

WIND INPUT FUNCTION

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1. INTRODUCTION

Wind is one of major sources for generating water surface gravity waves. To predict wind waves, an accurate wind input function must be first developed. The study of the mechanics of wind-wave generation started more than half century ago (Phillips, 1957; Miles, 1957; Cavaleri and Malanotte-Rizzoli, 1981). However, the wind wave prediction based on classical theories tends to underestimate the wave energy growth (Hasselmann et al. 1973). In the advanced ocean wave model WAM (WAMDI group, 1988) and a recently developed coastal wave model (Lin and Huang, 1996a, 1996b; Lin and Perrie, 1997a, 1997b, 1999), the wind input function is formulated based upon the state-of-the-art technology. These recent models are generally reliable in the prediction of wave height in the deep water but not always accurate for wave direction and in coastal regions. It appears that the ocean wave generation by winds is still not fully understood. For instance, oceanographic scientists have noticed that wind generated waves do not always propagate along the down wind direction and, sometimes, the angle between wind and waves can be very large (Wang et al. 2000). This angle difference between wind and wave propagation directions can have a significant impact to the surface wave dynamics, especially in the coastal region where the wave propagation is more dependent on the bottom topography. In the present study, a new formulation of the wind input function at the ocean model scale is proposed and investigated based on the physics and wind wave data collected in the intermediate depth water.

In WAM, the physical parameters applied in the wind input function have included wave frequency, σ , wave propagation direction, θ , wind speed, u_{wind} , wind direction, θ_{wind} , and wave energy density, $E(\sigma, \theta)$. A linear relationship is assumed among the wind speed, wave propagation angle, and wave frequency (WAMDI group, 1988). However, this linear relationship is not always the case because the wind wave generation is a very complex physical process. The improvement of the wind input function in wave prediction models should be based on the wind-forcing wave process to determine the relationship among basic physical parameters. Furthermore, two more physical parameters should be also considered: wave phase velocity, c_σ , and wave age. The use of the wave phase velocity is in the light that the wave growth shall be a function of the difference of wind speed and wave phase velocity, $u_{wind} - c_\sigma$, instead of the wind speed alone. The effect of wave age is understood from the fact that the low frequency wave energy density in a narrow direction band is difficult to grow while a young wave spectrum in the high frequency range and wider direction band grows more easily. In the present study, the wave age is represented by $\frac{c_\sigma}{u_{wind}}$ and a new wind input formula is proposed to be the function of the wave age along with the others. Finally, the mechanics about the angle between wind and wave for newly generated waves must be considered (Phillips, 1957). The wind input function is also investigated for the case of a large angle between wind and wave propagation directions and possible causes associated with the condition besides the Phillips' mechanics. To accurately predict the wind wave generation, it is essential to consider the nonlinear wave-wave interaction, wave breaking, and bottom friction, especially in the shallow water. These effects were discussed in detail in separated papers (Lin and Perrie, 1977a and b; Lin and Lin, 2004).

To illustrate the new wind input formula, a series of directional wave energy spectra measured offshore of Cape Fear, NC, available from 2000 to 2003 were selected for the following three cases: (1) moderate wind, with steady speed and direction, blowing seaward for the new wave generation, (2) moderate wind, with

steady speed and direction, blowing in the same direction as the old wave propagation, and (3) persistent strong wind with a constant angle to the mean wave direction.

2. FIELD DATA

Ocean waves usually appear in irregular forms and propagate in multiple directions. To study the physical phenomena of ocean waves, it is meaningful to use data collected in the field. In the present study, wind and wave measurements offshore of Cape Fear, NC, were selected for the investigation of wind input function. Wave data were collected as a series of directional wave energy spectra from five wave gauges maintained by the Wilmington Harbor Monitoring Program (<http://www.frf.usace.army.mil/capefear>). The wave data collection was conducted in a three-year period from September 2000 to June 2003. Figure 1 shows the location of the five wave gauges relative to Cape Fear. Table 1 presents the location, water depth, and data collection periods of the five wave gauges. Offshore sea surface wind data are available from two meteorological stations maintained by the National Data Buoy Center (<http://www.ndbc.noaa.gov>). Figure 2 shows the location map of the two meteorological stations and Table 2 presents the location, water depth, and data collection period of the two stations. The two meteorological stations also measured wave energy spectra in 2000-2003. However, because these wave spectra measured from the two meteorological stations do not contain the wave direction information, they were not used in the wind input function investigation.

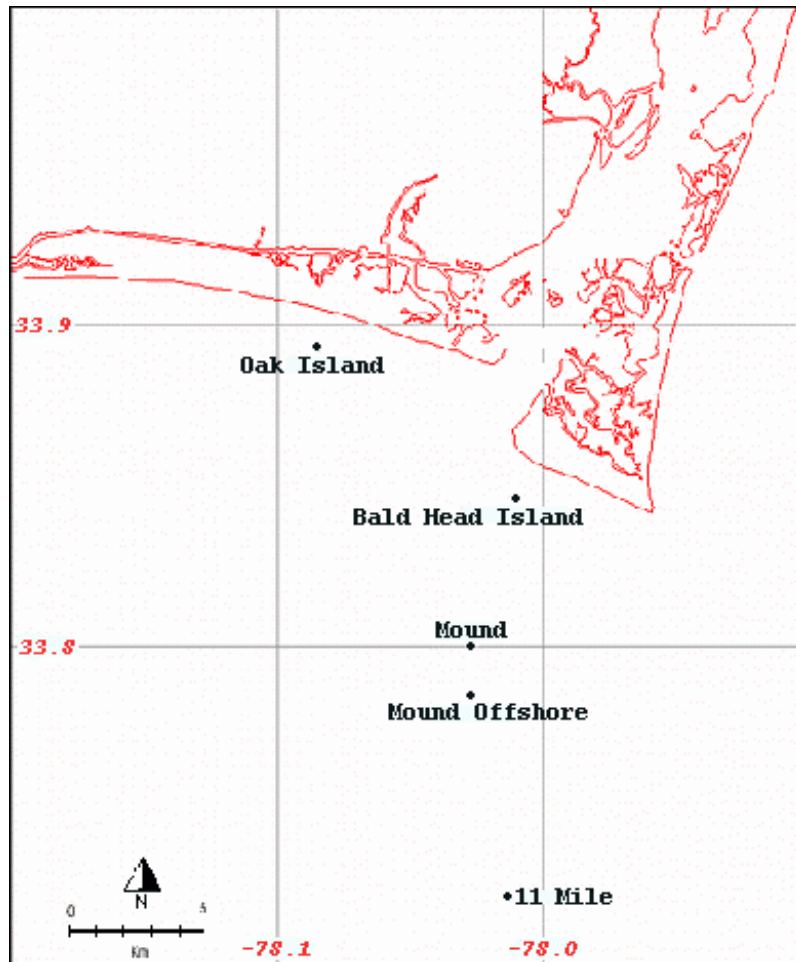


Figure 1. Location map of directional wave gauges

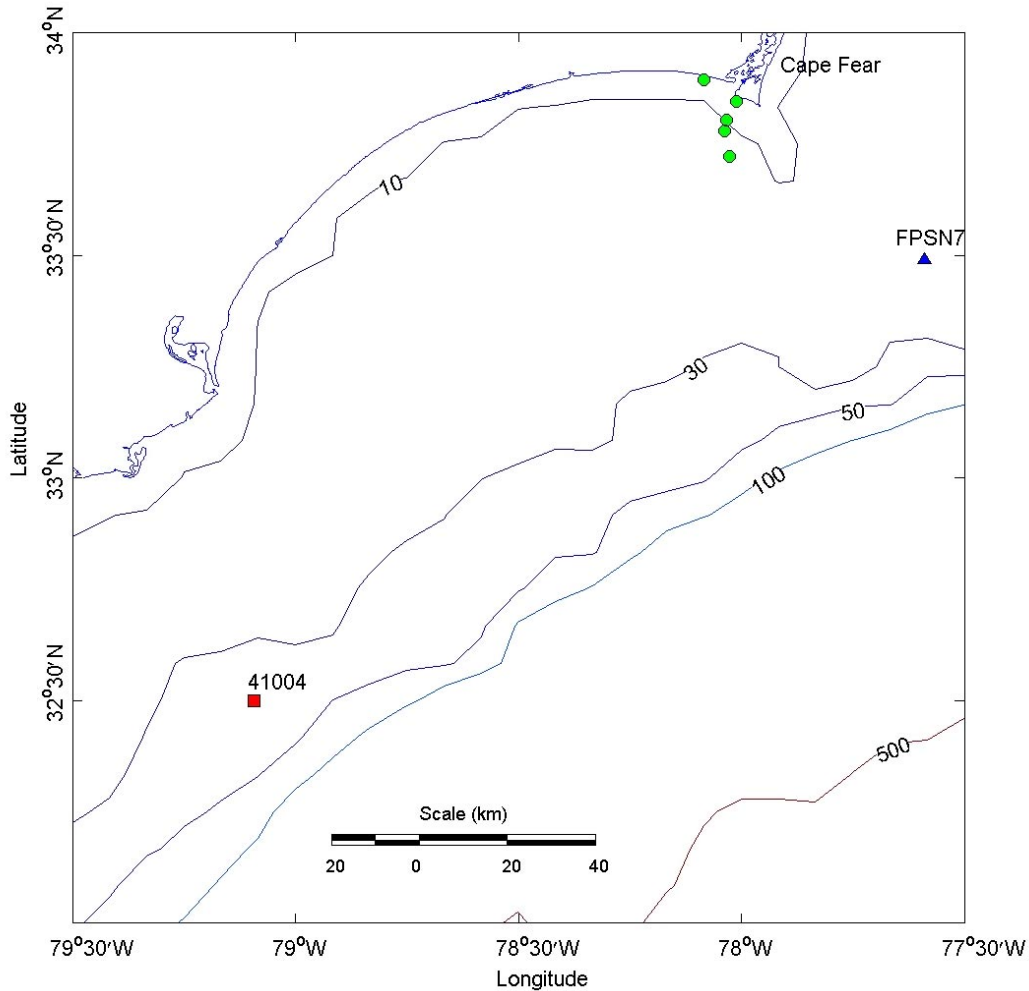


Figure 2. Location of two meteorological stations – Buoy 41004 (square) and Platform FPSN7 (triangle); also shown are directional wave gauges (green circles) and water depth contours in meters

Table 1 Directional Wave Gauge Information			
Station	Coordinates	Nominal depth (m)	Data collection period
Oak Island	33° 53' 40" N, 78° 05' 04" W	7	September 2000 – May 2003
Bald Head Island	33° 53' 02" N, 78° 00' 40" W	5.8	September 2000 – June 2003
Mound Crest	33° 48' 13" N, 78° 02' 02" W	7	July 2001 – June 2002
Mound Offshore	33° 46' 48" N, 78° 02' 15" W	12.8	August 2001 – July 2002
Mile 11	33° 43' 17" N, 78° 01' 32" W	12.8	September 2000 – June 2003

Table 2 Meteorological Station Information			
Station	Coordinates	Nominal depth (m)	Data collection period
FPSN7	33° 29' 24" N, 77° 35' 24" W	14	November 1984 – present
Buoy 41004	32° 30' 36" N, 79° 06' 00" W	38	June 1978 – present

3. WIND WAVE GENERATION

The wind input function, denoted as S_{in} , is a source term in the wave action conservation equation in recently developed New Coastal Wave Model (Lin and Huang, 1996a, 1996b; Lin and Perrie, 1997a, 1997b, 1999) as

$$\frac{\partial A}{\partial t} + \frac{\partial[(c_{gx} + u)A]}{\partial x} + \frac{\partial[(c_{gy} + v)A]}{\partial y} + \frac{\partial[c_{g\theta}A]}{\partial \theta} + \frac{\partial[c_{g\sigma}A]}{\partial \sigma} = S_{in} + S_{dp} + S_{nl}, \quad (1)$$

where A is the wave action density, defined as the ratio of spectral energy density to intrinsic frequency, σ ; c_{gx} , c_{gy} , $c_{g\theta}$, and $c_{g\sigma}$ are group velocities relative to x , y (eastern and northern axes of the water surface), θ (angle between north and each direction) and σ , respectively; u and v are the current velocity components in the x and y directions, respectively. S_{ds} is a sink term for the energy dissipation and S_{nl} is a functional term for nonlinear wave-wave interactions. The numerical scheme for solving the left hand side equation was described in Lin and Huang (1996a, 1996b). The theory and method of calculating S_{nl} were described in Lin and Perrie (1997a, 1997b, 1999). The importance and a generic formulation of S_{ds} will be discussed in a separate paper by authors (Lin and Lin, 2004). The present paper focuses on the investigation and formulation of S_{in} .

3.1 Phillips's Mechanics

As Hasselmann et al. (1973) pointed out that the wind wave generation predicted by Phillips's (1957) and Miles's mechanics (1957) is generally one order of magnitude smaller than the observed information. However, Phillips's mechanics presents an interesting condition that wind waves do not propagate in the down wind direction if the wave phase velocity is smaller than the wind speed. The angle between wind and wave propagation directions is determined from

$$\varphi = \begin{cases} \cos^{-1} \frac{c_{\sigma}}{u_{wind}}, & \text{if } c_{\sigma} < u_{wind} \\ 0, & \text{if } c_{\sigma} \geq u_{wind} \end{cases} \quad (2)$$

where

$$c_{\sigma} = \frac{\sigma}{k} \quad (3)$$

is the wave phase velocity and k is the wave number. If the wave phase velocity is equal to or greater than the wind speed, according to Phillips's mechanics, new wind waves will propagate in the down wind direction, but if the wave phase velocity is smaller than wind speed, then there is a angle, φ , between the wind wave propagation and wind directions. As an example, Figure 3 shows directional wave energy spectra observed from Mile 11, Mound Offshore, Mound Crest, and Bald Head Island between 09:38 and 11:35 GMT in November 29th, 2001, under a rather weak southeasterly wind (wind speed is 3m/sec and wind direction is from 140^o, relative to the north). The unit of the directional wave energy density is shown in m²sec/radian. These measured wave spectra clearly show that the wind wave growth is in the down wind direction.

In the case of a strong wind over the sea surface, if the wave phase velocity is smaller than the wind speed, waves can grow at an oblique angle to the wind direction. To illustrate the case, Figure 4 displays two directional spectra measured from Mound Crest and Mile 11 at 20:15 and 20:35 GMT, respectively, in November 16th, 2001, under a strong northeasterly wind (wind speed is 13m/sec and wind direction is 20^o).

These two spectra show that longer waves with a primary peak at about 0.11 Hz have propagated toward the coastline (shore normal at 180°) as results of refraction. For the primary peak of the spectrum from Mound Crest, $c_\sigma = 10.6$ m/sec and $\varphi = 35.1^\circ$ can be obtained from Equations (3) and (2), respectively, and the predicted wave propagation direction is 55.1° . For the primary peak of the spectrum from Mile 11, $c_\sigma = 12.2$ m/sec and $\varphi = 20.2^\circ$, and the predicted wave propagation direction is 40.2° . In these two spectra, the wind wave growth appears in the higher frequency range as a secondary peak at 0.2 Hz w and 120° . For this secondary peak at 0.2 Hz, $c_\sigma = 7.7$ m/sec and $\varphi = 53.1^\circ$ were determined from Equations (3) and (2), respectively, for both Mound Crest and Mile 11, and the predicted wave propagation direction is 73.1° . Surprisingly, observed wave directions vary more from the wind direction than predicted by Phillips. There must be other mechanics involved, which cause the wave propagation away from the down wind direction.

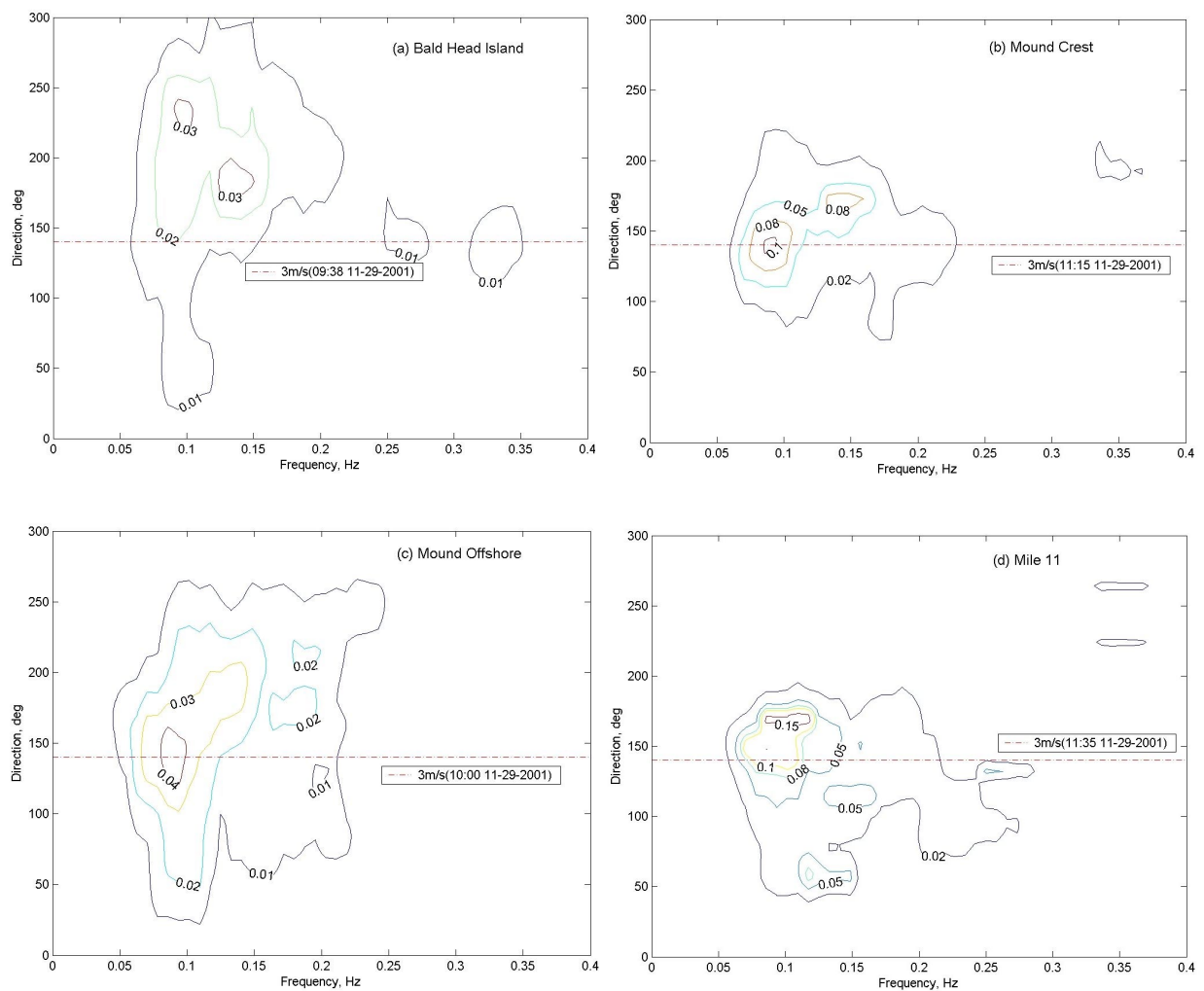


Figure 3. Measured directional wave spectra from (a) Bald Head Island, (b) Mound Crest, (c) Mound Offshore, and (d) Mile 11 between 09:38 and 11:35 GMT, November 29th, 2001 (wind speed is 3m/sec and wind direction is from 140° , shown as dash lines)

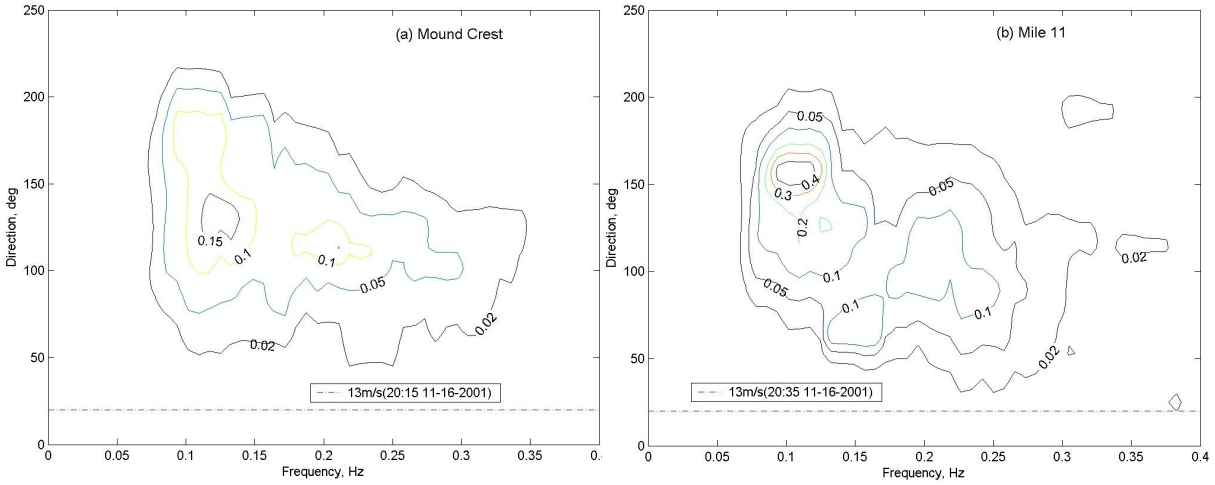


Figure 4. Measured directional wave spectra from (a) Mound Crest at 20:15 GMT and (b) Mile 11 at 20:35 GMT, November 16th, 2001 (wind speed is 13m/sec and direction is 20°, shown as dash lines)

3.2 The Nonlinear Effect of Wind Input to Wave Generation

Oceanographers and ocean research scientists have recently noticed that wind generated waves indeed may not propagate in the down wind direction and, sometimes, the angle of the difference between wind and wave directions can be greater than 60° (Wang, et al. 2000). This is particularly of interest for the wind wave generation, especially in the intermediate depth water. What are the physics, besides Phillips's mechanics, that can cause the wind-wave propagation to divert from the wind direction? It is likely due to the nonlinear wave-wave interaction that is represented by the functional term S_{nl} in Equation (1). First, we examined the five-wave interaction mechanics because many scientists have suggested, as have Lin and Perrie (1997b), the mechanics plays an important role in the spectral wave energy balance process. To investigate this, it is necessary to examine the relationship among the wind speed, spectral peak frequency, and the angle between wind and wave propagation directions. For fully developed sea states in a finite depth, h , the peak frequency, σ_{peak} , can be approximated by (Graber and Madsen, 1988)

$$\sigma_{peak} = \frac{g \tanh k_{peak} h}{u_{wind} \cos(\theta_{wind} - \theta_{wave})}$$

where k_{peak} is the wave number corresponding to the peak frequency. Figure 5 shows an example of this relationship for the demonstration: (1) the peak frequency of wind generated waves increases as the wind speed decreases, and (2) the peak frequency of wind generated waves increases as the angle between wind and wave propagation directions increases. If the mechanics is due to five-wave interactions, the dominated mechanics must transfer the energy from lower to higher frequencies. At Mile 11, where $h=12.8$ m, and the peak frequency is 0.2 Hz, so $kh = 2.1$. This condition corresponds to the intermediate water regime and, as a result, the four-wave interactions should dominate. If the peak frequency is 0.11 Hz, then $kh=0.8$. It also corresponds to the intermediate water regime that four-wave interactions dominate. Furthermore, Figures 4a and 4b show that wave energy densities in both lower and higher frequency tails are not symmetrical along the wind direction. Therefore, five-wave interactions cannot be the major mechanics. In this case, the four wave interactions should dominate and, as a consequence, most wave energy must transfer from higher to lower frequencies along the down wind direction.

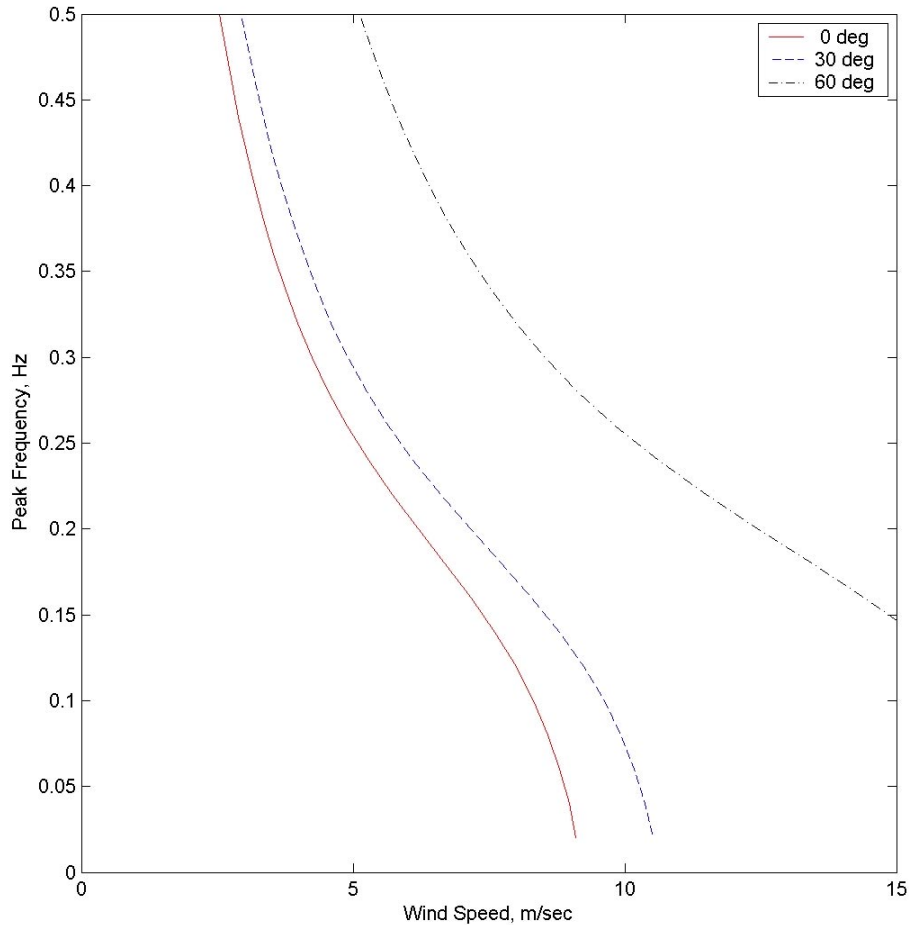


Figure 5. Relation of peak frequency, wind speed, and angle between wind and wave propagation directions

What is the mechanics responsible for wind waves to propagate away from the down wind direction besides the Phillips' mechanics? Based on the nonlinear mechanics by Lin and Perrie (1997), the long wave interacting with local wind waves is the dominant mechanics in the coastal region. The long wave can be a swell, an edge wave, or a bottom topography wave. In Figure 4, the long wave is a swell from the open ocean that has absorbed the energy of local wind waves by four-wave interactions. Therefore, the swell gained wave energy from local wind waves but it retained its original propagation direction. To further demonstrate the case, the wind and wave information collected from Mound Offshore, Bald Head Island, and Oak Island were used in the investigation. Figure 6a displays the directional wave spectrum collected from Mound Offshore at 19:00 GMT in November 16th, 2001, during a strong northeasterly wind (wind speed is 13 m/sec and wind direction is 20°). The directional spectrum shows a swell with the spectral peak at 0.12 Hz and propagation direction from the SSE (160°). Figure 6b displays the directional wave spectrum collected at 19:38 GMT of the same day from Bald Head Island. It indicates that the swell is associated with the peak at 0.12 Hz and propagation direction is more from the south (170°). Figure 6c displays the directional spectrum at 19:55 GMT of the same day from Oak Island. This spectrum also shows the swell is more from the south (170°). It is evident in Figure 6a, 6b, and 6c that the swell gradually turns its direction from the SSE toward the north from the open ocean to coastal region as a result of wave refraction. The swell does not follow the local wind direction. On the other hand, the spectral tail or higher frequency energy densities have experienced less wave refraction and are affected more by the local wind. Because much greater energy content is associated with the swell than in the spectral tail, as shown in Figure 6, the swell must have absorbed the energy of higher frequency wind waves through the four-wave interactions (Lin and Perrie, 1997b).

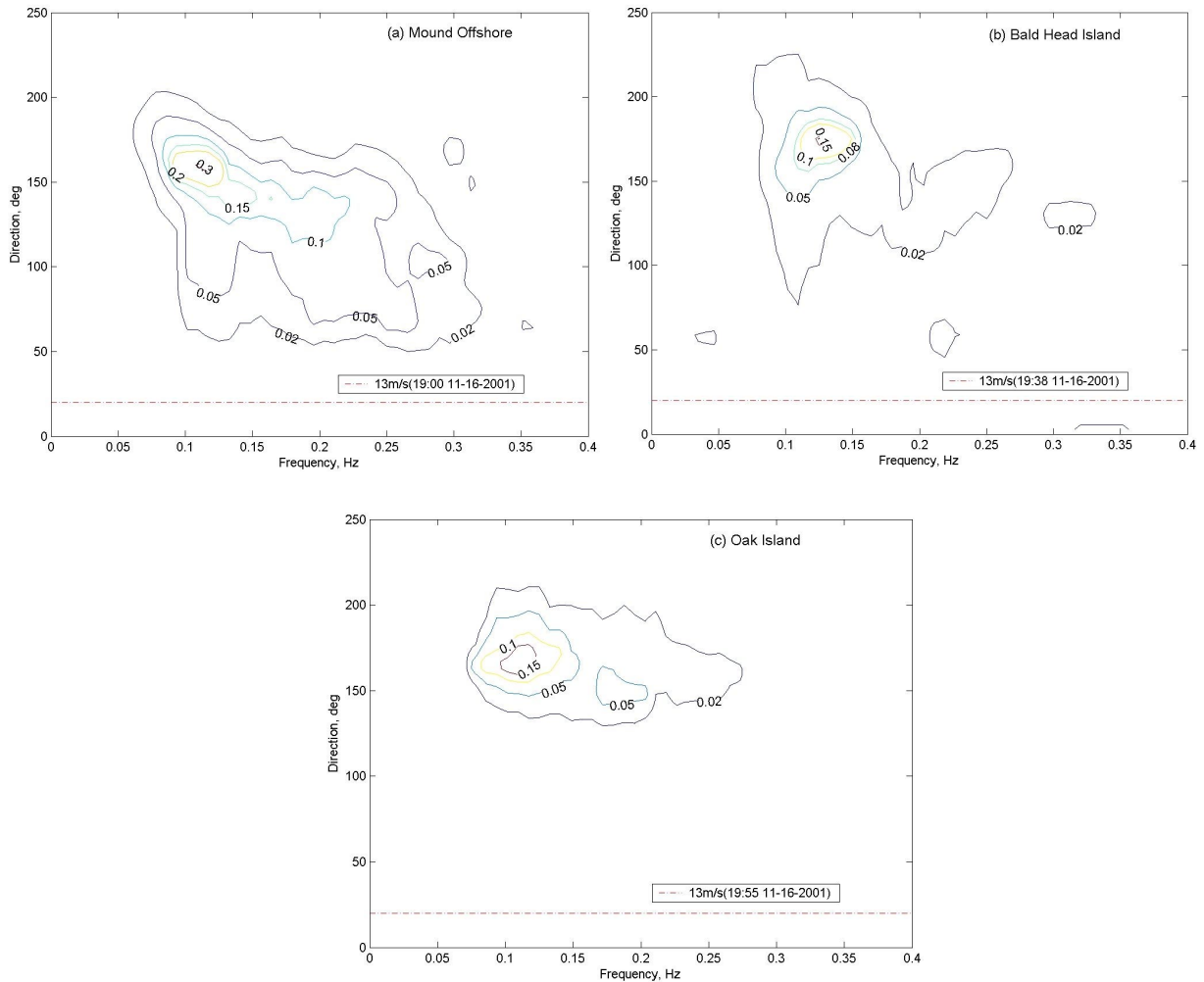


Figure 6. Measured directional wave spectra from (a) Mound Offshore at 19:00 GMT, (b) Bald Head Island at 19:38 GMT, and (c) Oak Island at 19:55, November 16th, 2001 (wind is 13m/sec from 20^o, as dash lines)

4. The NEW WIND INPUT FUNCTION

Section 3 demonstrates that wind generated waves can be strongly influenced by an initial spectrum through nonlinear four-wave interactions, transferring the energy from higher to lower frequencies. In the cases where the local wind direction is different from a swell or longer waves in intermediate depth water, the growth of waves appears in the swell absorbing the energy from higher frequency wind waves. Although the wind wave generation has been studied more than half century, it is still not fully understood. Existing wind input functions applied in recent wave models are still based on either empirical or semi-empirical formulas (Hasselmann et al. 1973; WAMDI group, 1988). The lack of sufficient wind and directional wave data in the past has limited the investigation of the wind input function. In the present study, the formulation of the wind input function is determined by how its physical process was interpreted by the authors and is expressed as

$$S_{in} = \frac{a_1 \sigma}{g} F_1(\bar{u}_{wind} - \bar{c}_\sigma) F_2\left(\frac{\bar{c}_\sigma}{\bar{u}_{wind}}\right) E_{PM}^*(\sigma) \Phi(\theta) + \frac{a_2 \sigma}{g} F_1(\bar{u}_{wind} - \bar{c}_\sigma) F_2\left(\frac{\bar{c}_\sigma}{\bar{u}_{wind}}\right) F_3\left(\frac{\bar{c}_\sigma}{\bar{u}_{wind}}\right) E(\sigma, \theta), \quad (4)$$

$$\text{where } F_1(\bar{u}_{wind} - \bar{c}_\sigma) = \begin{cases} |\bar{u}_{wind} \cos(\theta_{wind} - \theta) - c_\sigma(\sigma, \theta)|, & \text{if } \bar{c}_\sigma < \bar{u}_{wind} \\ 0, & \text{if } \bar{c}_\sigma \geq \bar{u}_{wind} \end{cases},$$

$$F_2\left(\frac{\bar{c}_\sigma}{\bar{u}_{wind}}\right) = \begin{cases} \cos \varphi, & \text{if } \bar{c}_\sigma < \bar{u}_{wind} \\ 1, & \text{if } \bar{c}_\sigma \geq \bar{u}_{wind} \end{cases},$$

$$F_3\left(\frac{\bar{c}_\sigma}{\bar{u}_{wind}}\right) = \begin{cases} \log_{10}\left[\left(\frac{c_\sigma}{u_{wind}}\right)^{-1}\right], & \text{if } \bar{c}_\sigma < \bar{u}_{wind} \\ 0, & \text{if } \bar{c}_\sigma \geq \bar{u}_{wind} \end{cases},$$

$$\text{and } E_{PM}^*(\sigma) = \frac{a_1 g^2}{\sigma^5} \exp(-0.74 \frac{\sigma_0^4}{\sigma^4}).$$

Here, $E_{PM}^*(\sigma)$ is the functional form of Pierson-Moskowitz spectrum, $\sigma_0 = g/u_{wind}$ is the Phillips's constant, and

$$\Phi(\theta) = \frac{8}{3\pi} \cos^4(\theta - \theta_{wind}) \quad \text{for } |\theta - \theta_{wind}| < \pi/2.$$

is a normalized directional spreading. Function F_1 is due to the wind stress effect, F_2 is due to Phillips' mechanics with φ being defined in Equation (2), and F_3 is due to the wave age effect. For long waves (old waves), the phase velocity is generally large and $F_3 < 1$. In the condition that if $c_\sigma \geq u_{wind}$, then, $F_3 = 0$. For short waves (young waves), the phase velocity is generally small and $F_3 > 1$. A least squares routine is used to estimate coefficients a_1 and a_2 in the following two steps:

$$\text{Step 1: Compute } \Delta = \sum_{i=1}^N (S_{data} - S_{in})_i^2, \quad (5)$$

where S_{data} is the wind wave energy from the observational data, and subscript i is the index of frequency and direction bands.

$$\text{Step 2: Solve } \frac{\partial \Delta}{\partial a_1} = 0, \text{ and } \frac{\partial \Delta}{\partial a_2} = 0. \quad (6)$$

By using the data collected in Mile 11 and Mound Offshore in the intermediate depth water, and therefore neglecting the dissipation from bottom friction, in the case of new wave generation under moderate seaward winds, and eliminating strong wave breaking, $a_1 = 0.000003$ and $a_2 = 0.00005$ were obtained from Equations (5) and (6). Figure 7, as an example of applying Equation (4) for a depth of 18 m, shows the wave energy growth in the frequency spectrum for the first 5 hours under the steady wind of 5 m/sec (wind direction is 90°) over a calm sea. Figure 8 shows the directional wave spectrum at the end of the 5-hour simulation. The results in both Figures 7 and 8 do not consider nonlinear wave-wave interactions, wave breaking, and bottom friction effects. Therefore, higher frequency waves at the end of the 5-hour simulation should already break and longer waves should appear as a result of wave-wave interactions transferring wave energies from high to low frequencies.

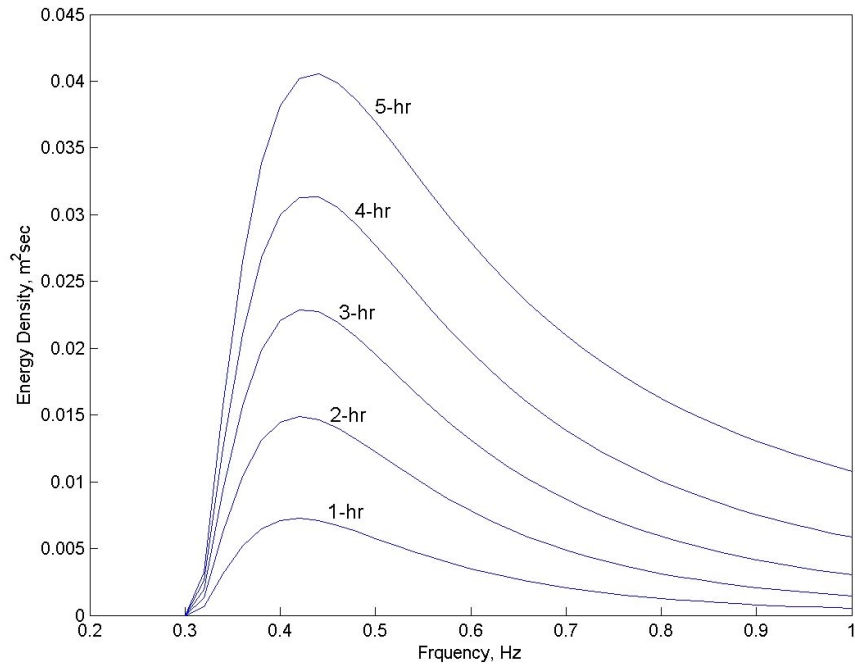


Figure 7. Wave energy growth in the frequency spectrum for the first 5 hours under the steady wind condition (wind speed is 5 m/sec, from 90°) over a calm sea in water depth of 18 m

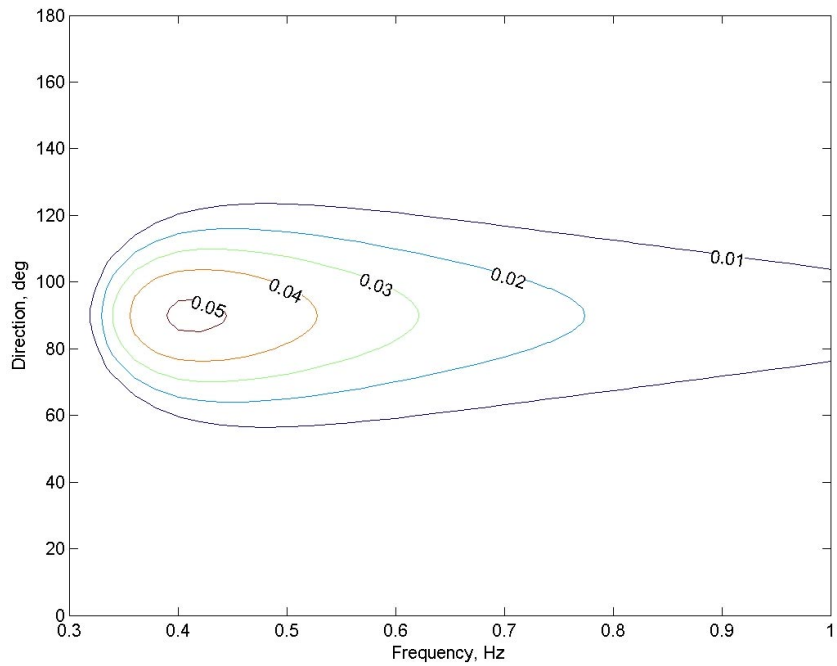


Figure 8. Directional wave spectrum (in the units of $\text{m}^2\text{sec/radian}$) from a 5-hour simulation under the steady wind condition (wind speed is 5 m/sec and direction from 90°) over a calm sea in water depth of 18 m

5. SUMMARY

The new wind input source in the New Coastal Wave Model (Lin and Huang, 1996a and b; Lin and Perrie, 1997a and b, 1999) is formulated as a function of the difference between wind speed and wave phase velocity, wave age, Phillips' mechanics, as well as those traditional parameters used in WAM, such as wave frequency, σ , wave propagation direction, θ , wind direction, θ_{wind} , and wave energy density, $E(\sigma, \theta)$. To accurately estimate the wind input function, one must consider the difference between wind speed and wave phase velocity, instead of wind speed alone. It is also essential to include the wave age effect because long waves such as a swell are more difficult to grow while young waves can grow easier. In addition, to include the Phillips' mechanics and the effect of nonlinear wave-wave interactions (Lin and Perrie, 1997a and b), the new wind input is able to predict the wave propagation direction divergent from the wind direction. It has been noticed recently that the wind wave growth in the ocean is not always in the same direction as the wind, and sometimes the difference of direction between the two can be large. The diversion of the wind wave propagation from the wind direction may significantly impact the wave generation. In the case of a strong wind over young waves with the wave phase velocity smaller than the wind speed, the corresponding wave propagation direction will be different from the wind direction, especially in intermediate depth water. Phillips suggested (1957) that the angle between wind and wave propagation directions is $\cos^{-1} \frac{c\sigma}{u_{wind}}$. In the case of a mild wind where the wind speed is less than the wave phase velocity of the spectrum, the wind wave propagation will be in the down wind direction. Besides the Phillips' mechanics, the wind wave propagation direction is also affected by the initial spectrum and a new nonlinear four-wave interaction mechanics suggested by Lin and Perrie (1997b). For instance, when a swell propagates from the open ocean to coastal regions, it can absorb the energy from local wind waves through nonlinear four-wave interactions and the mean wave propagation will be strongly associated with the original swell direction.

Based on the understanding of physical processes of the wave growth at sea, a new wind input function formula is expressed in Equation (4), which is a function of wind speed, wave phase velocity, wave age, and other basic parameters ($\sigma, \theta, \theta_{wind}$, and $E(\sigma, \theta)$) with empirical constants $a_1 = 0.000003$ and $a_2 = 0.00005$ being estimated from a least square method based on the observational data. To more accurately estimate the wind wave, one must couple the nonlinear wave-wave interaction term (Lin and Perrie, 1997a, 1999) and wave breaking function (Lin and Lin, 2004).

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References

- Cavaleri, L. and Malanotte-Rizzoli, P. 1981. Wind Wave Prediction in Shallow Water: Theory and Applications. *J. Geophys. Res.*, 86, No. C11, 10,961-10,973.
- Graber, H.C. and Madsen, O.S. 1988. A Finite-Depth Wind-Wave Model. Part I: Model Description, *J. Phys. Oceanogr.*, 18, 1465-1483.
- Hasselmann, K., Barnett, T.P., Bouws, E., Carlson, H., Cartwright, D.E., Enke, K., Ewing, J.A., Gienapp, H., Hasselmann, D.E., Krusemann, P., Meerburg, A., Muller, P., Olbers, D.J., Richter, K., Sell, W. and Walden, H., 1973. Measurements of Wind-Wave Growth and Swell Decay during the Joint North Sea Wave Project (JONSWAP), *Dtsch. Hydrogr. Z. Suppl.*, 12, A8.

- Lin, R.-Q. and Huang, N.E. 1996a. The Goddard Wave Model, Part I. The Numerics, *J. Phys. Oceanogr.*, 26, 833-847.
- Lin, R.-Q. and Huang, N.E. 1996b. The Goddard Wave Model, Part II. Kinematics, *J. Phys. Oceanogr.*, 26, 848-862.
- Lin, L. and Lin, R.-Q. 2004. Wave Breaking Function. 8th International Workshop on Wave Hindcasting and Forecasting, North Shore, Oahu, Hawaii (Accepted).
- Lin, R.-Q. and Perrie, W. 1997a. A New Coast Wave Model Part III. Nonlinear Wave-wave Interactions, *J. Phys. Oceanogr.*, 27, 1813-1826.
- Lin, R.-Q. and Perrie, W. 1997b. A New Coastal Wave Model Part V. Five-wave Interactions, *J. Phys. Oceanogr.*, 28, 2169-2186.
- Lin, R.-Q. and Perrie, W. 1999. Wave-wave Interactions in Finite Water Depth (A New Coastal Wave Model, Part IV), *J. Geophys. Res.*, 104, No. C5, 11193-11213.
- Miles, J. W., 1957. On the Generation of Surface Waves by Shear Flows. *J. Fluid Mech.* 3, 185-204.
- Phillips, O. M., 1957. On the Generation of Waves by Turbulent Wind. *J. Fluid Mech.* 2, Part 5, 417-445.
- WAMDI group, 1988. The WAM Model - A Third Generation Ocean Wave Prediction Model. *J. Phys. Oceanogr.*, 18, 1775-1810.
- Wang, W.D., Hwang, P.A., Kaihatu, J.M., and Rogers, W.E. 2000. Bimodal Directional Distribution of Ocean Waves in Mixed Seas. The 6th International Workshop On Wave Hindcasting and Forecasting, Monterey, California. 56-64.