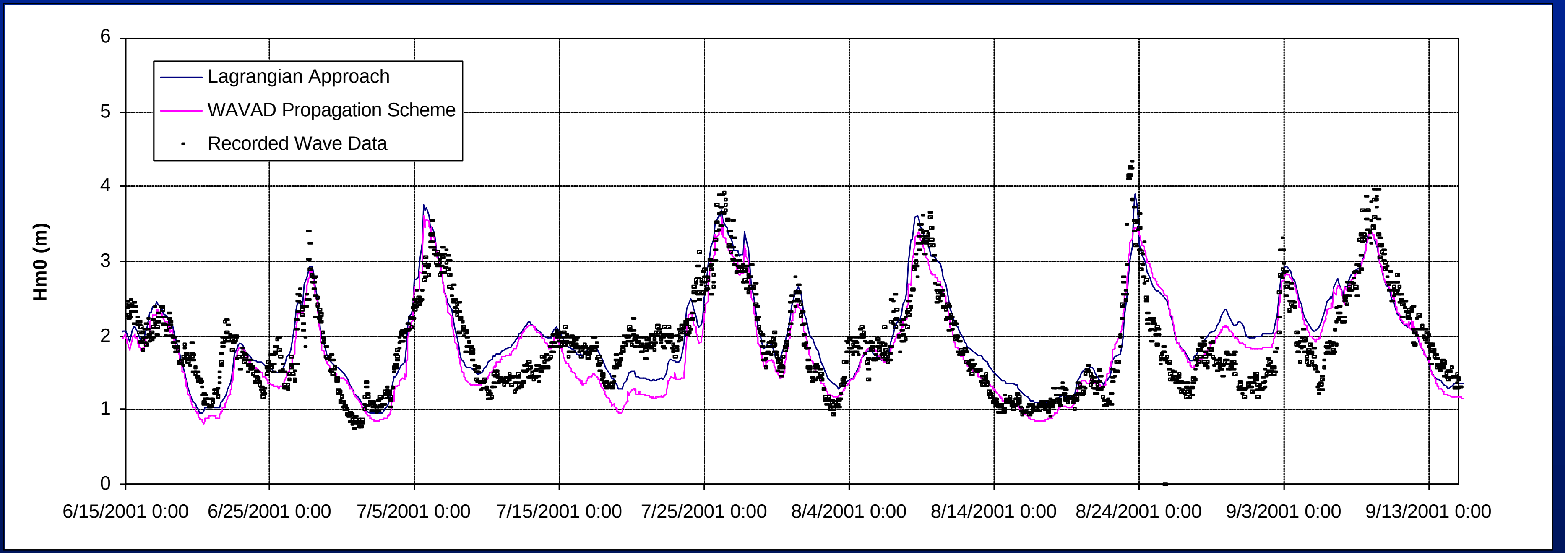
- NCEP/NCAR Re-Analysis Winds
- Only simplistic modification to winds in equatorial region based on buoy data & scatterometer (linear factor)
- One year – 2001



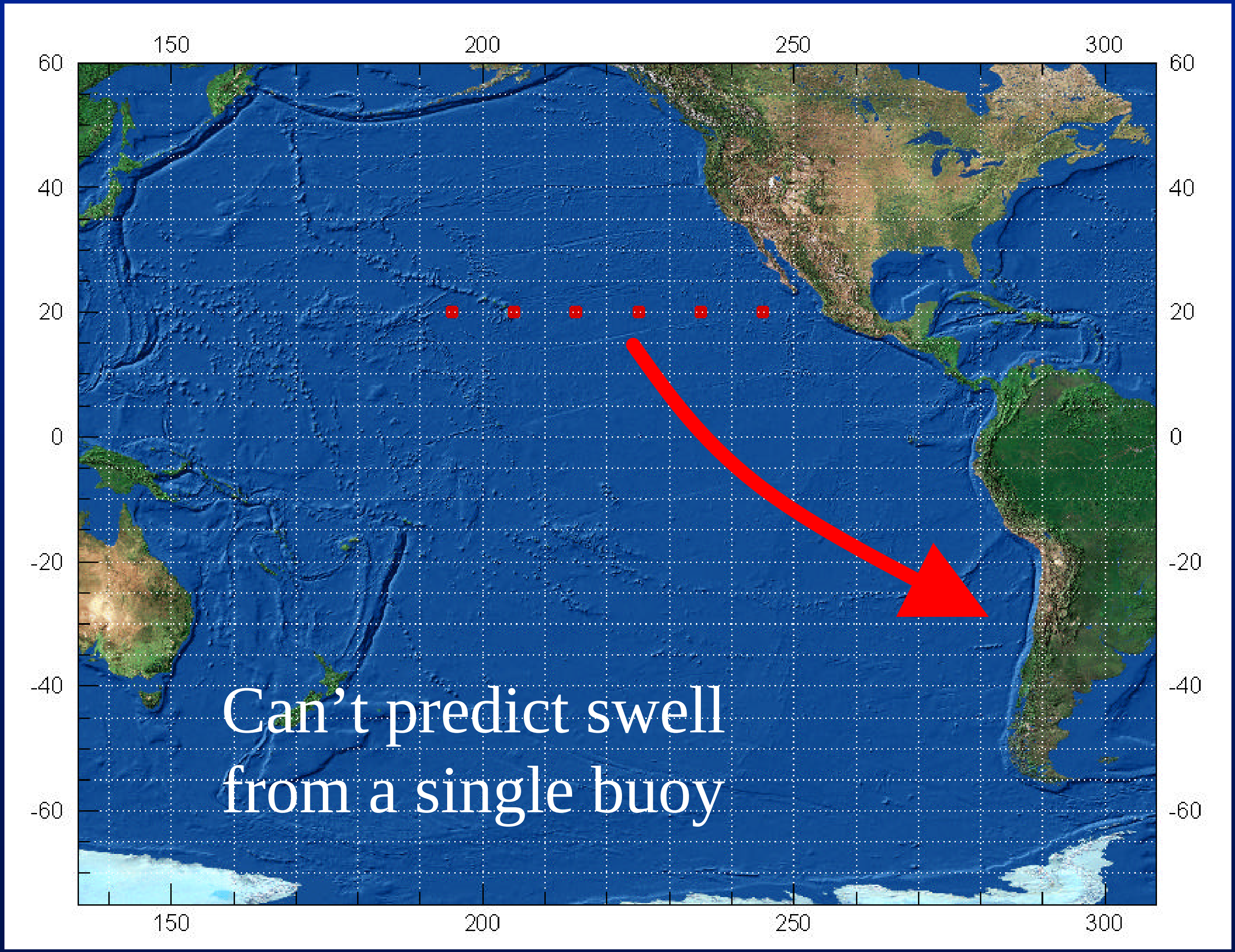
WAVAD
Bias: -0.01
SI: 0.23
R: 0.91
Bias: -0.06
SI: 0.19
R: 0.87
Bias: 0.02
SI: 0.22
R: 0.91
Bias: 0.14
SI: 0.18
R: 0.71
Bias: -0.08
SI: 0.27
R: 0.62
Note: sheltering issues, refraction
Use of higher order wave-ray scheme had no significant impact on results
150
200
250
300
60
40
20
0
-20
-40
-60

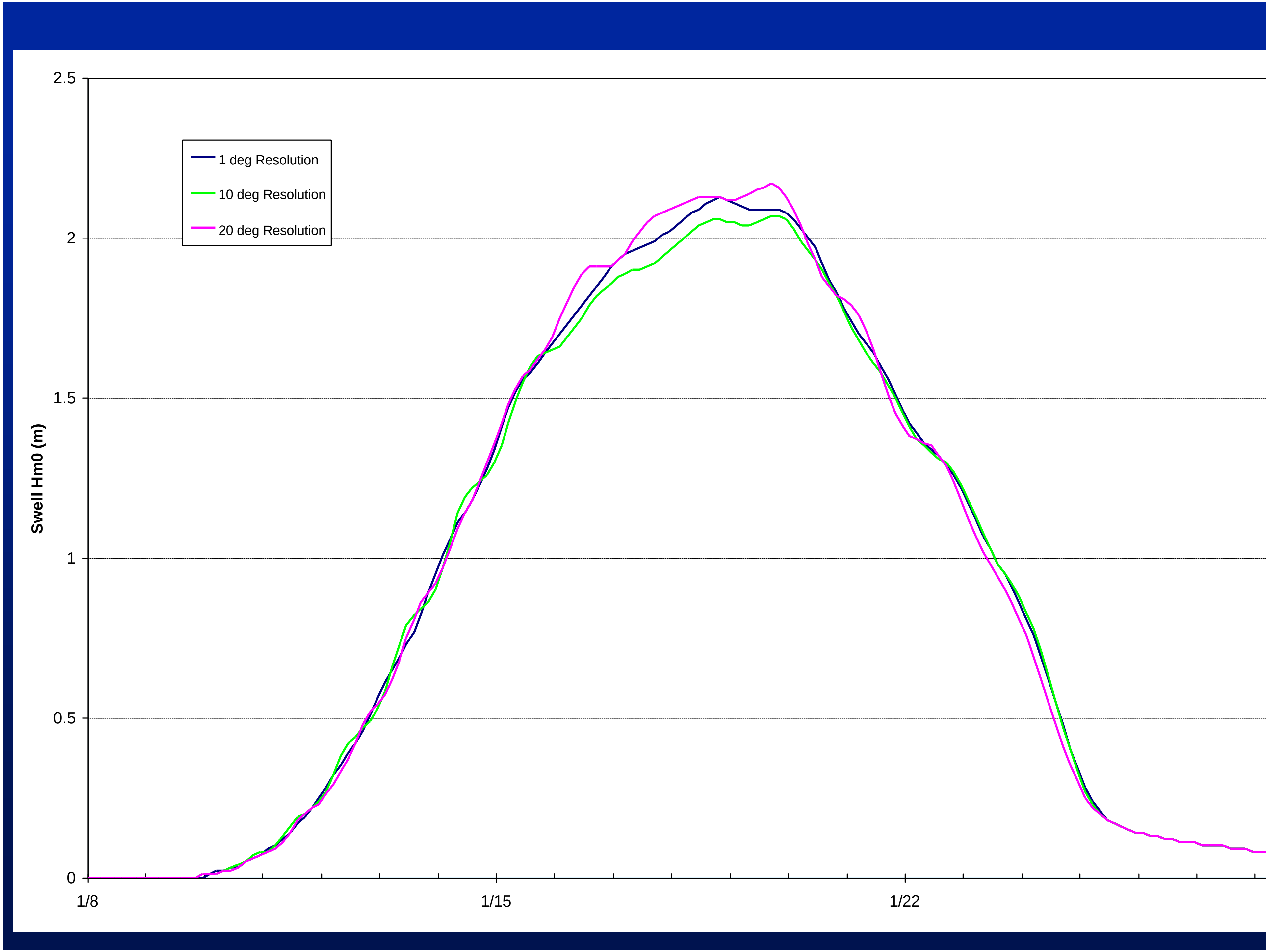

WW3
Bias: -0.15
SI: 0.19
R: 0.94
Bias: -0.04
SI: 0.16
R: 0.90
Bias: 0.10
SI: 0.17
R: 0.95
Bias: -0.28
SI: 0.15
R: 0.78
Bias: 0.25
SI: 0.24
R: 0.76
Note: sheltering issues
150
200
250
300
60
40
20
0
-20
-40
-60

Typical Wave Height Comparison with Line Integral



Swell Predictions from a Buoy Network





2.5
2
1.5
1
0.5
0
Swell Hm0 (m)
1 deg Resolution
10 deg Resolution
20 deg Resolution
1/8
1/15
1/22

Conclusions

- An outline of semi-Lagrangian wave-ray approach to wave propagation has been shown with extensions to higher order and prediction of swell from a line source.
- Line source approach didn't yield overall improvements under realistic sea states in the Pacific Ocean
  - *May aid in defining specific swell events*
  - *Useful method for isolating source term effects and evaluating diffusion in propagation schemes*
  - *Can assess areas of "influence" when assimilating buoy data*

A Diversion……..

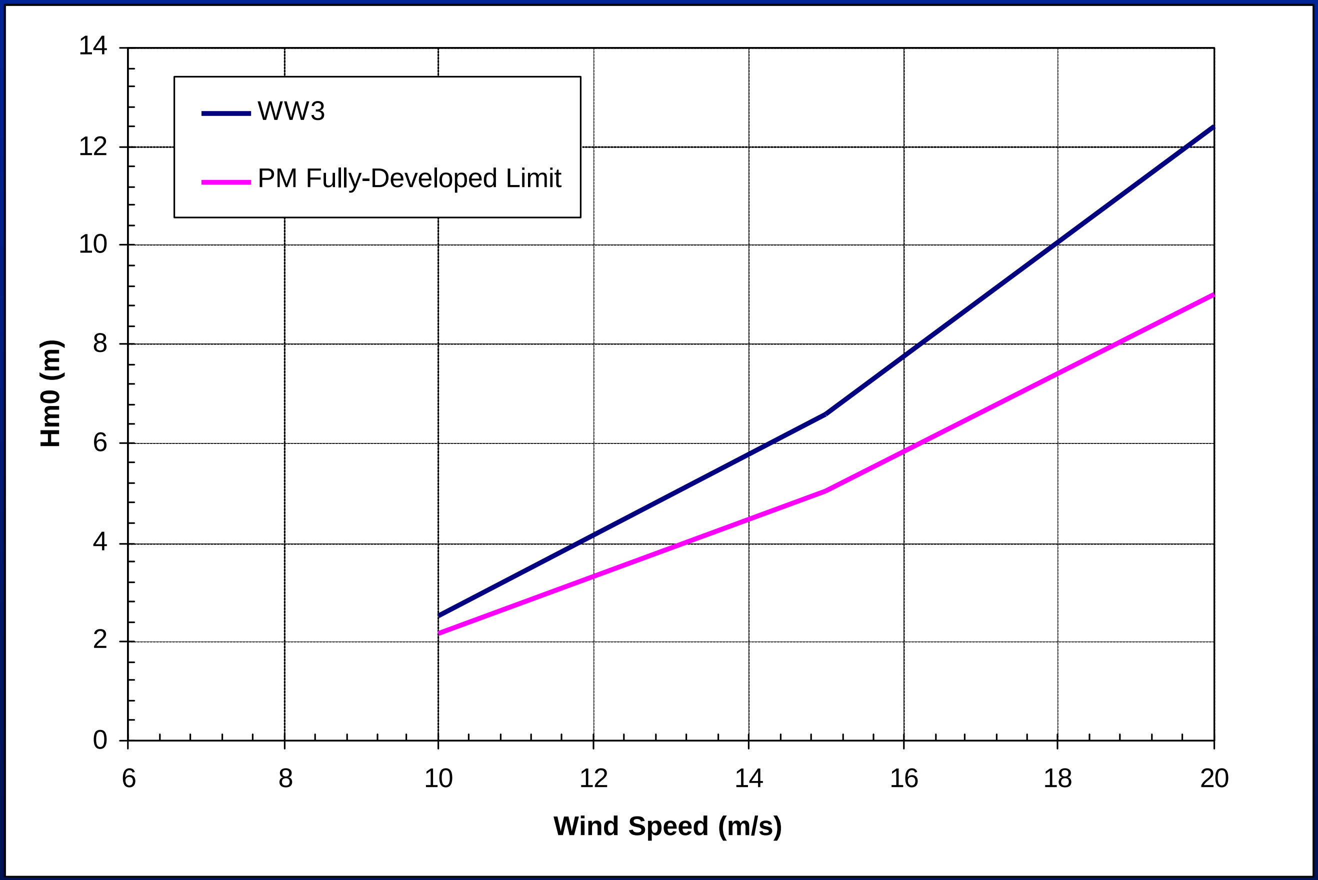
With discussion of Boltzman integrals, etc. coming up over the next couple of days…….

$$\frac{\partial n_1}{\partial t} = \iiint G\left(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4\right) \times \delta\left(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4\right) \times \delta\left(\omega_1 + \omega_2 - \omega_3 - \omega_4\right)$$
$$\times\left[n_1 n_2\left(n_3 + n_4\right) - \left(n_1 + n_2\right) n_3 n_4\right] d\vec{k}_2 d\vec{k}_3 d\vec{k}_4$$

Source Term Comparisons

- Observations when carrying out model comparisons
  - *WAVAD*
  - *WaveWatchIII (v2.22, operational parameters)*

Comparisons to Pierson-Moskowitz Limit



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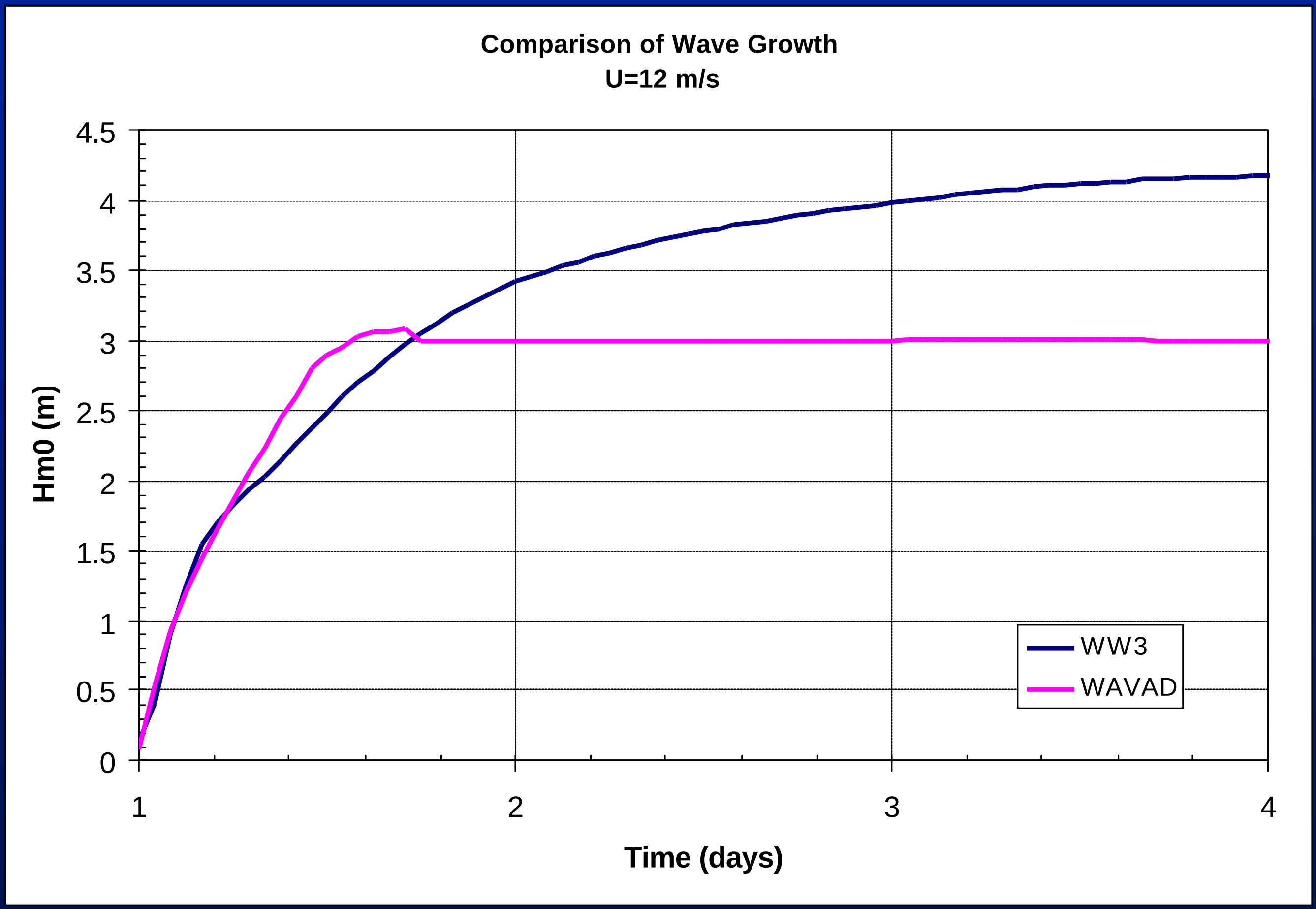
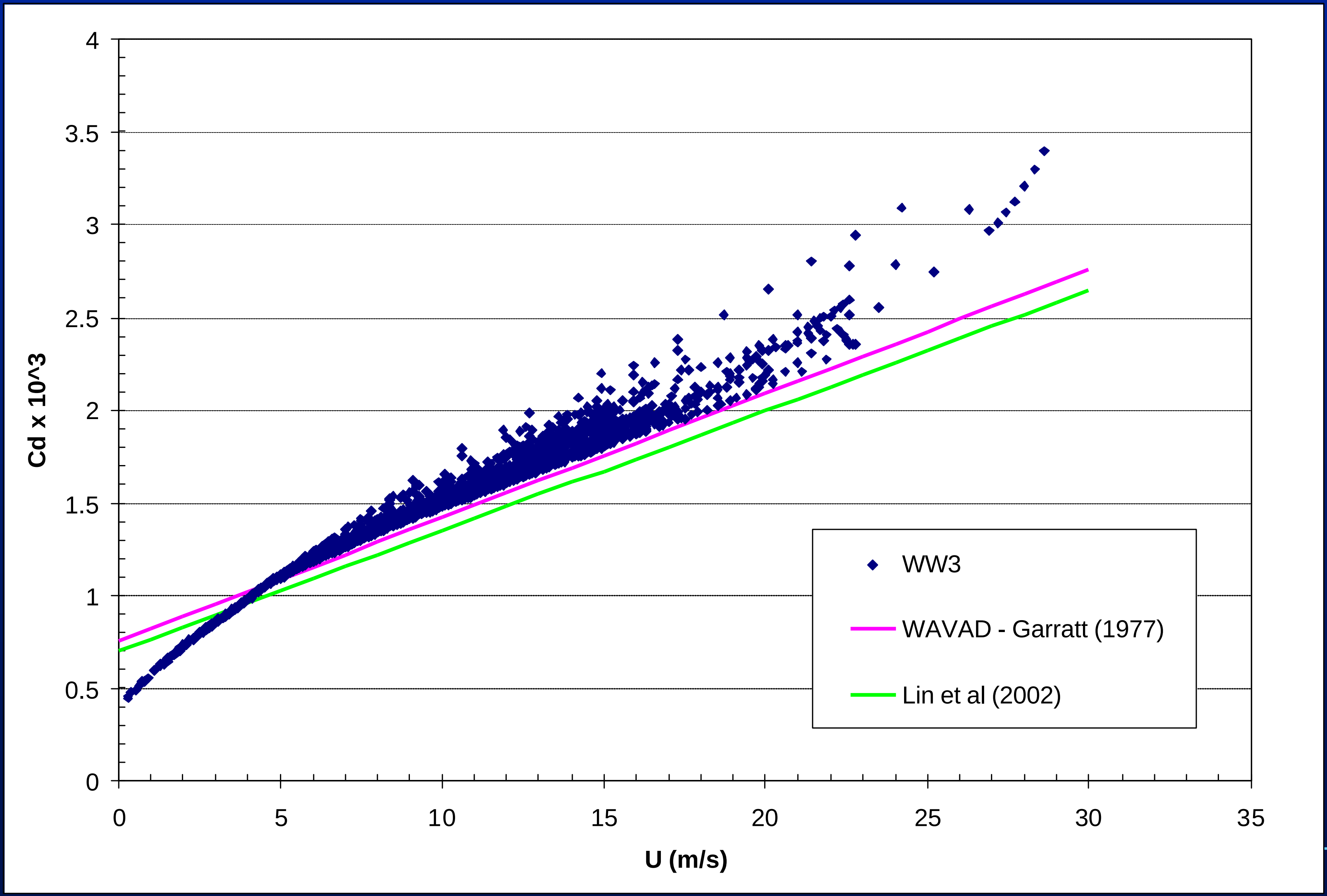
See Rogers (2002)

Rate of Development

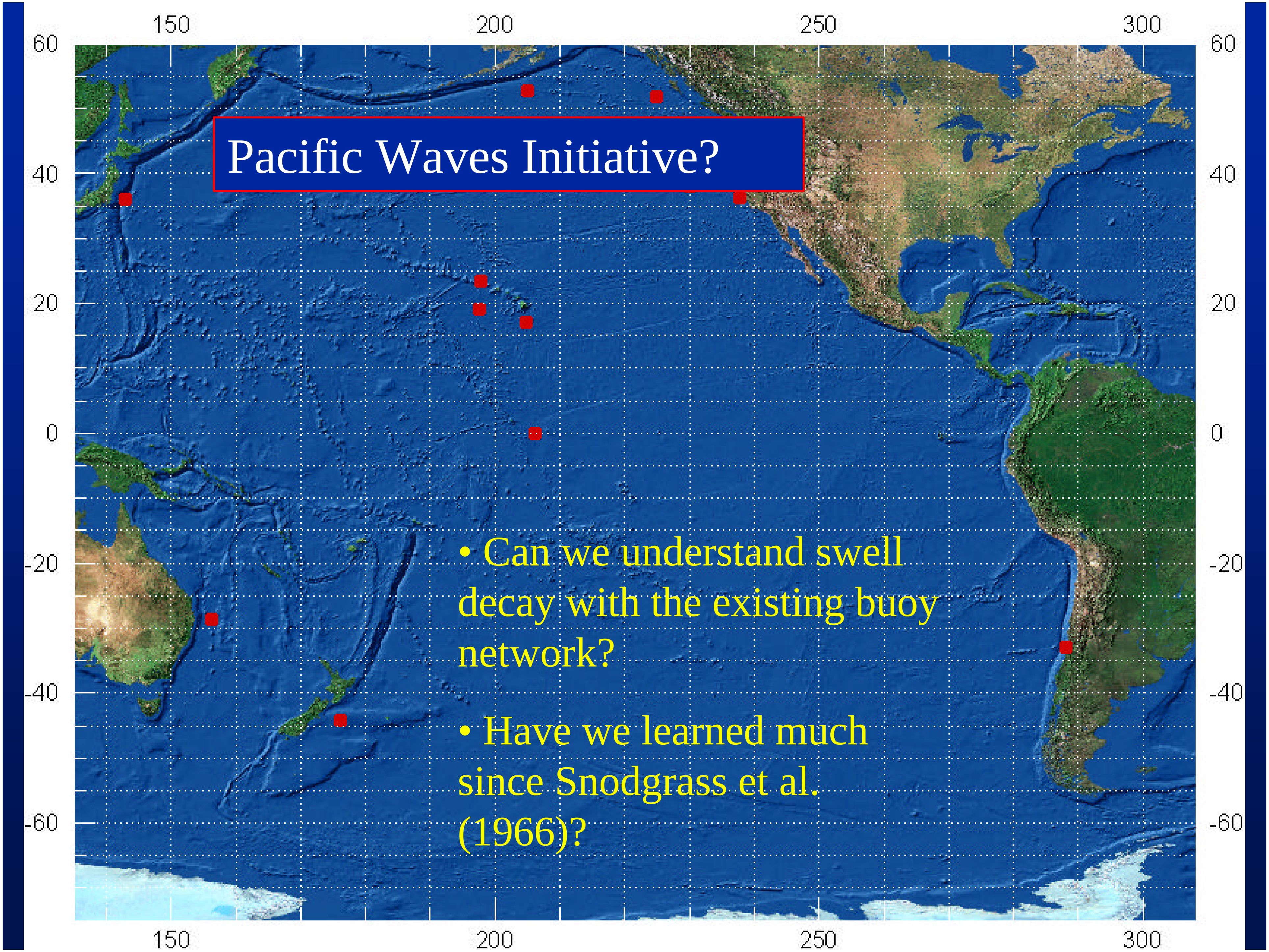


Drag Coefficient



Observations Regarding Source Terms

- Wide difference in behaviour exhibited
- Implications towards our understanding of wave physics?
- Interesting that similar results can be provided under such disparate source behaviour


Pacific Waves Initiative?
• Can we understand swell decay with the existing buoy network?
• Have we learned much since Snodgrass et al. (1966)?
150
200
250
300
60
40
20
0
-20
-40
-60