A GENERALIZED WAVE-RAY APPROACH FOR PROPAGATION ON A SPHERE AND ITS APPLICATION TO SWELL PREDICTION

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1. INTRODUCTION

A generalized wave-ray approach for simulating the propagation of ocean waves has been developed. This approach, based on a semi-Lagrangian numerical scheme, considers a solution for the intersection of wave rays propagating from a grid boundary of arbitrary size into a central grid point, minimizing the number of mathematical operations required in each propagation step. This method also explicitly allows for angular variations along great circle paths and depth effects, and permits an alternative long-distance approximation for swell propagation. This alternative method is equivalent to treating waves arriving at a point as a summation of contributions from a line source.

The work described in this paper has been motivated by two primary objectives:

- Development of a more natural form for incorporating depth refraction in models. Although most wave generation modeling is carried out at spatial scales where underwater topography and refraction processes are not well represented, there can be distinct advantages to the use of nested higher resolution grids that allow for improved simulation of wave refraction processes. In many wave models forumulated on a Cartesian grid system, the additional curvature due to refraction causes numerical problems and limits are placed on curvature. The proposed scheme addresses these shortcomings.

- Improved swell prediction. The prevalence of ocean swells and the importance of reliable swell prediction has been long recognized (e.g. Barber and Ursell, 1948). Low frequency energy has impact on many coastal processes and activities, including harbour design and sediment transport. Moored vessels in a harbour respond more readily to long period wave action than to shorter periods, and quantification of the magnitude and frequency of occurrence of these wave conditions is important to the assessment of general marine operability. Ocean swells follow great circle paths around the globe, which are simply extensions of wave rays.

The propagation scheme has been implemented in a wave model that is devised on the basis of second-generation physics. A series of comparisons have been carried out by means of simplistic test cases as well as simulations of wave conditions in the Pacific Ocean.

2. DEVELOPMENT OF GENERALIZED WAVE-RAY APPROACH

2.1 The Wave Ray Approach

Most computer codes for wave propagation today are based on adaptations of Cartesian x-y propagation methods, first propagating in the x-direction (longitude) and subsequently the y-direction (latitude). In order to adapt these for propagation on a sphere, the apparent turning of the waves as they follow great circle paths and the divergence/convergence of the waves along adjacent great circles must also be considered. In typical latitude-longitude grid systems, convergence effects are added to propagation in the north-south direction and curvature effects are added to the east-west direction. Although the Cartesian propagation methods have had reasonable success in ocean-scale applications, they often exhibit significant problems when refraction effects are included. In such situations, the additional curvature due to refraction is so large that very small time steps must be used in order to maintain stability in the simulation. Often curvature limits are employed in an attempt to minimize the impacts on time steps, but this is at the cost of accuracy at shallow sites where such limits are applied.

It is clear that, particularly for the case of transition to shallow water, there is an advantage to reformulating the propagation scheme used in time-stepping models to an alternative form that retains stability for arbitrary bathymetry. This is possible if a semi-Lagrangian method is used to trace the path all the way to the intersection of a given spectral component with the grid boundary between to adjacent grid cells, as is done in shallow-water models for arbitrary bathymetry. In this case an equation for the solution of the intersection can be written as

$$(x, y, \alpha)_{\text{intersection}} = \Phi(x_0, y_0, \delta x_0, \delta y_0, -\alpha_0)$$

where α is the direction of wave propagation and x and y are the longitude and latitude of the intersection point at the boundary, x_0 and y_0 are the values at a given grid point, and $-\alpha_0$ is the negative angle of wave propagation (i.e. the direction of the "back-tracked" ray) at the grid point. The heading along a great circle path can be written as

$$\alpha = 2\pi - \cos^{-1}(Y / X)$$

where

$$Y = \sin(y) - \sin(y_0)\cos(D)$$
$$X = \sin(D)\cos(y_0)$$

and D is the distance along the great circle between (x,y) and (x_0,y_0). Since both the location of the intersection and the angle at the intersection are unknown, an iterative solution must be used to solve this equation. The method of bisection was chosen to provide a stable adaptive method for any number of angles and any grid spacing. This solution method can also easily be extended to the treatment of additional curvature due to refraction. Since this part of the solution need only be computed once, it does not influence the overall efficiency of the code significantly.

By the method of construction, each intersection point falls between two definable grid points and angles. Hence, at any chosen time the energy at the intersection point (which is the energy that ends up at the grid point some elapsed time later) can obtained via a bilinear interpolation of the form

$$\hat{E}(f,\theta)_{\text{intersection}} = \lambda_{11}\hat{E}_{i\pm n,\, j\pm m}(f,\theta_1) + \lambda_{12}\hat{E}_{i\pm n',\, j\pm m'}(f,\theta_1) + \lambda_{21}\hat{E}_{i\pm n,\, j\pm m}(f,\theta_2) + \lambda_{22}\hat{E}_{i\pm n',\, j\pm m'}(f,\theta_2)$$

where n and n' are the counters for the grid displacement of the two reference grid points in longitude and m and m' are counters for the grid displacement of the two reference grid points in latitude. In this equation, the usual directional spectral density $E(f,\theta)$ has been replaced by $\hat{E}(f,\theta) = E(f,\theta)\cos y$, where y is latitude as previously defined. As might be expected, one additional multiplier is required to extend this to the case of a fixed time step. This multiplier will depend on the ratio of the propagation distance over a time step to the total distance between the grid point and the intersection point, i.e.

$$\lambda_t = \frac{c_g \Delta t}{D}$$

where Δt is the model time step and c_g is the group velocity of the waves. A simple linear upwind differencing scheme for a spectral component at a grid point could be written as

$$\hat{E}_{i,j}(f,\theta)^{(n)} = (1-\lambda_t)\hat{E}_{i,j}(f,\theta)^{(n-1)} + \lambda_t \hat{E}(f,\theta)_{\text{intersection}}$$

A comparison of this propagation form to a comparable Cartesian linear propagation shows that it should be slightly more efficient than the previous methods, but its real advantage is in its ability to operate in regions of arbitrary bathymetry without arbitrary restrictions on the curvature of the rays.

2.2 Semi-Lagrangian Approach to Swell Propagation

An interesting extension of the method described above can be used to relate the waves arriving at any point on the earth to the waves passing a fixed, non-intersecting line, given that the source terms are negligible over the propagation distance. In order to focus specifically on situations where the source terms might be expected to be relatively small, it is usually advisable to restrict this treatment to a single line through which swell from a very remote source area propagates toward a point of interest. In this case, the energy arriving at a point can be estimated via the relation

$$E(x_0, y_0, f, \theta, t) = E(x, y, f, \theta(x, y, x_0, y_0), t - D/c_g)$$

where the notation $\theta(x, y, x_0, y_0)$ is used to denote the wave angle at location x,y that passes through point x_0,y_0 on a great circle path. Two points that should be noted here are 1) since group velocity is a function of frequency the time shift in this equation over a large distance from one discretized frequency to the next can be quite significant and 2) the form of this equation is such that the length of line required to cover a fixed range of angles arriving at x_0,y_0 increases approximately linearly with the distance between x,y and x_0,y_0. This last point is important to recognize, since it means that data from a single point can rarely be used effectively as a boundary condition for incoming waves. Instead, unless it is absolutely known that the gradients in the wave field are essentially zero, it is far preferable to use multiple points along a line, particularly for propagation distances that are larger than 2 or 3 degrees. As an example of this, a line along 30^o north latitude would have to be 900 nautical miles in length to cover a single angle band of 15^o at a point located at 60^0 south latitude.

This line source approach is implemented in the numerical model through internal storage of spectral information along pre-defined lines of latitude as the model marches forward in time. Typically, a storage latitude of 30° or 35°N is considered for south-propagating wave energy and 30° or 35°S for north-propagating energy.

Figure 1 shows the great circle paths extending from a line at 30°N directed to a given coastline location at 30°S. The starting locations along the latitude line source are spaced at one-degree intervals and the points along each great circle route are spaced one day apart at the propagation velocity of a swell component with a frequency of 0.05 Hz. It may be noted that wave energy propagating over a large region of the North Pacific affects a relatively narrow directional aperture at the site.

At regular intervals in the overall wave model (typically 3 hours), low-frequency spectral data is stored along the line of latitude. During a given time step at the particular site of interest, the wave model then looks back in time at the stored spectral information, and replaces the computed wave energy densities in the relevant directional and frequency bins with the energy propagated from the line, representing the integration of the wave information along the line. In this manner, a revised estimate of the spectral energy density at the site is derived using low-frequency energy propagated without numeric diffusion.

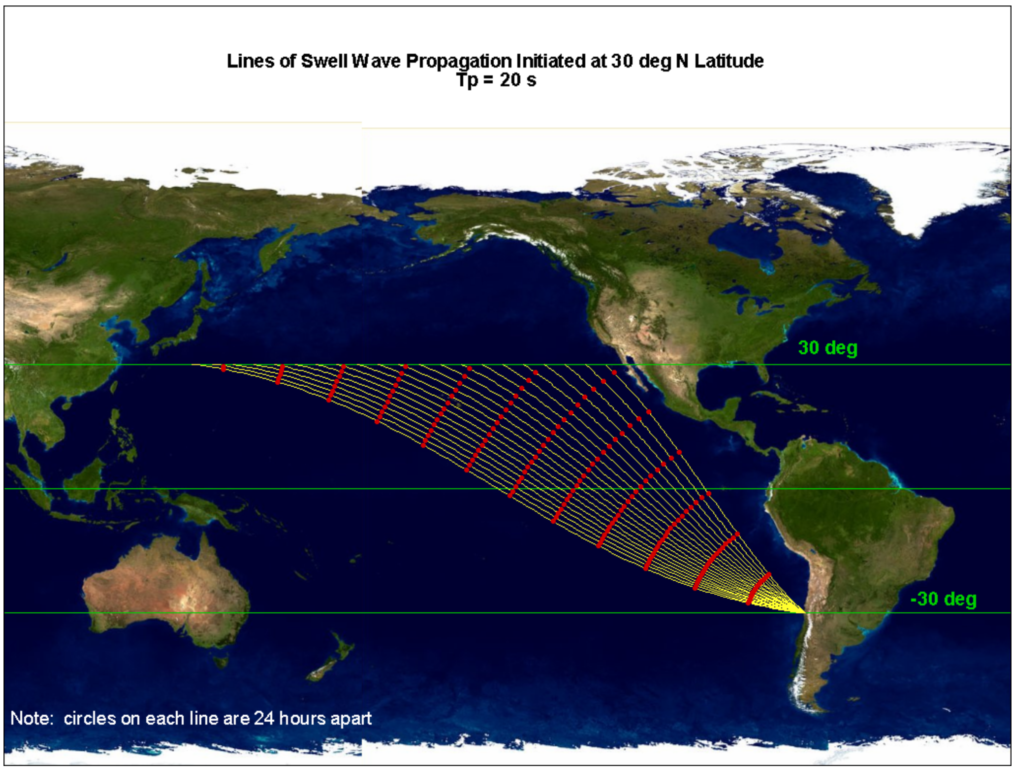

Figure 1. Northern Pacific Swell Propagation to a Location in the South Pacific

This approach, as it does not consider source effects along the lines of propagation to the site of interest, takes advantage of the consideration that swell effects are primarily the result of the passage of storm (eg. extratropical cyclones) events in the mid-latitudes and not tropical meteorological phenomena.

The spectral information along the line can also be stored as a file during the course of a simulation, for use in post-processing the two-dimensional spectrum at other sites. This file storage can be relatively compact as only certain wave directional and frequency bins need be stored for the points along the line.

The effects of islands can be readily incorporated into the method of characteristics approach through simply ignoring certain great circle paths. Similarly, sub-grid blocking can be considered through variation in the wave energy transmission.

2.3 Extension of the Methodology to Higher Order Accuracy

The wave ray approach can be readily extended higher order accuracy through improved interpolation of the wave energy propagating to a given grid point. Various schemes for this interpolation could be utilized, including spline and multi-point polynomial approaches. The adopted method was relatively simple in concept, and focused on reducing diffusion in the direction of wave propagation while retaining diffusion (albeit in an uncontrolled fashion) in lateral directions. In this approach, the great circle arc followed by a given wave packet is followed upwind two grid cells and downwind one grid cell. Linear interpolations are performed at the grid intersection locations, and a cubic polynomial interpolation performed through the various grid intersection points. This is illustrated in Figure 2, where the energy density at location P* in the next time step is estimated from the current energy density at location P^{int}, interpolated from energy density at locations P1, P2, P3 and the current energy density at P*.

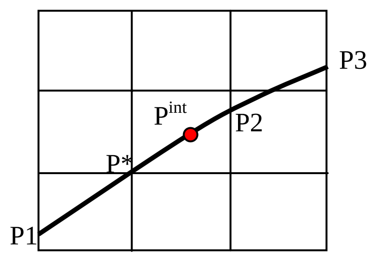

Figure 2. Higher Order Polynomial Interpolation

3. MODEL IMPLEMENTATION

The wave-ray approach was implemented in the WAVAD wave model, as summarized in Resio (1981) and Resio and Perrie (1989). WAVAD is a second generation (2G) spectral wave model that maintains an equilibrium between the wind source and non-linear wave energy flux with an assumed f^{-4} shape for the wave spectrum. The non-linear wave interactions are represented as a momentum flux to the forward face (frequencies less than spectral peak) of the spectrum based on a constant proportion of the energy transferred out of the mid-range frequencies. Wave-wave interactions also transfer energy to the high frequency region of the spectrum where it is assumed that energy is lost due to breaking processes.

WAVAD has been successfully applied on all of the world’s major ocean basins, as well as on numerous lakes and smaller waterbodies. The model formulation is robust and, unlike 3G models, does not require specific tuning or adjustment with different waterbody sizes.

4. SIMPLIFIED TEST CASES

A variety of simplified test cases were simulated in order to assess the way-ray approach and to inter-compare the various methodologies for swell wave prediction. In all cases, the simulations were conducted using an input grid with 1° resolution (latitude and longitude) representative of the bathymetry of the Pacific Ocean. Sub-grid obstacles (i.e. islands) were equally represented in all models.

4.1 Gaussian Wave Field from Northern Hemisphere

An initial test case was developed that involved a burst of wave energy created in the North Pacific Ocean, and wave height comparisons conducted at various locations in the South Pacific Ocean. The wave field consisted of a Gaussian spectrum with peak period of 12 seconds and a very narrow spectral spreading of 0.002 Hz initiated at 165°E, 45°N with a direction of 277° (meteorological convention) in a single one hour time step. The wave field had a Gaussian spread in space of 2 degrees. The wave height comparisons were carried out at various points along the theoretical great circle route followed by the wave energy. Figure 3 shows a snapshot of wave height contours for this theoretical test case. For the purposes of comparison, simulations were carried out for a variety of models and propagation schemes (source terms were not included):

- WAVAD with the first order scheme.
- WAVAD with cubic interpolation.
- WAVAD with the line source swell routine (line located at 35°N).
- Wavewatch III with the QUICKEST scheme in conjunction with Tolman (1999) spatial averaging.
- Wavewatch III with the first-order propagation scheme.

Figure 4 provides a summary of the estimated significant wave height for a location in the South Pacific Ocean (80°W, 30°S). It may be noted that WW3 with the QUICKEST scheme gave the largest significant wave height and sharpest definition of peak wave conditions. Other approaches provided a more diffusive wave field. The semi-Lagrangian approach with cubic interpolation gave an estimate of maximum significant wave height that fell between that of the first order schemes and the QUICKEST approach.

The results clearly varied with proximity of the storage line to the wave energy source, as indicated in Figure 5. As might be anticipated, the closer the line was to the wave energy burst, there was reduced opportunity for numerical diffusion of the wave results in the wave model propagation scheme.

4.2 Moving Storm in Northern Hemisphere

This test simulated the passage of an extratropical cyclone in the North Pacific Ocean, as shown in Figure 6. A cyclonic wind field was created that had an overall radius of influence of 1000 km. The wind velocity at the center of the storm was zero and achieved a peak velocity of 25 m/s at a distance of 50 km from the centre. The wind velocity reduced linearly from this maximum to a velocity of 10 m/s at a distance of 1000 km from the center. The storm was initiated at a location of 160°E, 30°N, had a forward speed of 20 kph and a bearing of 70°E.

Figure 7 gives a comparison of estimated significant wave height for identical location as above (80°W, 30°S). As may be anticipated, the first order scheme provided the smallest wave heights, although the differences between the various schemes was not as prominent as with a single burst of wave energy.

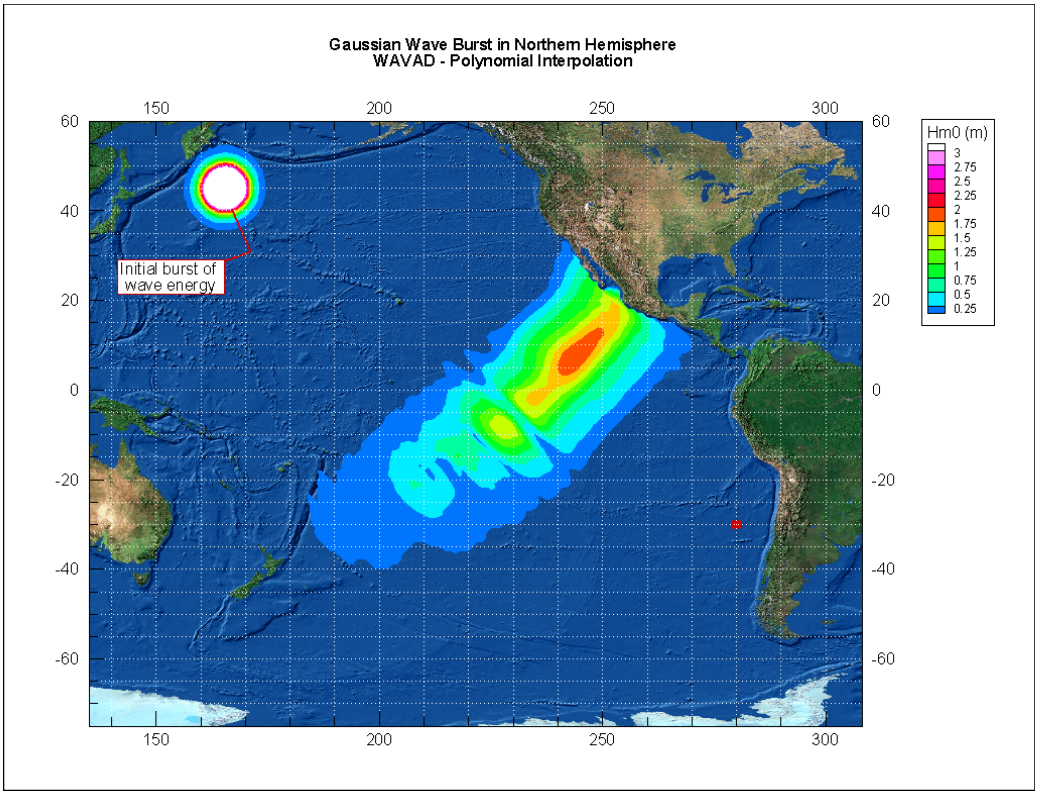

Figure 3. Wave Height Contours for Wavewatch III Results for QUICKEST Propagation Scheme

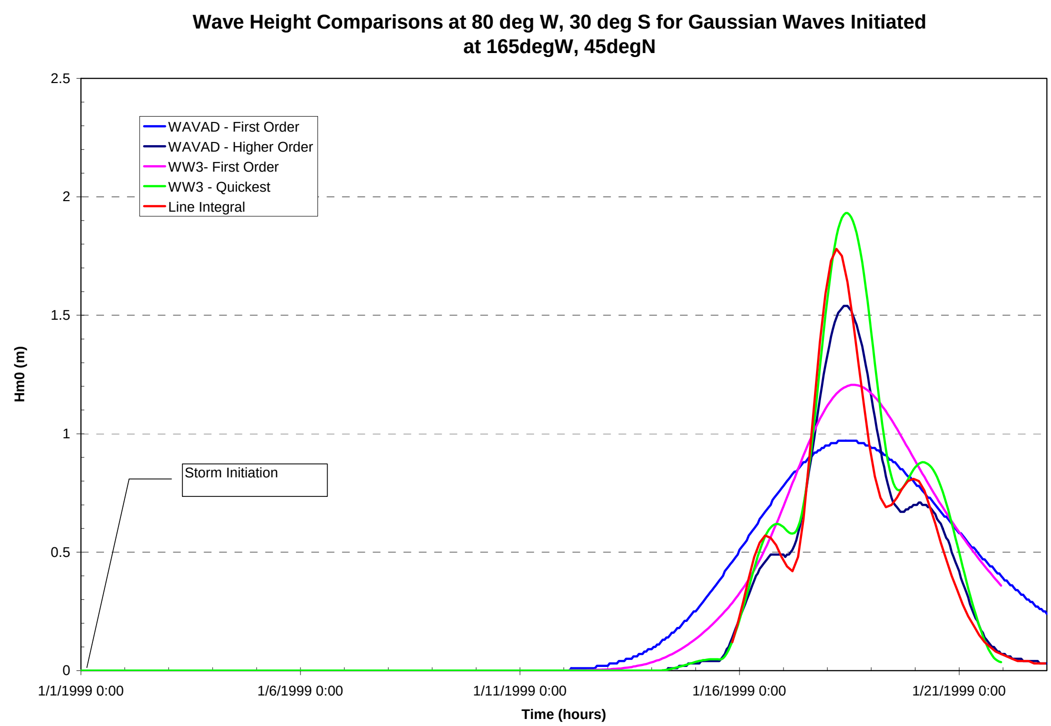

Figure 4. Inter-comparisons of Significant Wave Height at 30°S, 80°W

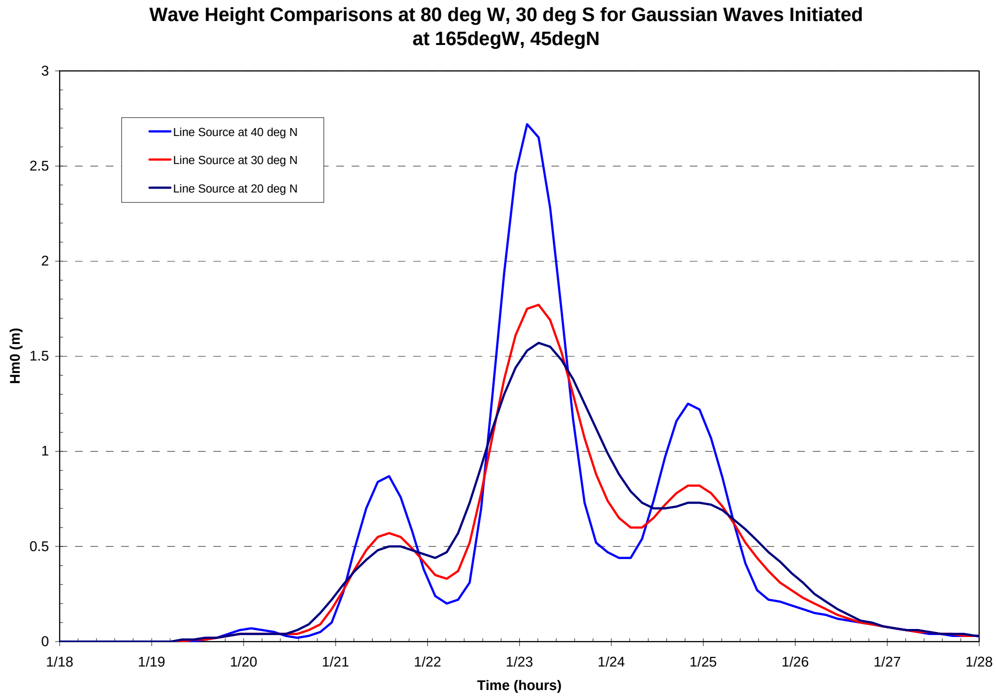

Figure 5. Inter-comparisons of Significant Wave Height at 30°S, 80°W for Varying Line Location

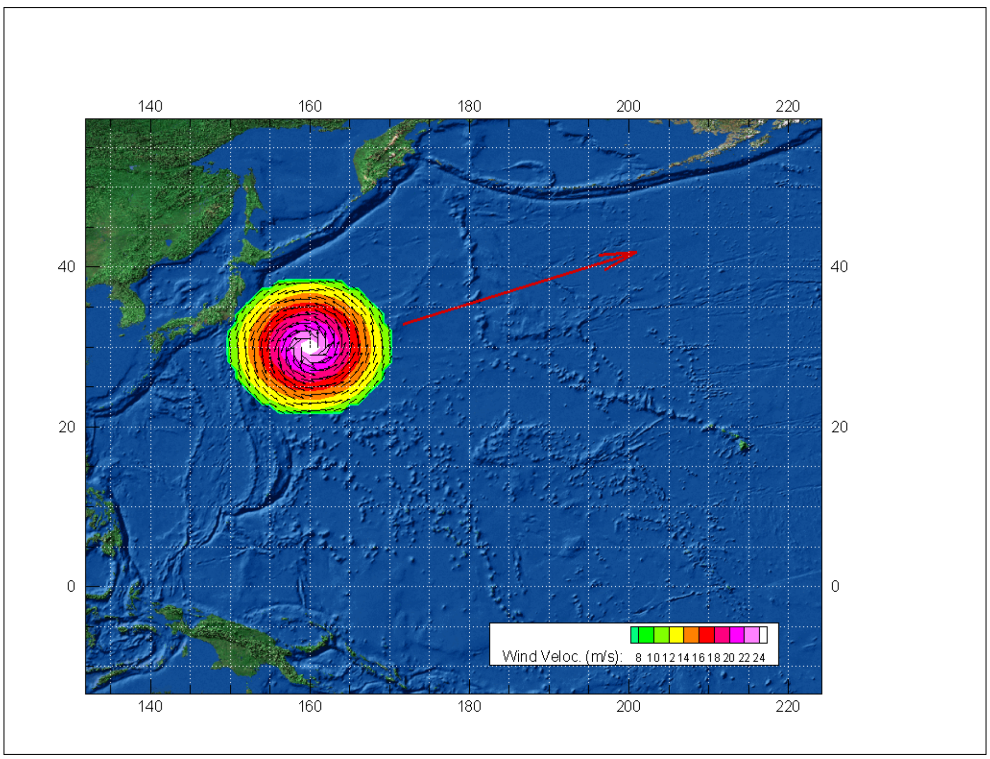

Figure 6. Snapshot of Wind Field in Moving Storm

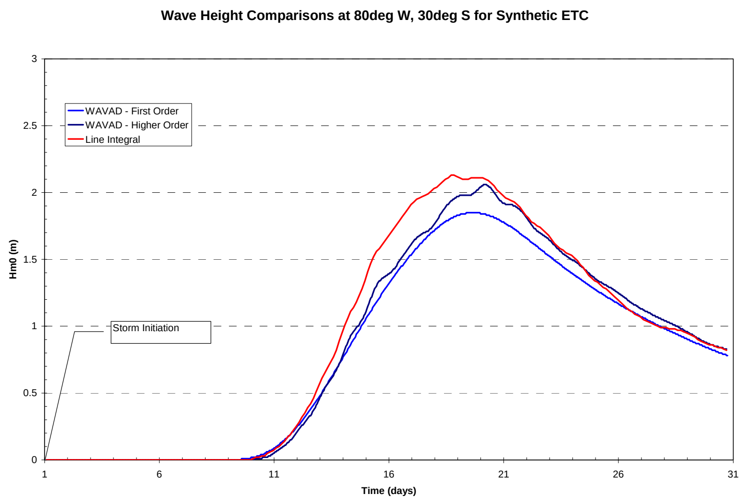

Figure 7. Wave Height Comparisons at 80°W, 30°S

5. COMPARISONS WITH REALISTIC SWELL CONDITIONS IN THE PACIFIC OCEAN

A series of numerical model runs were carried out for the purpose of investigating the propagation of low frequency waves in the Pacific Ocean driven by realistic wind conditions.

A numerical model grid of the Pacific Ocean was created that covered a domain extending from 120° E to 66° W and from 64° S to 56° N, at a resolution of 1.0°. The model bathymetry for much of the Pacific Ocean was derived from a global (ETOPO30) database. A total of twenty-three frequencies were used for the hindcasts in conjunction with a directional resolution of 15° (24 direction bands). The frequencies were spaced at a constant ratio of 1.1 starting at 0.039 Hz.

Wind fields derived from the NCEP/NCAR Reanalysis Project data set (Kalanay et al., 1996) were used as the primary driving mechanism for the wave model. The U (east-west) and V (north-south) wind fields at 10 m elevation above ground were extracted from the NCEP/NCAR global database for a grid network that covered the entire Pacific Ocean. The NCEP data are available on a 6-hourly basis for a grid resolution of approximately 1.875° longitude by an average 1.905° latitude (actual grid is Gaussian), and were interpolated onto the more refined WAVAD input grid using a four-point bi-linear interpolation scheme. This approach provided a driving wind field for the model at six-hour intervals. The wind field in the equatorial region (20°N to 20°S) were adjusted upwards based on both scatterometer (QUIKSCAT) and buoy wind observations. This adjustment was greatest at the equator and decreased gradually north and south.

Comparisons with recorded data were conducted at a number of wave buoys around the Pacific, including adjacent to the Northern American coastline, Hawaii, and Chile. Reasonable agreement was achieved at the various buoys, as typified by the quantile-quantile plots shown in Figures 8 and 9. It was noted, however, that use of the line source approach to propagate swells without diffusion did not lead to an overall improvement in model performance.

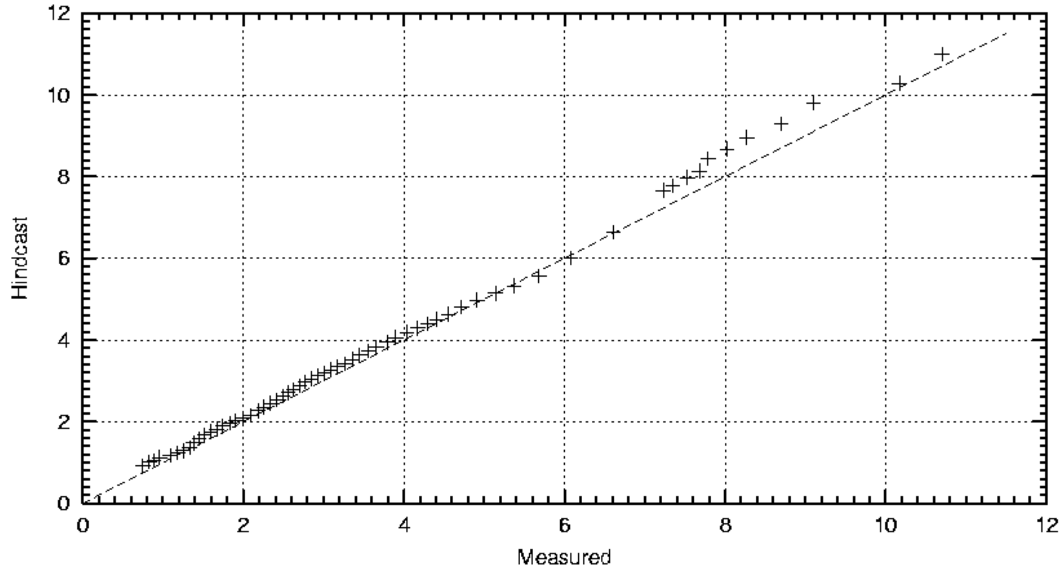

Figure 8. Quantile-Quantile Plot for Hm0 (m) at NDBC Buoy 46006 (40.84° N 137.49° W)

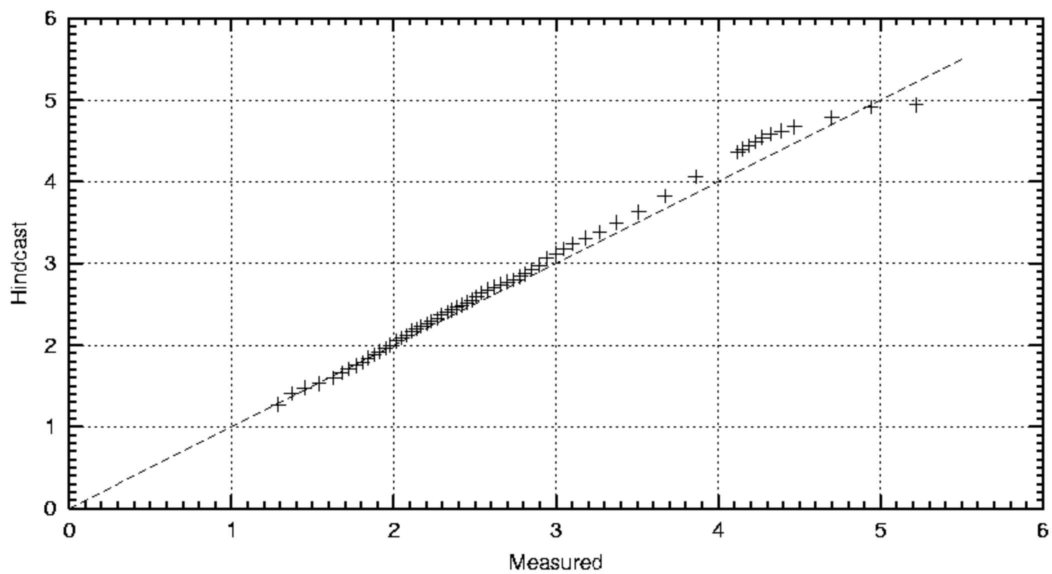

Figure 9. Quantile-Quantile Plot for Hm0 (m) at NDBC Buoy 51002 (17.14° N 157.79° W)

Figure 10 shows a sample time series comparisons to recorded wave data for the period from June 15 to September 15, 2001 at a buoy (NDBC 45059) located offshore of California. In this plot, time series for both the way-ray approach and the line source method are shown. The general finding from the comparisons of this type was that the line source approach did not yield significantly improved swell prediction, especially as compared to other potential sources of error in numerical modeling of wave generation (such as wind field specification and source physics).

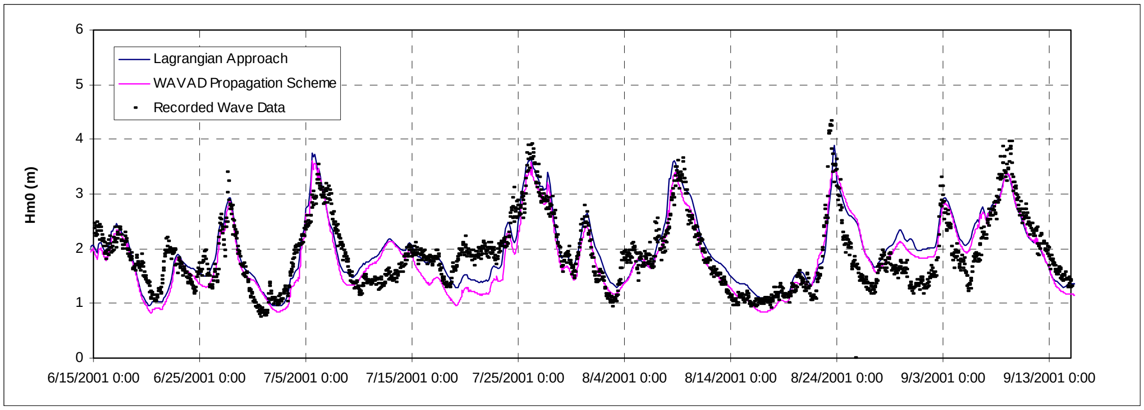

Figure 10. Sample Time Series Comparison [Buoy 45059 37.98°N 130.00°W]

7. IMPLICATIONS FOR SWELL WAVE PREDICTION

One of the interesting elements that can be readily explored with the line source approach is the potential for use of directional wave buoys as a mechanism for swell prediction at a given site. As an example of this, a swell propagation line was defined at 20°N in order to estimate swells at an arbitrary site of interest (30°S) in the South Pacific. The theoretical cyclonic storm defined previously was then used to force the model in a number of simulations in which the number and spacing of the Lagrangian wave rays was varied.

Figure 9 provides a time series comparison for the various numerical model runs, with wave ray spacings varying from one degree to twenty degrees. There was a noticeable degradation in the estimated wave height between wave ray resolutions of five to ten degrees. Figure 10 shows the relative position of the source points considered with 10 degree ray spacing.

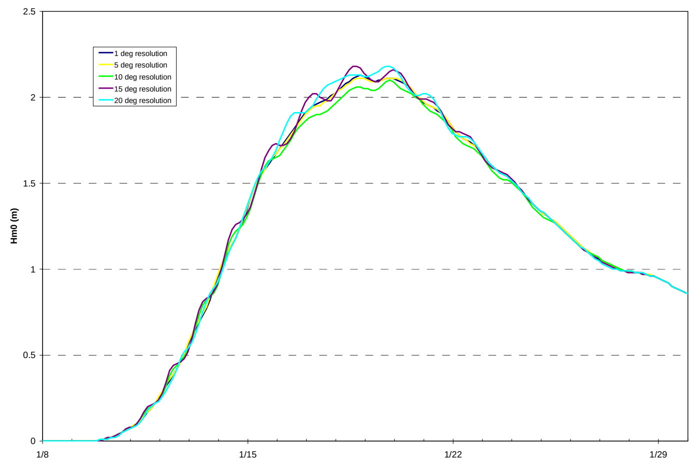

Figure 9. Hindcast Significant Wave Height at 80°W,30°S for Varying Numbers of Wave Rays

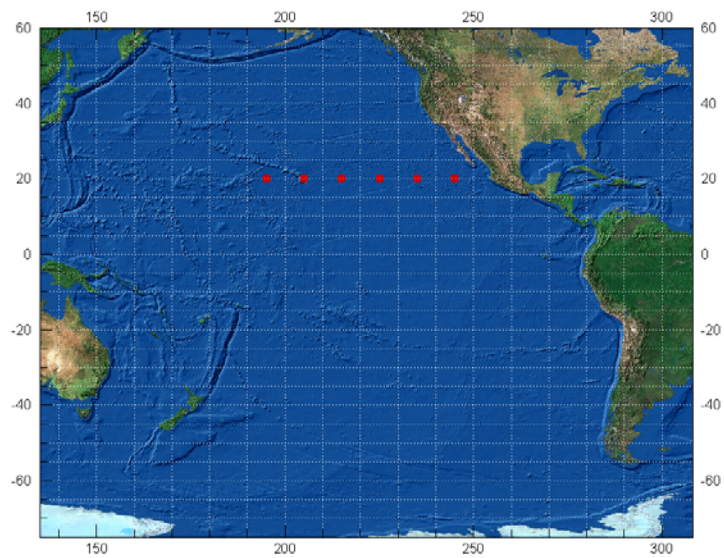

Figure 10. Wave Ray Spacing at 10 Degree Resolution

8. CONCLUSIONS

The theoretical development of a generalized ray-tracing approach to simulating wave propagation has been shown that provides an improved approach to incorporating the effects of wave refraction processes into a numerical wave model. This wave ray approach has an interesting extension into a line source method for estimating swell propagation without numerical diffusion over regions where source terms are negligible.

A series of simulations were carried out to assess the performance of the alternative propagation schemes, ranging from the application of idealistic test conditions to reproduction of realistic wave conditions in the Pacific Ocean. Despite the improvements shown in the idealized tests and reasonable agreement achieved with measured wave conditions, the line source method did not yield significant improvements over more diffusive numerical techniques. Greater uncertainty is evident in both wind forcing as well as associated with the model source physics in swell-dominated environments.

The proposed line source propagation scheme may have application in defining the requirements for a swell prediction system and consideration of data assimilation procedures, and provide a useful tool to assess the impact of long-distance swell decay in numerical wave models.

9. REFERENCES

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