Numerical Aspects of Wave Modeling

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Scope of lecture

- Focussing on relatively large scale models :
  - Full spectral
  - Random Phase
- So this will not include :
  - Phase coupled models as used on the beach.
  - Diffraction issues.
- Much focussed on recent NCEP work.

Large Scale Models 1

- Several types of large scale models can be distinguished. Main categories are :
  - Ray models
  - Grid models
- Furthermore, three types of grid models can be distinguished :
  - Finite difference approaches.
  - Semi-Lagrangian grid models.
  - Finite element approaches.
- The focus here will be mostly on finite difference grid models.

Large Scale Models [1]

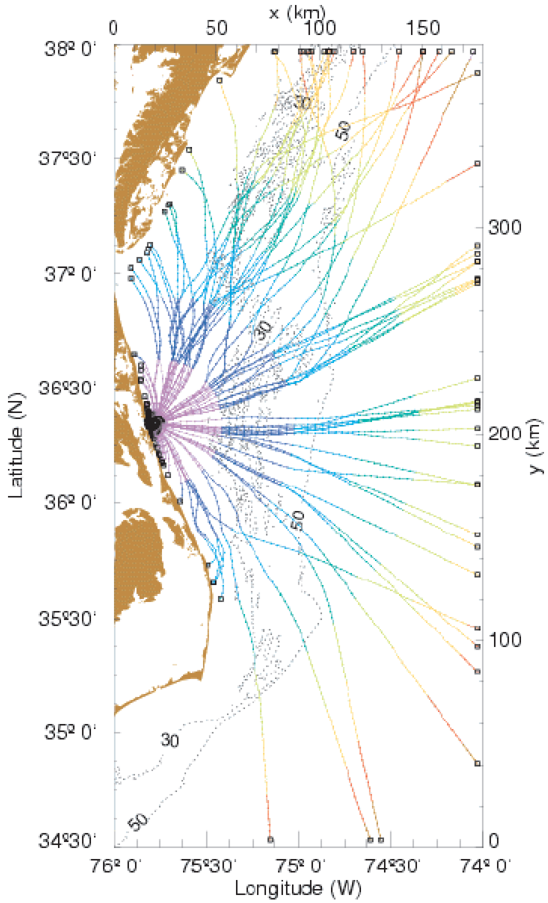
- …veral types of large scale models c… …uished. Main categories ar…
  - …dels
  - Gr…
- Furthermore, … grid models can be distinguished…
  - Finit… …e ap…
  - …grangian grid m…
  - …e element approaches.
- …he focus here will be mostly on finite difference grid models.

AND ANYTHING IN BETWEEN

" A MATTER OF TASTE "

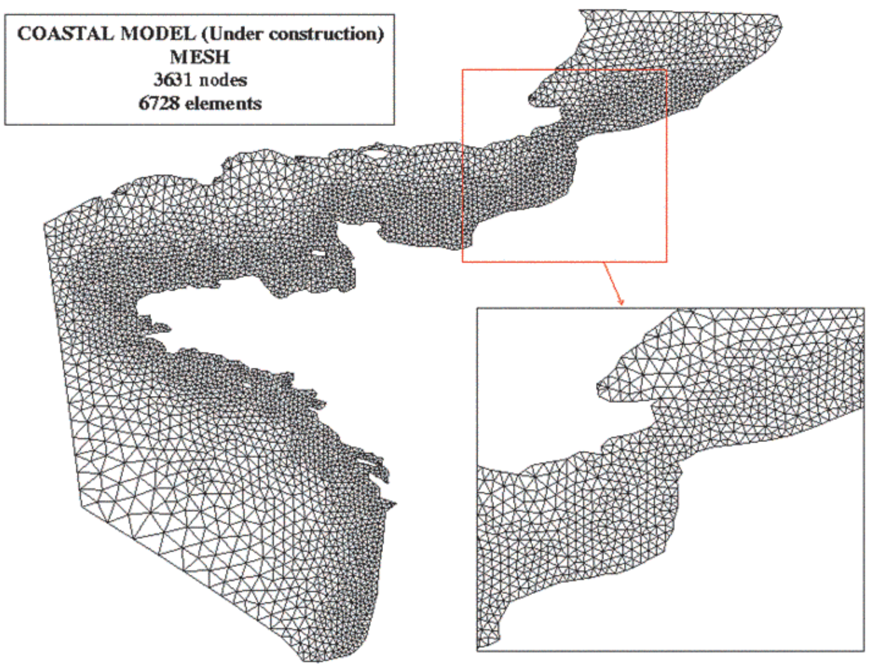
Large Scale Models 2

- Ray models, either for selected output points using backtracking along characteristics, or for (sets of) mono-chromatic boundary conditions. Source term inclusion not straightforward. Figure courtesy Fabrice Ardhuin.



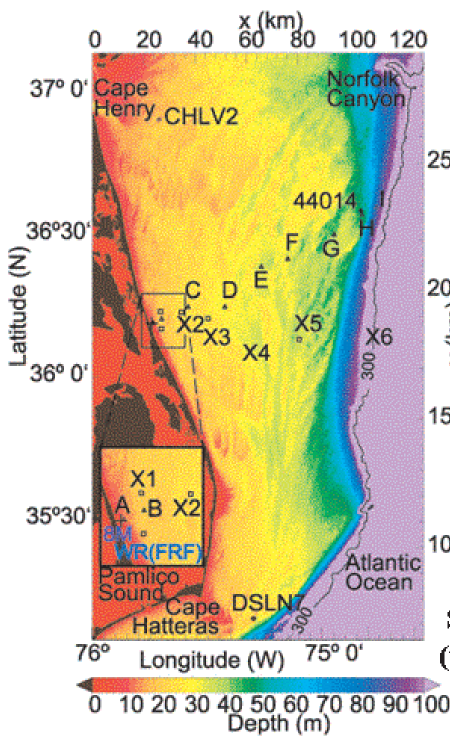
Large Scale Models 3

- Finite element grid models, example from TOMAWAC model (courtesy Fabrice Ardhuin and Michel Benoit).



Large Scale Models

- Semi Lagrangian models (Lavrenov, CREST), Example from CREST, courtesy of Fabrice Ardhuin.

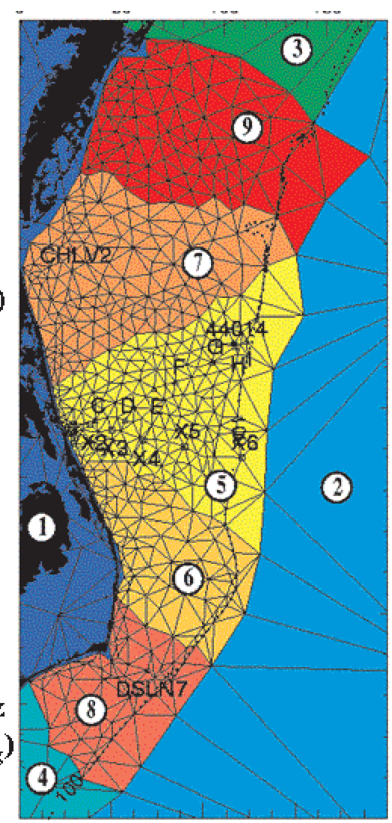


D1. validation

Bathymetry and CREST (JPO, May 2001) model grid

570 grid points
19 frequencies
(0.05 – 0.15 Hz)
72 directions
(5 deg. resolution)
Δt = 10 minutes

→ 8× real time on SGI IP27 300 MHz (with S_{fric} and S_{Bragg}) using 850 Mb



Large Scale Models 5

- Spectral wave models are based on a version of the spectral energy/action balance equation

$$\frac{\partial F(f,\theta;\mathbf{x},t)}{\partial t} + \nabla_{\mathbf{x},f,\theta}\, c\, F(f,\theta;\mathbf{x},t) = S(f,\theta;\mathbf{x},t)$$

propagation

source / sink

Propagation inherently linear, includes shallow water effects and mean currents (refraction, straining), but not diffraction.

Direct parameterization of all source terms in the above equation. Nonlinear interactions part of source terms.

No assumptions on spectral shape.

third generation models

Large Scale Models[6]

- For some applications, a quasi-steady model approach can be used, solving

$$\cancel{\frac{\partial F(f,\theta;\mathbf{x},t)}{\partial t}} + \nabla_{\mathbf{x},f,\theta}\, c\, F(f,\theta;\mathbf{x},t) = S(f,\theta;\mathbf{x},t)$$

Resulting equation is elliptic, requires matrix inversion to solve.

Cheap solution: solve only in one direction, which needs to be the dominant wave direction (HISWA, ...).

Full solution can be done economically using iterations (SWAN).

intermediate scales

Large Scale Models [7]

- Most spectral grid models solve the entire equation :

$$\frac{\partial F(f,\theta;\mathbf{x},t)}{\partial t} + \nabla_{\mathbf{x},f,\theta}\, c\, F(f,\theta;\mathbf{x},t) = S(f,\theta;\mathbf{x},t)$$

Resulting equation is hyperbolic, can be solved forward in time by discretizing the equation (or using characteristics).

Fractional step methods separate source term and propagation for economical reasons.

Implicit numerical schemes generally not feasible due to number of degrees of freedom (unless "single sweep" alg.).

Large Scale Models

- The fractional step method treats propagation and source terms consecutively :

$$\frac{\partial F(f,\theta;\mathbf{x},t)}{\partial t} + \nabla_{\mathbf{x},f,\theta}\, c\, F(f,\theta;\mathbf{x},t) = 0$$

$$\frac{\partial F(f,\theta;\mathbf{x},t)}{\partial t} = S(f,\theta;\mathbf{x},t)$$

Additional fractional steps are possible and may increase economy of model (WAVEWATCH).

Fractional steps make hybrid models possible (CREST: full ray for propagation, full grid for sources).

Model Resolution[1]

- Discretizing spectral and physical space results in a large number of degrees of freedom in a wave model. This almost always leads to a conflict between accuracy and economy.
- Increasing the spatial resolution has a dramatic impact on economy. Reducing the grid increment by a factor 2, increases computational costs by a factor of 8!
- If more computational resources become available, do you increase spectral or spatial resolution ?

Model Resolution ^{2}

- Recent trend: increase spatial resolution, leave spectral resolution as is. Is this wise ?
- Increase spectral resolution :
  - Calculation of source terms (Snl), may require upgrade of source terms.
  - General representation of spectrum.
  - Swell dispersion and GSE.
- Increased spatial resolution :
  - Utilize details in wind fields.
  - Bathymetry, coastlines, islands.

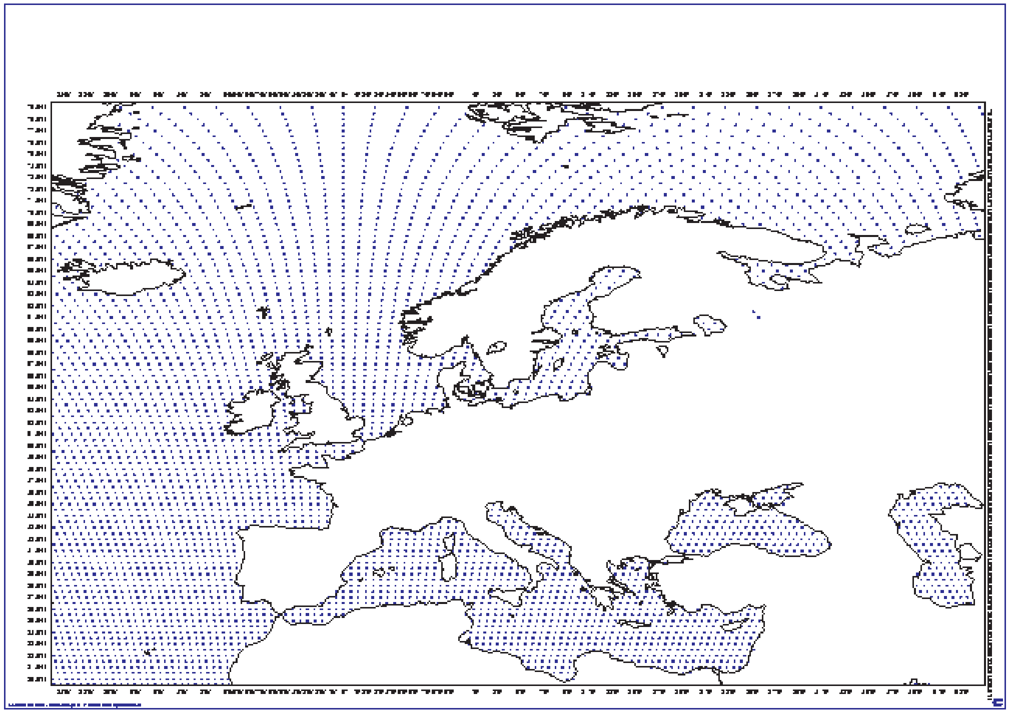
Model Resolution 3

- At NCEP, we are leaning towards keeping spatial resolution of global model the same, but increase spectral resolution.

- Alternatives to increasing spatial resolution :
  - Drop regular longitude-lattitude grids.
    - Bidlot et al (1997) (next slide).
    - Finite elements (TOMAWAC) (previous slide).
    - Conformal mapping (Van Vledder).
    - Earth as cube (Purser and Rancic 1998).
  - Sub-grid treatment of islands.
  - Regional models.

Model Resolution

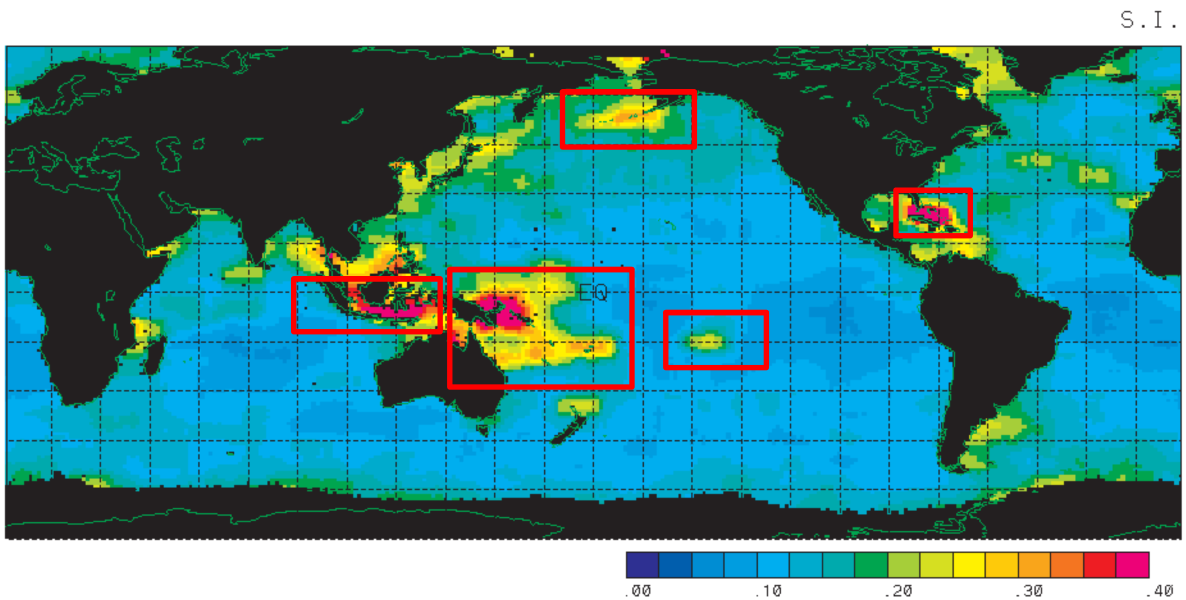
- Thinned grid as used at ECMWF shown below. Finite element and irregular grids shown previously.



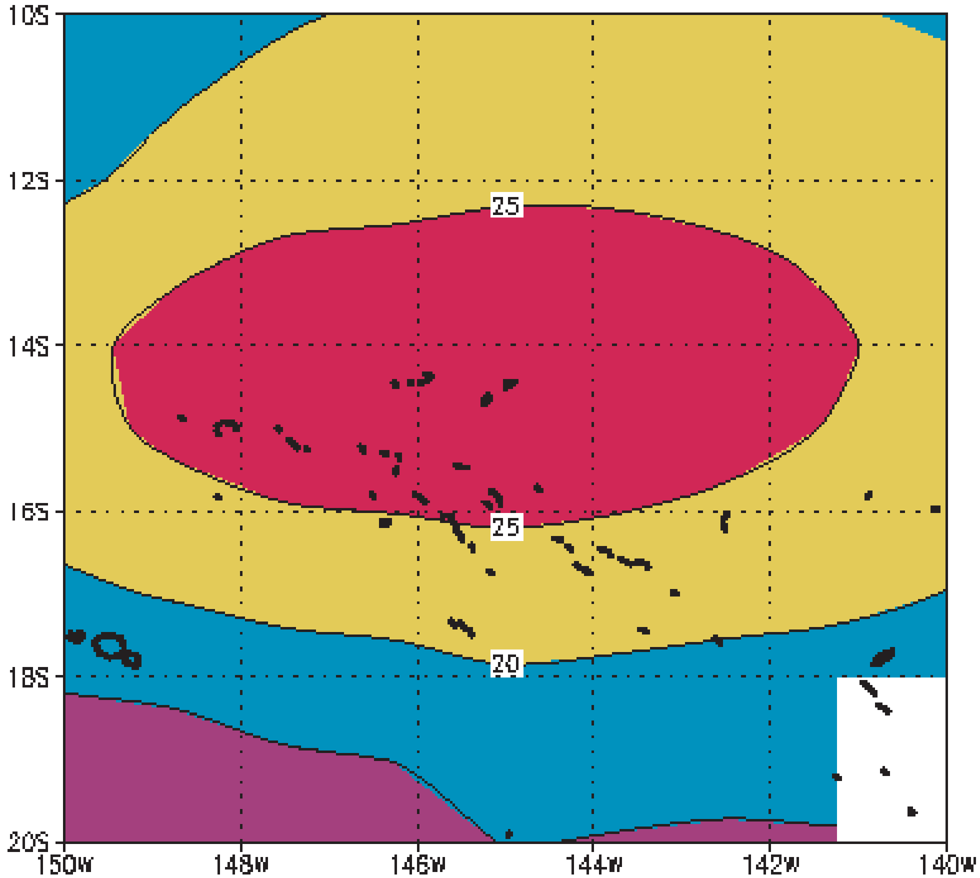
Bidlot and Holt, Coastal Engineering 1999

Model Resolution 5

- Scatter index for Hs (rms error / avg obs) against ERS-2 altimeter data clearly shows where unresolved islands result in highly increased model errors. Some clear examples in boxes below. (Sep.-Nov. 2000)



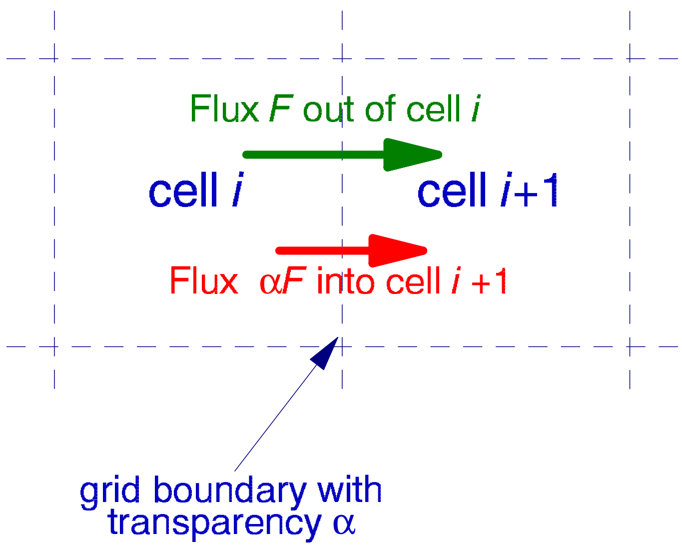
Model Resolution



French Polynesia on the left is a prime example of how extremely high model resolutions will need to be to get rid of model errors due to unresolved islands. This is obviously not desirable in a large scale wave model.

Model Resolution

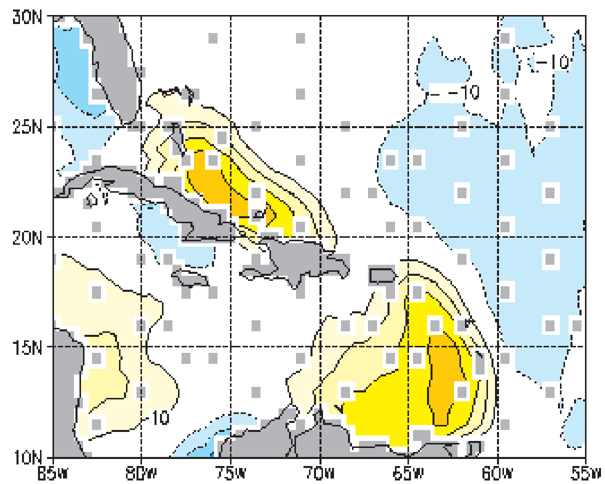
- A more gratifying way to deal with this is to threat such islands as sub-grid obstacles, as is presently done in SWAN and WAVEWATCH III.



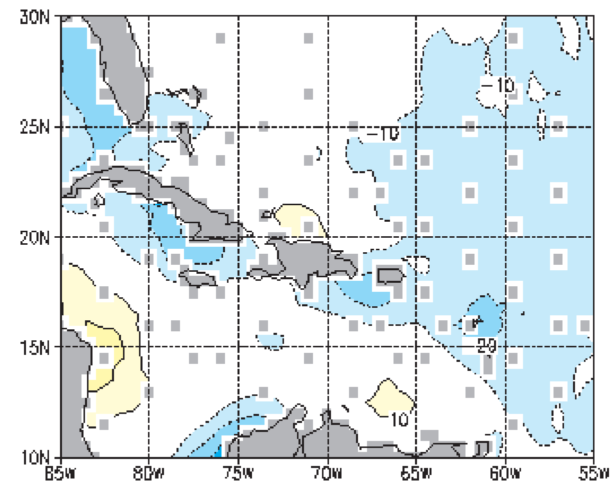
- This can be implemented in a simple way by defining transparencies of cell boundaries for ingoing fluxes.
- Similar to treatment of Great Barrier Reef in Hardy et al. (2000, Ocean Engineering).

Model Resolution

bias without sub-grid islands



bias with sub-grid islands



Even for a fairly high resolution regional model (NCEP's 25 km WNA model), modeling unresolved islands has a massive local impact.

Source Term Integration 1

- In the source term step of a wave model, the following equation is solved :

$$\frac{\partial F(f,\theta;\mathbf{x},t)}{\partial t} = S(f,\theta;\mathbf{x},t)$$

- The solution of this equation in third-generation wave models is complicated, because time scales of the source term balance are very small, particularly at higher frequencies.

Source Term Integration 2

- WAM introduced a fairly robust semi-implicit time integration scheme for source terms (first order Taylor expansion for full implicit scheme).

  Justification: reach equilibrium at high frequencies with physically unrealistically large numerical time steps.

  Original scheme central in time, more recent scheme forward in time.

- This approach is not sufficient to guarantee numerically stable solutions.

  So-called "limiter" added to WAM 1-3.

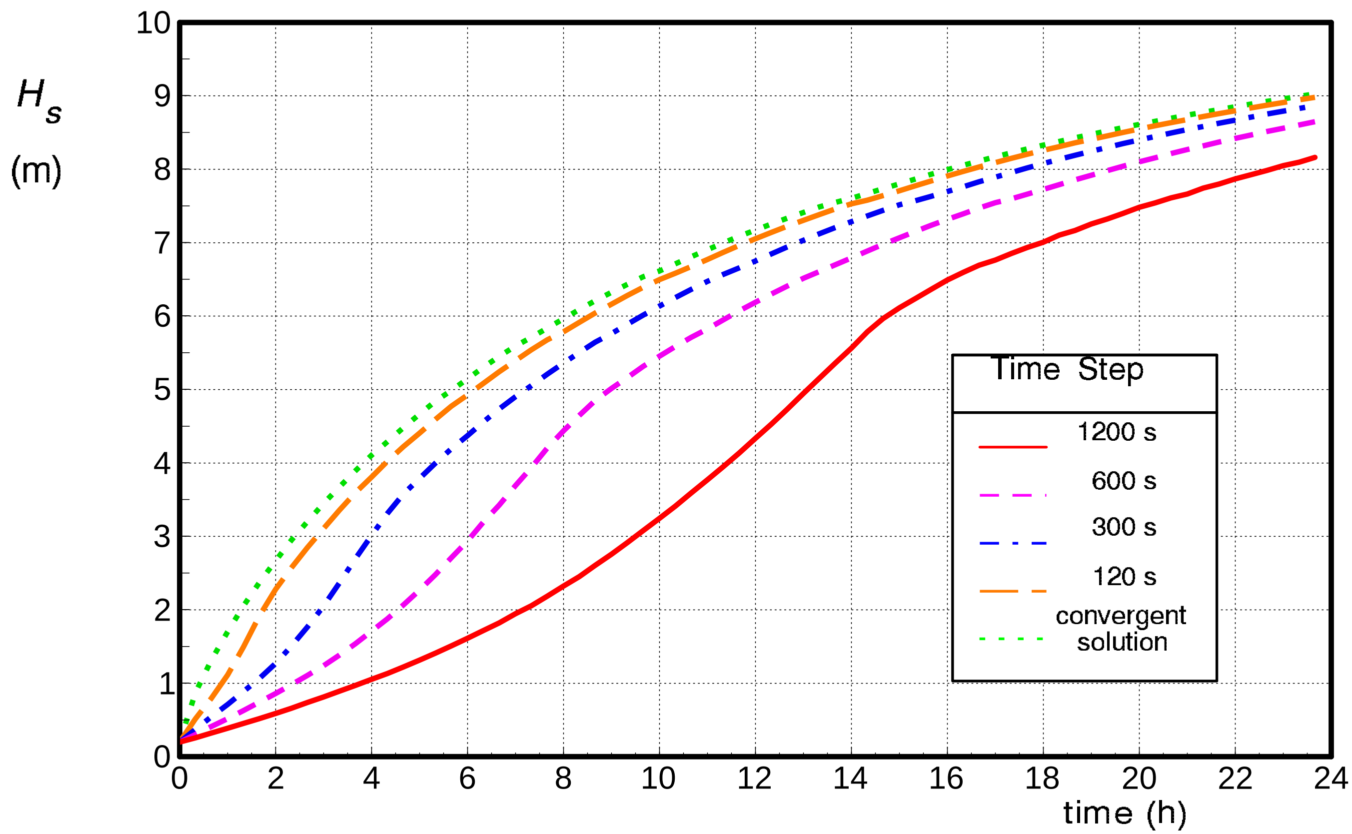
Source Term Integration 3

- The limiter in WAM 1-3 limits the change of the spectrum per time step to about 10% of the Pierson Moskowitz high-frequency spectral shape.

  Was shown in Tolman (1992) to seriously suppress initial growth in WAM-3 environment.

  Makes model results sensitive to actual time step.

Source Term Integration 4



WAM-3 physics and limiter for 20 m/s wind speed.

Source Term Integration

- WAVEWATCH solution (1992): use limiter to calculate maximum allowed time step (local and instantaneous).

  Reproduces convergent solution.

  In large scale model much cheaper, as in majority of grid points this allows for much larger integration time steps (WAM: 20 min fixed time step, WAVEWATCH NCEP global model 40 min average time step).

  Cannot be used in WAM as WAM structure requires the physics time step to be identical for all grid points.

Source Term Integration

- Developments in WAM :

  In WAM cycle 4 (early 1990's) a time-step dependence was added to the limiter. This removed most of the time step dependence in the model.

  Hersbach and Janssen (1999) reformulated the limiter to improve short-fetch model behavior.

  - Reformulated for proper scaling.
  - Non-convergent, part of solution.
  - Excellent model results.

$$|\Delta F|_{max} = 3.0 \times 10^{-7} g\, \tilde{u}_* \, f^{-4} f_c \, \Delta t$$

Source Term Integration

- Possible convergent limiter :

  Assume that there is a scale separation between signatures of physics and of numerical instability.

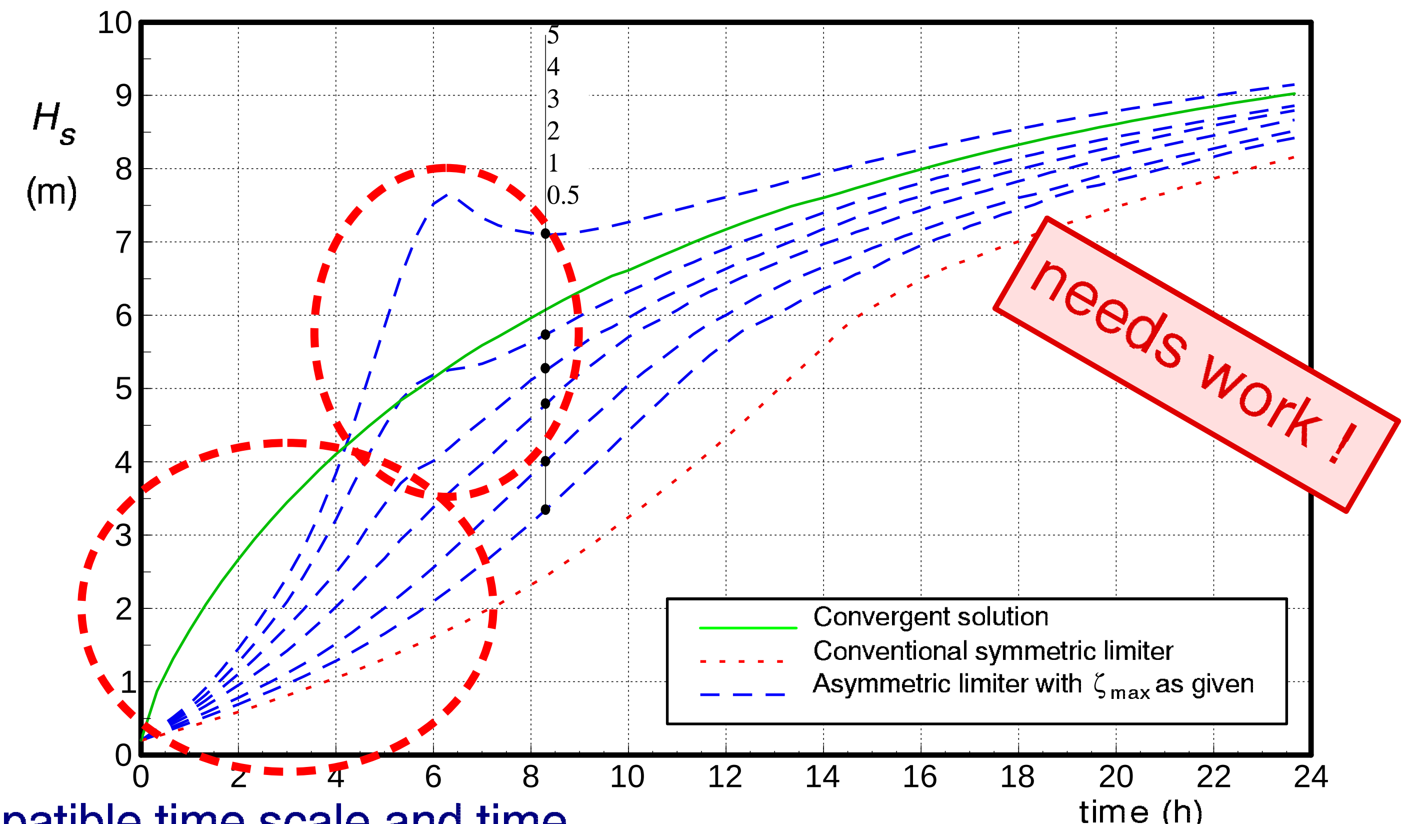
  Relax limiter by applying it asymmetrically around the estimated physical behavior.

  - Simple test with WAM cycle 3 physics and numerics shows feasibility.
  - Will need much more work before operationally applicable.

$$(\zeta - 1) L_o < \Delta F < (1 + \zeta) L_o$$

$$\zeta = \overline{\Delta F / L_o}$$

Source Term Integration [8]



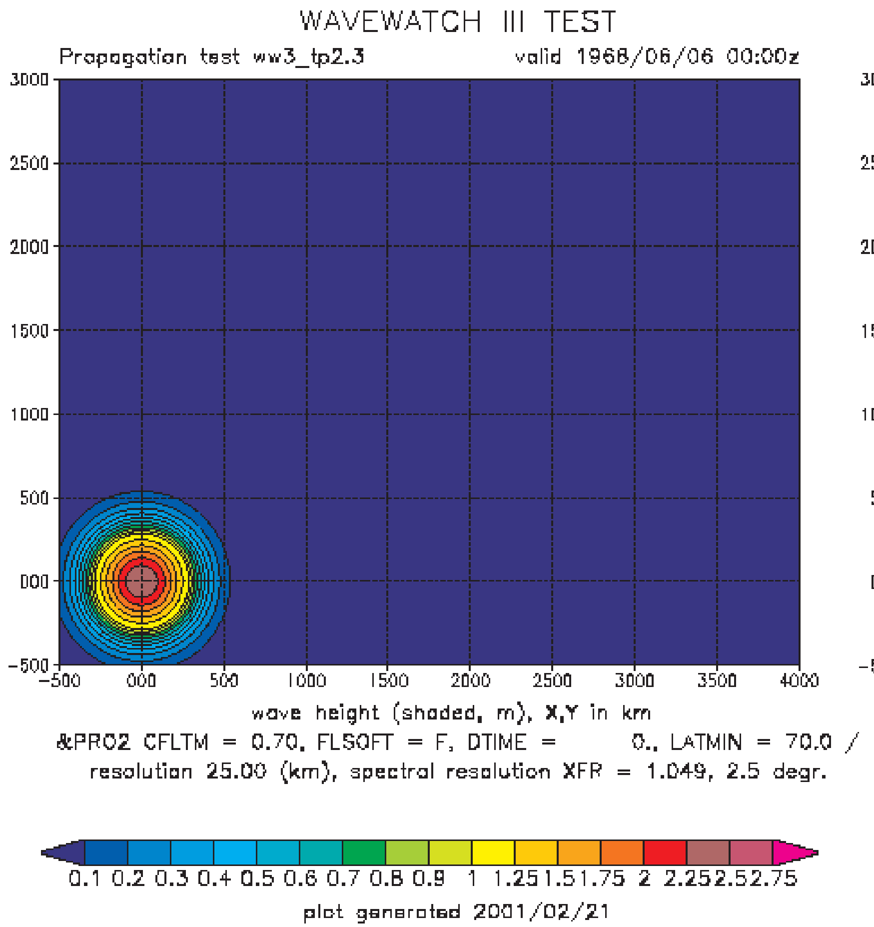
Incompatible time scale and time step make it unavoidable that problems remain in initial growth, and further down the line

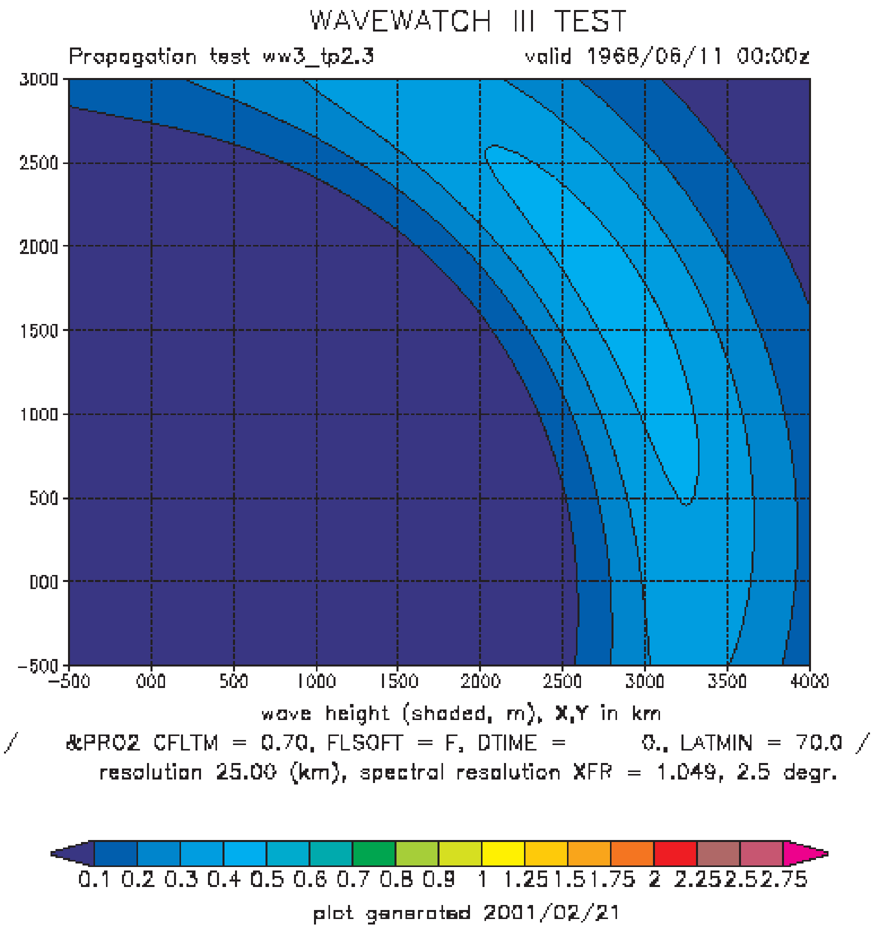
Tolman, GAOS 2002

Wave Propagation 1

- Although refraction can be locally important, and provide problems due to coarse model resolutions, long-distance swell propagation provides the major numerical propagation problem in a large scale wave model.
- First order scheme as in WAM: diffusive, anisotropic.
- Straightforward higher order schemes: less diffusive, more isotropic, but garden sprinkler effect makes them useless for practical applications.
  - Booij and Holthuijsen (1987): diffusion tensor
  - Tolman (2002) averaging or divergence.

Wave Propagation 2

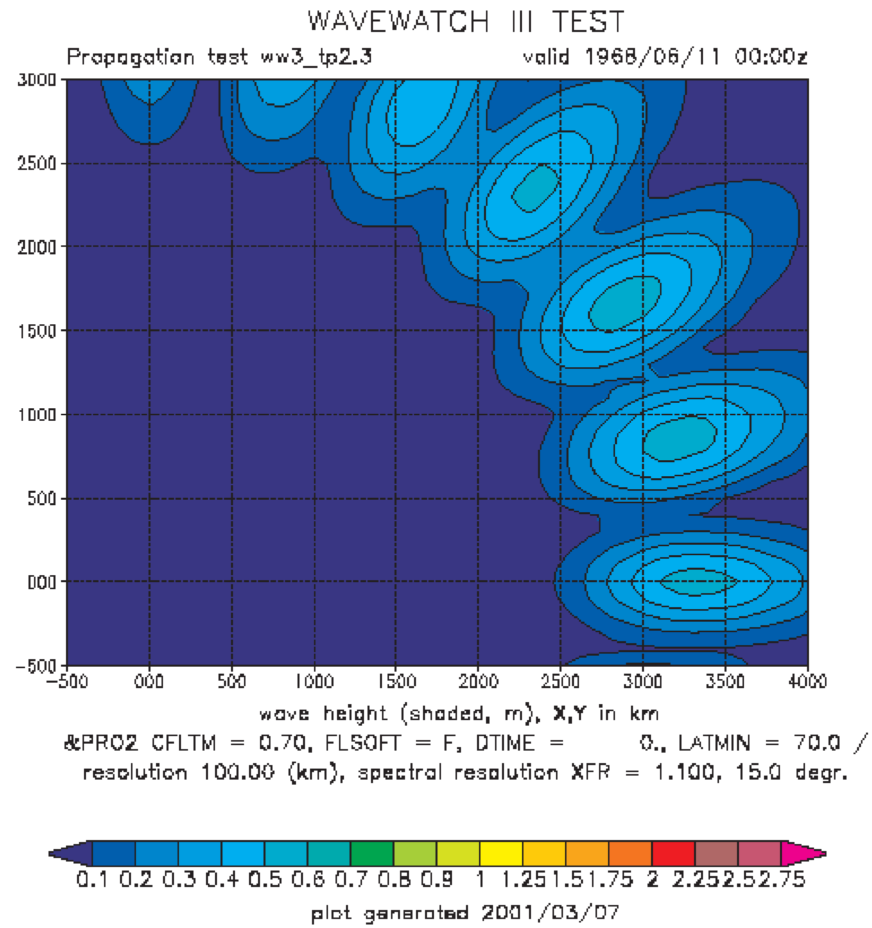




Exact solution: continuous dispersion of swell energy over a large area.

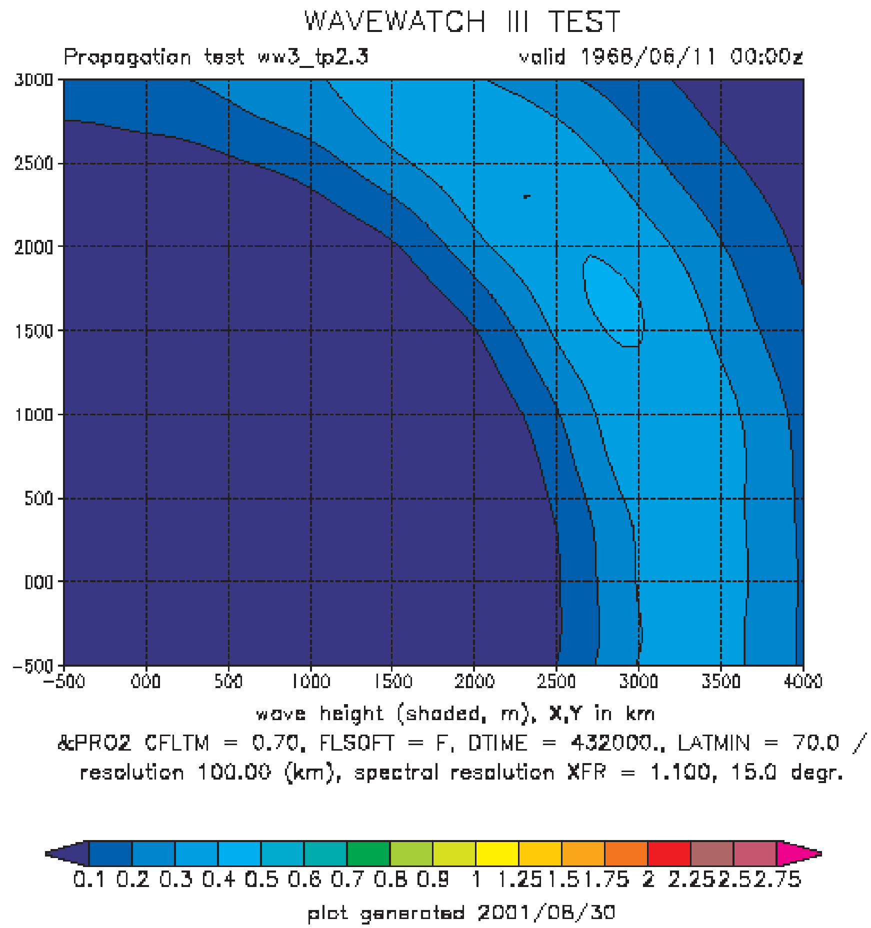
Wave Propagation 3

- Third order accurate Ultimate-Quickest scheme (Leonard) of WAVEWATCH III.
- Obvious garden sprinkler effect, spectral discretization results in disintegration of swell field.
- Better scheme gives worse results. Essentially useless in this form.



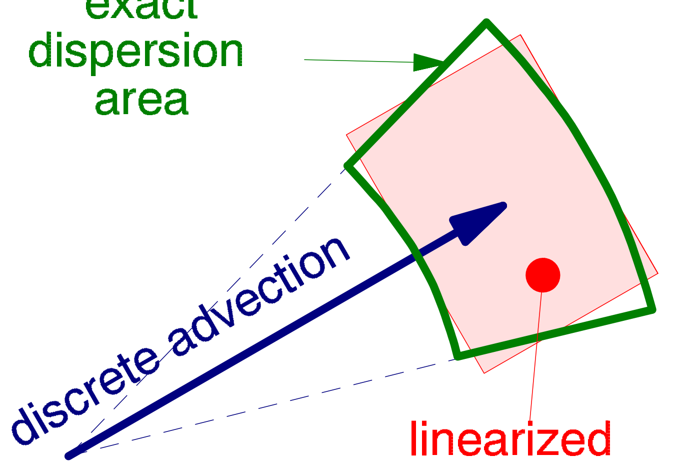
Wave Propagation

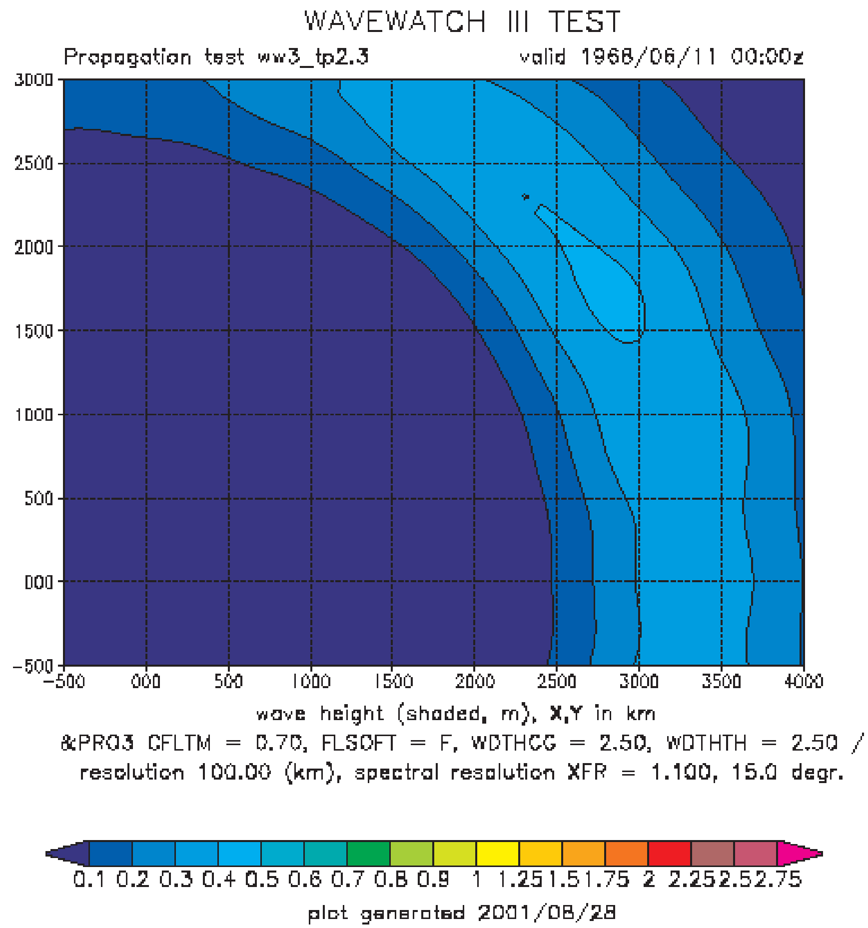
- UQ scheme with Booij and Holthuijsen (1987) diffusive dispersion correction.
- Major improvement over plain UQ scheme, tunable.
- Due to explicit schemes, stability becomes a major issue at small grid steps (order 25 km).



Wave Propagation 4

- UQ scheme with simple pre- or post-averaging of fields.
- Virtually identical results as previous, tunable, cheap.

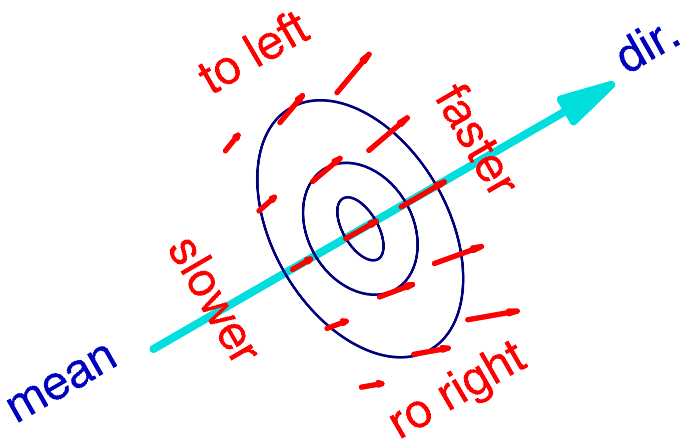


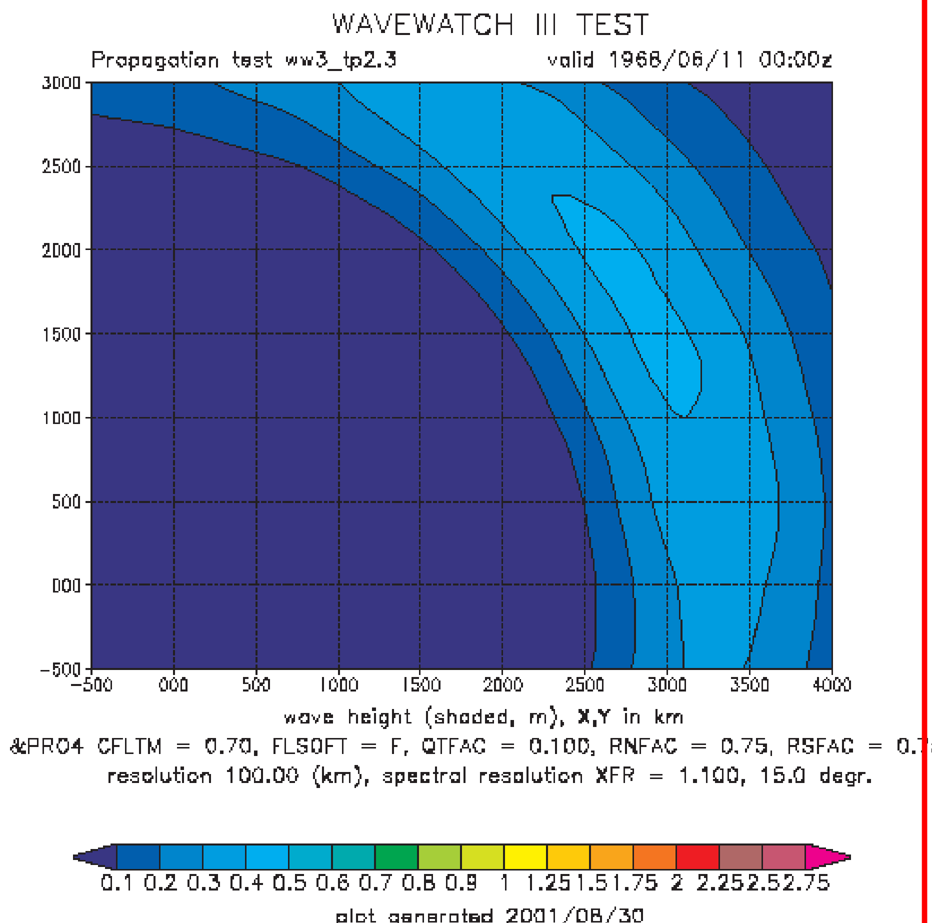


WAVEWATCH III v. 2.22.

Wave Propagation

- UQ scheme with divergent advection velocities.
- Real 2-D correction, no stability issue, application to real world still a challenge.

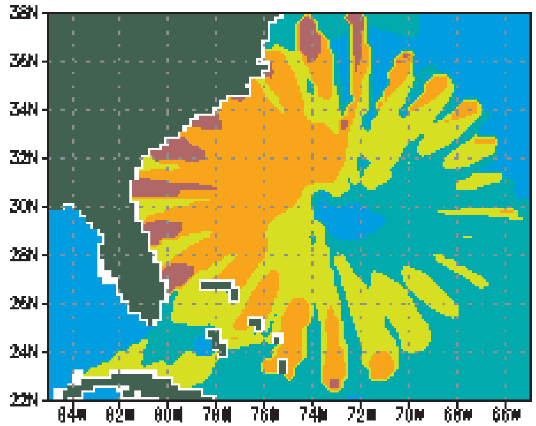


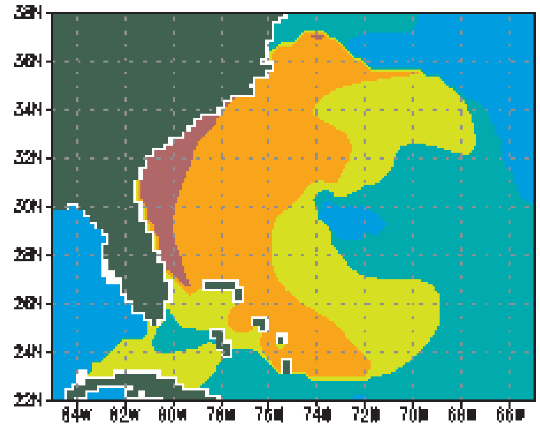


experimental method

Wave Propagation

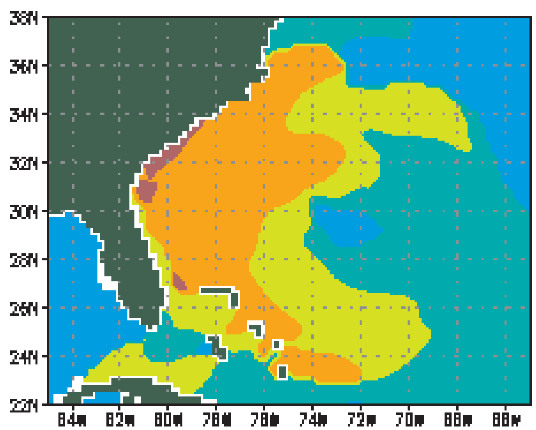
UQ plain
1.00





UQ + dif.
1.75

UQ + avg.
1.09



NAH model speed up is 37%. for all other models it is 4%

Peak periods from 7 to 10s from hurricane Florence at 00z Sept. 13 2000 from NCEP's NAH model. Relative computational costs in red.

Advanced Computing 1

- On modern computer hardware maximum computational model performance is achieved using two paradigms :

  Vectorization.

  Parallelization.

- Due to fractional step methods used in wave models, in combination with multi-dimensional problem, a code can be designed that effectively vectorizes and parallelizes at the same time.
- With a properly designed model architecture, particularly time resolving finite difference spectral grid models are therefore highly suitable for arbitrary computer architectures.

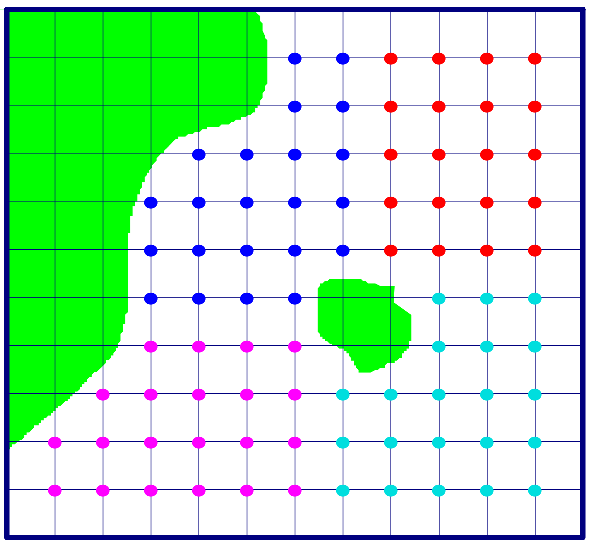
Advanced Computing ^{2}

- Vectorization takes place in the innermost loops in the model source code. Optimization is generally performed on a subroutine by subroutine basis by properly ordering loops.
- Parallelization takes place in the outermost loops of the wave model, and is impacts the overall layout of the model.
- On distributed memory computers ("MPPs"), the distribution of the wave model over processors is a fundamental model design issue.
- Traditionally, physical space is distributed over processors, without splitting up individual spectra.

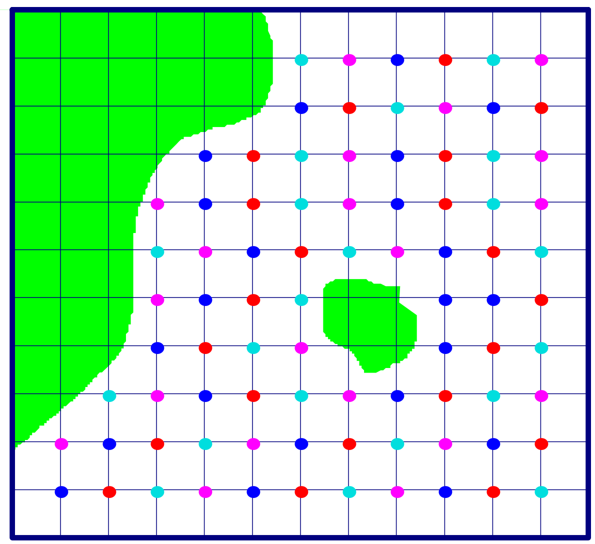
Advanced Computing 3

- Traditionally data and work is distributed over processors using a blocking (domain decomp.) or scattering technique.

"blocking"



"Scattering"



Advanced Computing 4

- Blocking :

  Only data at block bound. needed.

  Total amount of data comm. is a function of # of processes.

  Algorithm de-pends on actual prop. scheme.

- Scattering :

  Full data transpose needed.

  Total amount of data comm. nearly constant.

  Algorithm inde-pendent of prop. scheme.

  Load balancing easier.

  WAVEWATCH

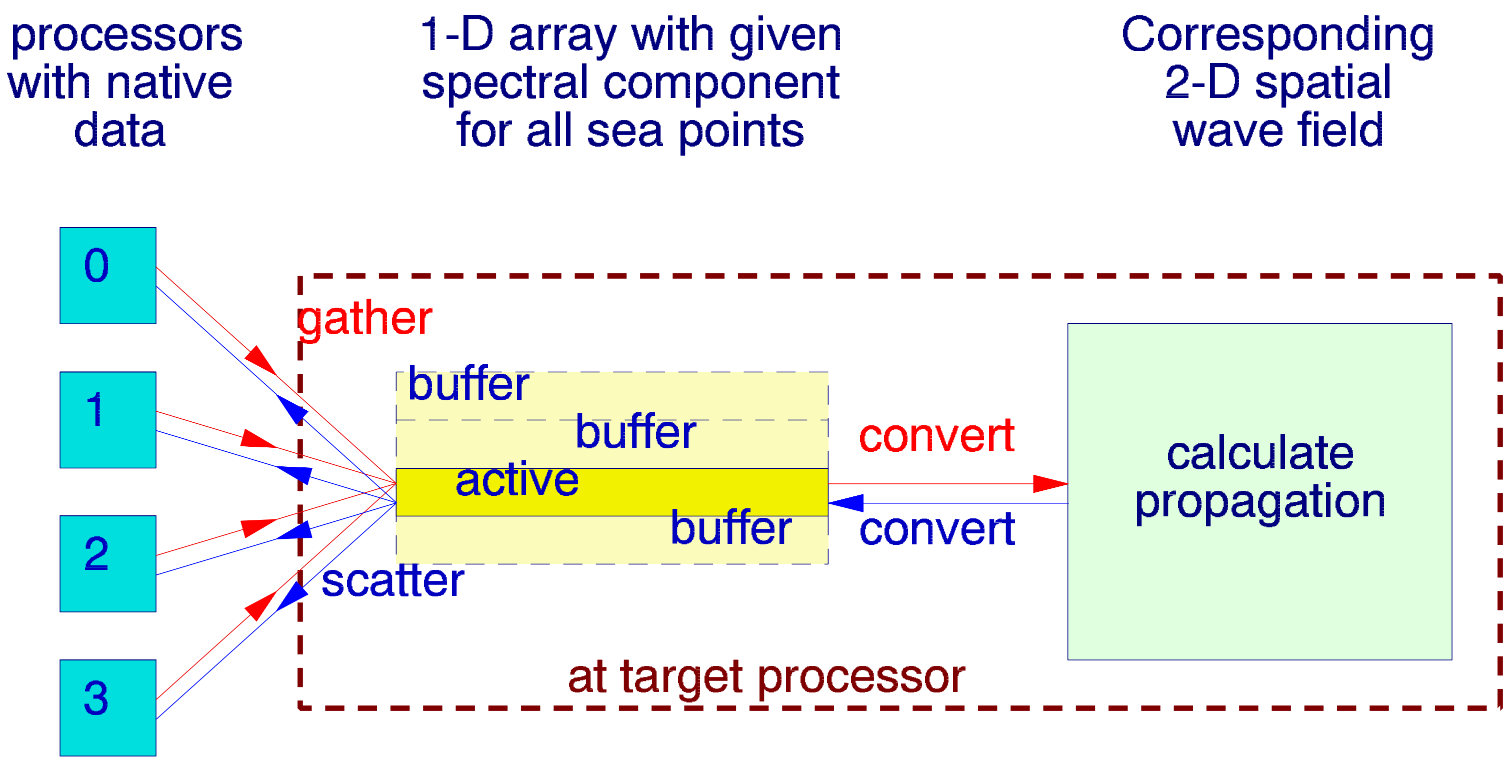
Advanced Computing

- Blocking represents the conventional approach, and is used in some adaptations of WAM. Favors small number of processors.
- Scattering is used in WAVEWATCH because :

  Maximum flexibility and transparency of code (future physics and numerics developments).

  Feasibility based on estimates of amount of communication needed.
- Scattering requires fast communication, and favors large number of processors.
- Scattering benefits from buffering techniques (next slide).

Advanced Computing 6



requires non-blocking communication

WAVEWATCH III

Conclusions

- There is no "one size fits all" numerical approach to large scale wave modeling.
- The law of conservation of trouble is alive and well :

  better propagation scheme = worse results.

  higher spatial resolution = worse GSE.
- There are still many careers to be made in this field.

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