

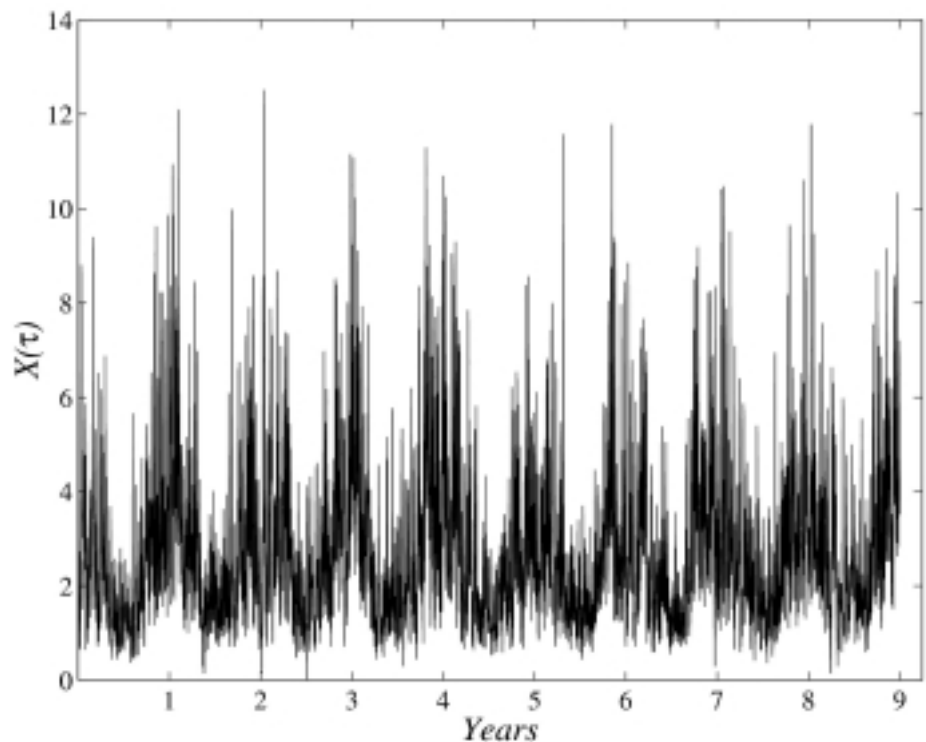
# A METHODOLOGY FOR INTEGRATING WAVE DATA FROM DIFFERENT SOURCES PERMITTING A MULTISCALE DESCRIPTION OF WAVE CLIMATE VARIABILITY

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## 1. INTRODUCTION

Long-term wave data (e.g. spectral parameters) present themselves as time series of data, exhibiting random variability, serial correlation, seasonal periodicity and, possibly, a long-term climatic trend. The first three features are obvious in any long-term time series of wave data; see, for example, Figure 1, where a nine-year long time series of three-hourly sampled significant wave height  $H_s$  is shown. The long-term trend is a disputable character that may be seen after a careful statistical analysis of the annual mean values of  $H_s$  (see, for example, Athanassoulis and Stefanakos, 1995; WASA Group, 1998; Carter, 1999 and references cited there). In any case, the availability of multi-year long time series of data is a prerequisite for investigating the presence of a multi-year trend. In the present work, we shall disregard this question, since the data we have at our disposal are not long enough to resolve this feature.

To obtain a time series like the one shown in Figure 1 (nine years in situ measurements) is a time-consuming and expensive procedure. Nevertheless, this kind of data is necessary for a number of important applications such as, for example,



*Figure 1 — Nine-year (1980–1989) long time series of significant wave height  $H_s$  from Haltenbanken, Norway, (65.08°N, 7.57°E) [buoy measurements].*

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coastal morphodynamics (sediment transport and beach erosion) and direct numerical simulation of nonlinear long-term responses of offshore structures. Satellite altimeter data, that are being continuously collected, cannot substitute for the time series data in this kind of application, since the satellite footprint is moving across the ocean. Thus, the question arises of whether it is possible to estimate the different features of long-term time series of  $H_s$ , combining data from different sources, e.g. satellite altimeter data in conjunction with a restricted amount of buoy measurements.

The above question can be treated by exploiting an appropriate modelling of the time series (Athanasoulis and Stefanakos, 1995), which makes it possible to distinguish the different time scales (features) and, eventually, to associate the random variability and the correlation structure (hourly scale) with the buoy measurements, and the seasonal periodicity with the satellite measurements. In this way, the first two features can be estimated by means of a restricted amount of buoy data (say, one year), after a deseasonalization of these data by means of monthly mean values and monthly standard deviations obtained from several years of satellite altimeter measurements.

By further exploiting the time series modelling, it is possible to derive a many-year long time series by simulation, which combines all the basic statistical structure of the wave data. The whole methodology can be considered as an efficient way of blending (integrating) already available satellite data with a short (thus affordable and feasible) period of in situ measurements, to obtain an artefact of a long-term measured time series.

The structure of this paper is as follows. In section 2, the underlying stochastic modelling is presented and reformulated in accordance with the needs of the present work. In section 3, various statistics are introduced, defined by means of a time series of in situ measurements, which will be found to be comparable with appropriate statistics based on satellite measurements. The structure of satellite measurements that can be associated with a given site is discussed in section 4, where appropriate statistics are also defined. Systematic comparisons of the various statistics based on the two data sources, revealing which ones are interchangeable, are presented in section 5. After this assessment, the whole methodology for integration of satellite and in situ measurements leading to the construction of a simulated long-term time series is recapitulated in section 6. A general discussion and some conclusions concerning the extent of applicability and the necessary precautions in using the present approach are presented in section 7.

## 2. MODELLING AND ANALYSIS OF $\alpha$ -HOURLY LONG-TERM TIME SERIES

<sup>1</sup> We use the terminology ' $\alpha$ -hourly' (time series) in order to denote any time series of measurements with time step  $\Delta\tau = \alpha$  hours. Usually, spectra or spectral parameters are recorded (or calculated) every 1, 3, 6 or 12 hours, thus  $\alpha = 1, 3, 6, 12$ . However, any value  $1 \leq \alpha \leq 12$  is possible.

<sup>2</sup> Let it be noted that the whole methodology presented herein can be equally well applied to the case where a climatic trend  $\bar{X}_{tr}(\tau)$  is present, if the data (from the same or other sources) allow us to identify such a trend.

Let us denote by  $X(\tau_i)$ ,  $i = 1, 2, \dots, I$ , the  $\alpha$ -hourly<sup>1</sup> many-year long time series of significant wave height  $H_s(\tau)$  or an appropriate transform thereof. Usually, the shifted logarithms of  $H_s(\tau)$  are considered, i.e.  $X(\tau) = \log[H_s(\tau) + c]$ , where  $c$  is a small positive constant between 0.2 m and 1 m. The constant  $X(\tau)$  is introduced to avoid zeros and minimize the skewness of the probability distribution of  $X(\tau)$ . The log-transformed data are often approximately Gaussian, which greatly facilitates the analysis and the simulation procedure. According to the modelling introduced by Athanasoulis and Stefanakos (1995, 1998), such a time series  $X(\tau)$  allows the following decomposition (see also Stefanakos, 1999):

$$X(\tau) = \bar{X}_{tr}(\tau) + \mu(\tau) + \sigma(\tau) W(\tau) \quad (1)$$

where  $\bar{X}_{tr}(\tau)$  is any possible long-term (climatic) trend,  $\mu(\tau)$  and  $\sigma(\tau)$  are deterministic periodic functions with a period of one year, and  $W(\tau)$  is a zero-mean, stationary, stochastic process. The functions  $\mu(\tau)$  and  $\sigma(\tau)$  are called seasonal mean value and seasonal standard deviation, respectively. In the sequel, we shall consider that  $\bar{X}_{tr}(\tau) = \bar{X} = const$  and this constant will be incorporated into  $\mu(\tau)$ <sup>2</sup>.

Thus, in the present work, decomposition (1) will be rewritten as:

$$X(\tau) = \mu(\tau) + \sigma(\tau) W(\tau) \quad (2)$$

The principal aim of the present work is to examine if, and how, it is possible:

- (i) To obtain reasonable estimates of  $\mu(\tau)$  and  $\sigma(\tau)$  by means of satellite data and, if the answer to this question is positive;
- (ii) To obtain the statistical characteristics of the residual process  $W(\tau)$  by exploiting the satellite-based estimates of  $\mu(\tau)$  and  $\sigma(\tau)$ , and a short-period (say, one year) of in situ measurements.

The methodology will be checked a posteriori by studying the stationarity of  $W(\tau)$  and making a comparison with the corresponding results of direct  $\alpha$ -hourly time-series analysis, in cases for which long-term in situ measurements are available.

The time series  $X(\tau)$  is usually reindexed, in order to properly treat variability at different time scales, by using the double Buys-Ballot index  $(j, \tau_k)$ , where  $j$  is the year index and  $\tau_k$  ranges within the annual time (Athanasoulis and Stefanakos, 1995). In the present work, a triple index of similar philosophy is introduced, denoted by  $(j, m, \tau_k)$ . The first component  $j$  is again the year index. The second component  $m$  is a month index, ranging through the set of integers  $\{1, 2, \dots, M = 12\}$ . The third component  $\tau_k$  represents the monthly time, the index  $k$  ranging through the set of integers  $\{1, 2, \dots, K_m\}$ , where  $K_m$  is the number of  $\alpha$ -hourly observations within the  $m$ -th month. Clearly, the meaning of the symbol  $\tau_k$  in the triple index  $(j, m, \tau_k)$  used herewith is different from the meaning of the same symbol in the double index  $(j, \tau_k)$  used in previous studies (e.g. Athanasoulis and Stefanakos, 1995; 1998).

According to the new, three-index notation, the time series  $X(\tau)$  is reindexed as follows:

$$\{X(j, m, \tau_k), \quad j = 1, 2, \dots, J, \quad m = 1, 2, \dots, M, \quad k = 1, 2, \dots, K_m\} \quad (3)$$

Note that there is a one-to-one correspondence between single index  $i$  and the triple index  $(j, m, \tau_k)$ . Indeed, if the triple index  $(j, m, \tau_k)$  is given, then the corresponding single index  $i$  is obtained by means of the relation:

$$i = (j-1)K + \sum_{q=1}^{m-1} K_q + k \quad (4)$$

where  $K = \sum_{q=1}^M K_q$  is the total number of observations within a year. Conversely, if the single index  $i$  is given, then the corresponding triple index  $(j, m, \tau_k)$  is calculated as follows:

$$\bullet \quad j = \left[ \frac{i-1}{K} \right] + 1 \quad (5a)$$

•  $m$  is the unique integer for which:

$$\sum_{q=1}^{m-1} K_q + 1 \leq i - (j-1)K \leq \sum_{q=1}^m K_q \quad (5b)$$

and

$$\bullet \quad k = i - (j-1)K - \sum_{q=1}^{m-1} K_q \quad (5c)$$

The three indices  $j, m, \tau_k$  represent three different timescales, making it possible to explicitly define statistics with respect to each one of them, separately. In the following sections, use will be made of the subscripts 1, 2, 3 to denote various statistics (mean value and standard deviation) with respect to the corresponding (first, second and third) index. To clarify the structure of this notation, we present a number of examples, some of which will also be used in the sequel:

$$M_1(m, \tau_k) = \frac{1}{J} \sum_{j=1}^J X(j, m, \tau_k) \quad (6a)$$

$$S_1(m, \tau_k) = \sqrt{\frac{1}{J} \sum_{j=1}^J [X(j, m, \tau_k) - M_1(m, \tau_k)]^2} \quad (6b)$$

$$M_3(j, m) = \frac{1}{K_m} \sum_{k=1}^{K_m} X(j, m, \tau_k) \quad (6c)$$

$$S_3(j, m) = \sqrt{\frac{1}{K_m} \sum_{k=1}^{K_m} [X(j, m, \tau_k) - M_3(j, m)]^2} \quad (6d)$$

We also define two-index statistics, which are obtained by successively taking mean values with respect to two indices. For example:

$$M_{13}(m) = \frac{1}{K_m} \sum_{k=1}^{K_m} \frac{1}{J} \sum_{j=1}^J X(j, m, \tau_k) = M_{31}(m) \quad (6e)$$

### 3. STATISTICS OF TIME SERIES OF MONTHLY MEAN VALUES

It is a straightforward matter to define the time series of monthly mean values (MMV) of  $X(\tau_j)$ . In fact, Equation (6c) defines this time series by averaging  $\alpha$ -hourly observations over each month. In Figure 2, the MMV time series, obtained from the  $\alpha$ -hourly time series shown in Figure 1, is presented (continuous line). By averaging  $M_3(j, m)$  over all the examined years, we obtain the overall MMV (per month):

$$\tilde{M}_3(m) = \frac{1}{J} \sum_{j=1}^J M_3(j, m) = M_{31}(m) = M_{13}(m) \quad (7a)$$

The time series of monthly standard deviations (MSD) of  $X(\tau_j)$  is defined by means of the Equation (6d). See also Figure 2 (dashed line). Averaging  $S_3(j, m)$  over all the examined years, we obtain the overall MSD (per month):

$$\tilde{S}_3(m) = \frac{1}{J} \sum_{j=1}^J S_3(j, m) \quad (7b)$$

It should be noted that  $\tilde{S}_3(m)$  is not the standard deviation of the time series  $M_3(j, m)$ . The selection of  $\tilde{S}_3(m)$  as the representative quantity for the variability of

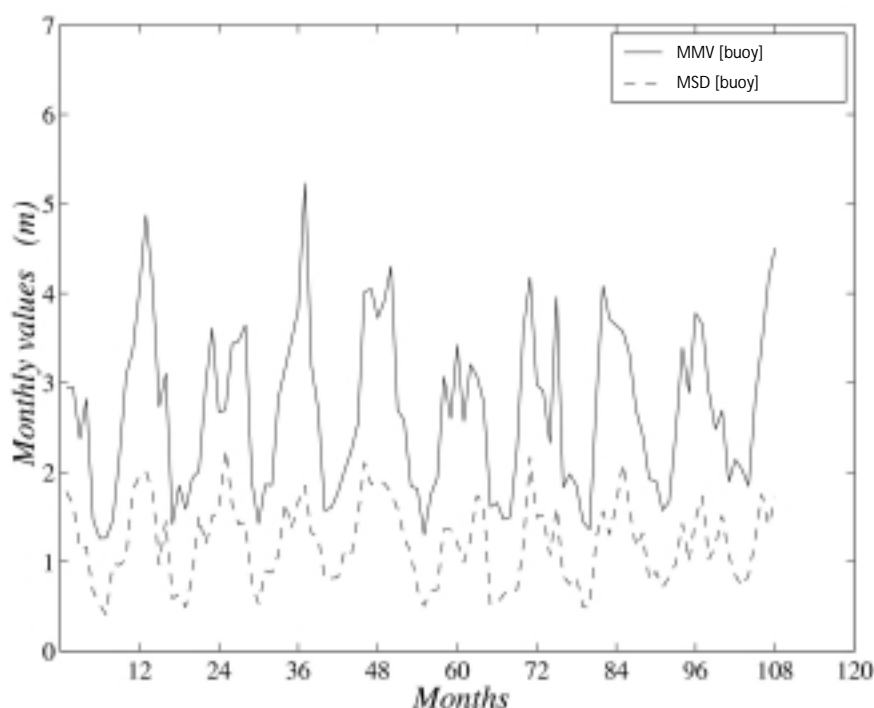


Figure 2—Time series of monthly mean values (MMV) and monthly standard deviations (MSD) from the Haltenbanken buoy measurements.

MMV  $M_3(j, m)$  about the overall MMV  $\tilde{M}_3(m)$  has been dictated by the data analysis. Indeed, after extensive numerical experimentation, it was found that it is exactly this quantity, i.e.  $\tilde{S}_3(m)$ , that can be related with (estimated by) an appropriately defined quantity obtained from satellite altimeter measurements.

#### 4. STATISTICS OF SATELLITE MEASUREMENTS

Let us now turn our attention to satellite altimeter measurements of  $H_s$ , obtained along specific (satellite dependent) ground tracks. Clearly, successive satellite observations are not referred to the same point in the sea. Thus, satellite wave data do not have the structure of a time series. If, however, we assume that the wave field is spatially homogeneous for an area  $S_A$ , surrounding a specific site of interest  $A^3$ , then we can associate to this site all satellite observations within the area (Tournadre and Ezraty, 1990). This set of observations (population) can be given the structure of a three-index data set:

$$\{X^{sat}(j, m, \chi_\ell), \quad j = 1, 2, \dots, J, \quad m = 1, 2, \dots, M, \quad \ell = 1, 2, \dots, L_m\} \quad (8)$$

where again  $j$  is the year index,  $m$  is the month index, and  $X_\ell$  is just a monthly counter, i.e. an index counting the number of observations within the area  $S_A$ , during the month  $m$  of the year  $j$ . Clearly, for given values of  $j$  and  $m$ , the individual values  $X(j, m, \tau_k)$ ,  $k=1, 2, \dots, K_m$ , and  $X^{sat}(j, m, X_\ell)$ ,  $\ell=1, 2, \dots, L_m$  are not directly comparable.

<sup>3</sup> The extent and shape of the area  $S_A$  are satellite dependent (sufficient data), and site dependent (local meteorological conditions).

Despite the structural differences between the data sets  $X(j, m, \tau_k)$  and  $X^{sat}(j, m, X_\ell)$ , it can be expected that appropriate statistics of  $X(j, m, \tau_k)$  can be approximated by analogous statistics of  $X^{sat}(j, m, X_\ell)$ , provided that the sea area  $S_A$  has been chosen appropriately. This expectation is based on the following assumptions concerning the time-space field of significant wave height  $H_s(\tau, \vec{r})^4$ :

- (i)  $H_s(\tau, \vec{r} = \text{const})$  is (approximately) stationary within each month<sup>5</sup>;
- (ii)  $H_s(\tau = \text{const}, \vec{r})$  is (approximately) homogeneous within the area  $S_A$ ; and
- (iii) a dispersion relation holds for the wave field  $H_s(\tau, \vec{r})$  (Tournadre, 1993, Section 5.3).

Some results concerning the correspondence of temporal and spatial scales of  $H_s(\tau, \vec{r})$  have been presented by Monaldo (1988, 1990), Tournadre (1993), and Krogstad and Barstow (1999).

<sup>4</sup> Observations  $X(j, m, \tau_k)$  and  $X^{sat}(j, m, X_\ell)$  are considered as two different samples from the field  $H_s(\tau, \vec{r})$ .

The triple-index notation greatly facilitates the definition of various statistics on  $X^{sat}(j, m, X_\ell)$ , and the comparison with analogous statistics on  $X(j, m, \tau_k)$ . We present below some definitions of MMV and MSD related with  $X^{sat}(j, m, X_\ell)$ :

Definitions (9a,b) and (10a,b) correspond to (6c,d) and (7a,b), respectively.

<sup>5</sup> Of course, in finer scales, short-duration energetic events (e.g. frontal passages) may occur that do not comply with the stationarity assumption. These events, which should be modelled using different (finer scale) stochastic processes, will not be considered herewith.

$$M_3^{sat}(j, m) = \frac{1}{L_m} \sum_{\ell=1}^{L_m} X^{sat}(j, m, \chi_\ell) \quad (9a)$$

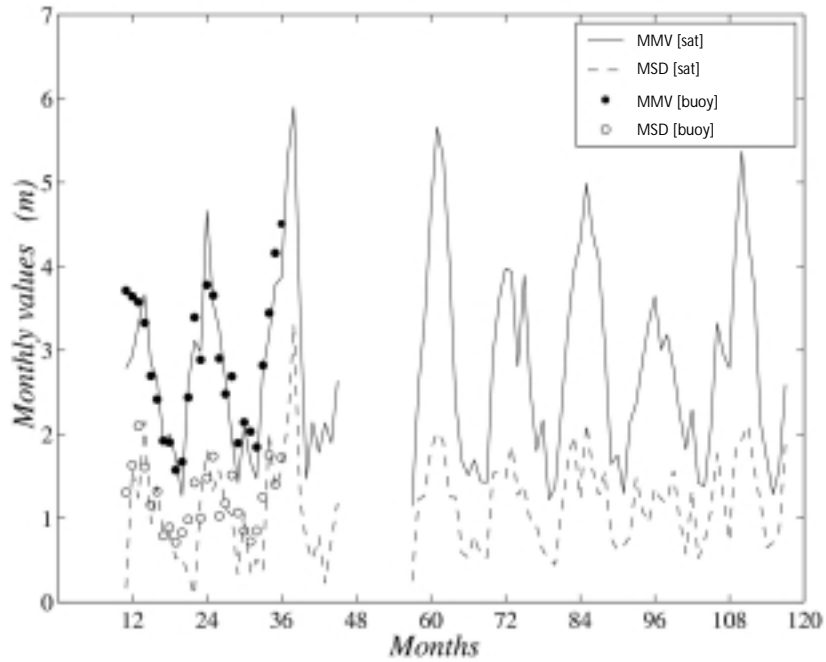
$$S_3^{sat}(j, m) = \sqrt{\frac{1}{L_m} \sum_{\ell=1}^{L_m} [X^{sat}(j, m, \chi_\ell) - M_3^{sat}(j, m)]^2} \quad (9b)$$

$$\tilde{M}_3^{sat}(m) = \frac{1}{J} \sum_{j=1}^J M_3^{sat}(j, m) \quad (10a)$$

$$\tilde{S}_3^{sat}(m) = \frac{1}{J} \sum_{j=1}^J S_3^{sat}(j, m) \quad (10b)$$

Clearly  $M_3^{sat}(j, m)$  and  $S_3^{sat}(j, m)$  are monthly time series generated by spatial averaging over the area  $S_A$ . In Figure 3, these time series, calculated from Geosat (1986-1989) and Topex (1992-1997) altimeter data, for an area near the Haltenbanken site, are shown. For the first two years, buoy measurements are also available, and they are depicted in the same figure by circles. As can be seen, the agreement between satellite monthly values and buoy monthly values is satisfactory. See also Figure 4, where results of the same analysis are shown for another site (NOAA 41001, 34.68°N, 72.64°W) in the western part of the North Atlantic Ocean. Note that this buoy is one of several buoys used in deriving the calibration

Figure 3—Time series of monthly mean values (continuous line) and monthly standard deviations (dashed line) from satellite measurements near Haltenbanken buoy (circles).



procedure applied to satellite altimeter data sets (see, for example, Krogstad and Barstow, 1999). Note also that the period of measurements for this NOAA buoy is 16 years (1982-1997), completely covering the period of the satellite measurements, which is the same as in the previous case. It seems, thus, reasonable to consider  $M_3^{\text{sat}}(j, m)$  and  $S_3^{\text{sat}}(j, m)$  as substitutes for  $M_3(j, m)$  and  $S_3(j, m)$ . However, for the needs of our study, only the weaker assumption that the monthly time series  $M_3^{\text{sat}}(j, m)$  and  $S_3^{\text{sat}}(j, m)$  are statistically equivalent to the monthly time series  $M_3(j, m)$  and  $S_3(j, m)$ , respectively, is necessary. On the basis of the above discussion, we can expect that  $\tilde{M}_3^{\text{sat}}(m) \approx \tilde{M}_3(m)$  and  $\tilde{S}_3^{\text{sat}}(m) \approx \tilde{S}_3(m)$ .

5. COMPARISON OF MONTHLY STATISTICS FROM BUOY AND SATELLITE DATA

In this section, the statistical equivalence of MMV  $\tilde{M}_3^{\text{sat}}(m)$  and  $\tilde{M}_3(m)$ , and MSD  $\tilde{S}_3^{\text{sat}}(m)$  and  $\tilde{S}_3(m)$  is established using the two aforementioned data sets (Haltenbanken and NOAA 41001).

In Figure 5, the monthly mean values from Haltenbanken are shown and compared. The nine values  $\{M_3(j, m), j=1,2,\dots,9\}$  for each month, obtained from buoy data, are depicted as filled circles, while the eight values  $\{M_3^{\text{sat}}(m), j=1,2,\dots,8\}$  for each month, obtained from satellite data, are depicted as open-faced stars.

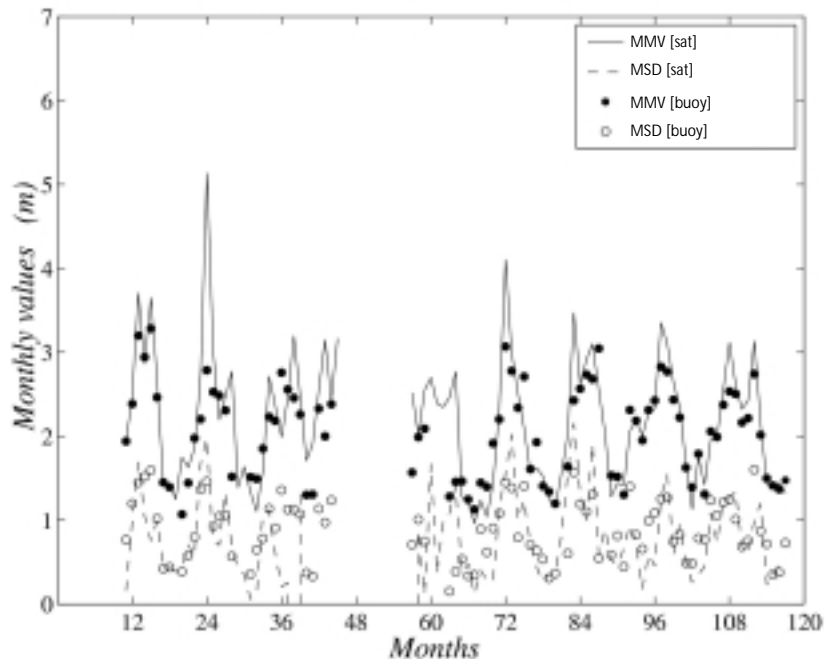


Figure 4—Time series of monthly mean values (continuous line) and monthly standard deviations (dashed line) from satellite measurements near NOAA 41001 buoy (circles).

These two sets of values (per month) are not comparable one-to-one since the periods of satellite and buoy measurements only partially overlap. However, based on the assumption of statistical periodicity of the wave climate, we expect that their mean values  $\tilde{M}_3(m)$  and  $\tilde{M}_3^{\text{sat}}(m)$  will be approximately the same, being two estimators of the same quantity. These two quantities are shown in Figure 5 by a solid and a dashed line, respectively. The agreement between  $\tilde{M}_3^{\text{sat}}(m)$  and  $\tilde{M}_3^{\text{sat}}(m)$  is, in general, impressively good, except for January and February. This discrepancy may (at least partly) be explained as follows:

After the end of the buoy measurement campaign in 1988, the winter wave climate in the Haltenbanken area, particularly during the months January to March, and for several years, had been surprisingly severe in comparison with the statistics of 1980-88 (buoy measurements period). This phenomenon has been confirmed by Barstow and Krogstad (1993) and other subsequent studies, using satellite and additional proprietary buoy data from the same area. This fact may be an indication of a long-term trend or periodicity in the wave climate in this area, but the amount of data is not yet enough to enable a statistical justification of such behaviour.

Similar comparisons are presented in Figure 6 for the monthly standard deviations from Haltenbanken. The conclusions are, in general, the same, though the agreement between  $\tilde{S}_3(m)$  and  $\tilde{S}_3^{\text{sat}}(m)$  is even better.

In Figure 7, the quantities  $\tilde{M}_3(m)$  and  $\tilde{S}_3(m)$ , as calculated from three different data sources, namely, buoy data, satellite data and model (WAM) data<sup>6</sup>, are shown. Again, the overall agreement is very good, with maximum discrepancy (less than 15 per cent) in the winter months.

A similar analysis has also been performed for the site of the NOAA buoy 41001. Figure 8 shows the overall monthly mean values  $\tilde{M}_3(m)$  and  $\tilde{M}_3^{\text{sat}}(m)$  (continuous line), and the overall monthly standard deviations  $\tilde{S}_3(m)$  and  $\tilde{S}_3^{\text{sat}}(m)$  (dashed line). The agreement between buoy and satellite results is even better in this case.

6 The model data used is a six-year long, six-hourly sampled time series of  $H_s$  (July '92-June '98) for the grid point (64.5°N, 7.5°E), obtained from the WAM model operating at ECMWF, Reading, UK.

## 6. A METHODOLOGY FOR INTEGRATION OF SATELLITE AND BUOY MEASUREMENTS

Having established that  $\tilde{M}_3^{\text{sat}}(m)$  and  $\tilde{S}_3^{\text{sat}}(m)$  are reasonable estimates of  $\tilde{M}_3(m)$  and  $\tilde{S}_3(m)$ , respectively, we are in a position to obtain the estimates  $\mu^{\text{sat}}(\tau)$  and  $\sigma^{\text{sat}}(\tau)$  as smoothed periodic extensions of the discrete estimates  $\tilde{M}_3^{\text{sat}}(m)$  and  $\tilde{S}_3^{\text{sat}}(m)$ . In this way, we can obtain the residual time series:

$$W^{\text{sat}}(\tau) = \frac{X(\tau) - \mu^{\text{sat}}(\tau)}{\sigma^{\text{sat}}(\tau)} \quad (11)$$

and compare it with the corresponding  $W(\tau)$  obtained by using the seasonal patterns  $\mu(\tau)$  and  $\sigma(\tau)$  estimated from the buoy data.

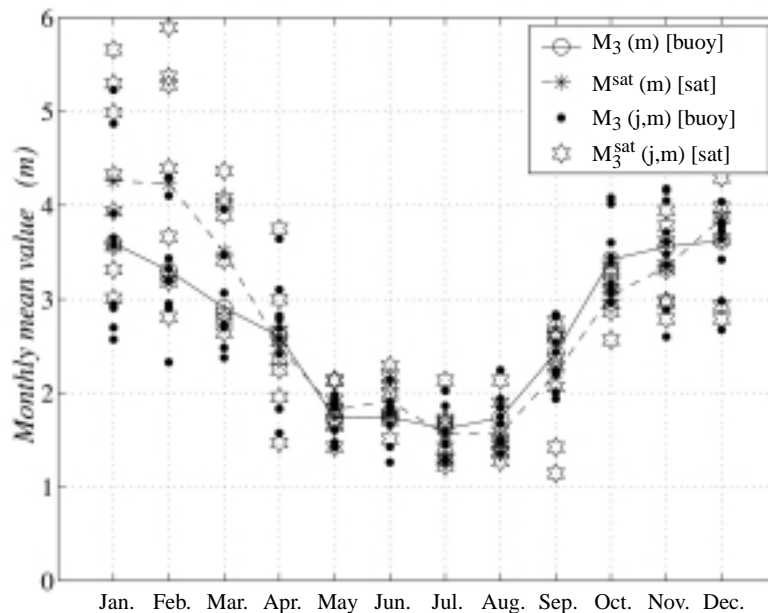


Figure 5—Comparison of monthly standard deviations of  $H_s$  from buoy (continuous line) and satellite (dashed line) measurements for the Haltenbanken site.

Figure 6—Comparison of monthly standard deviations of  $H_s$  from buoy (continuous line) and satellite (dashed line) measurements for the Haltenbanken site.

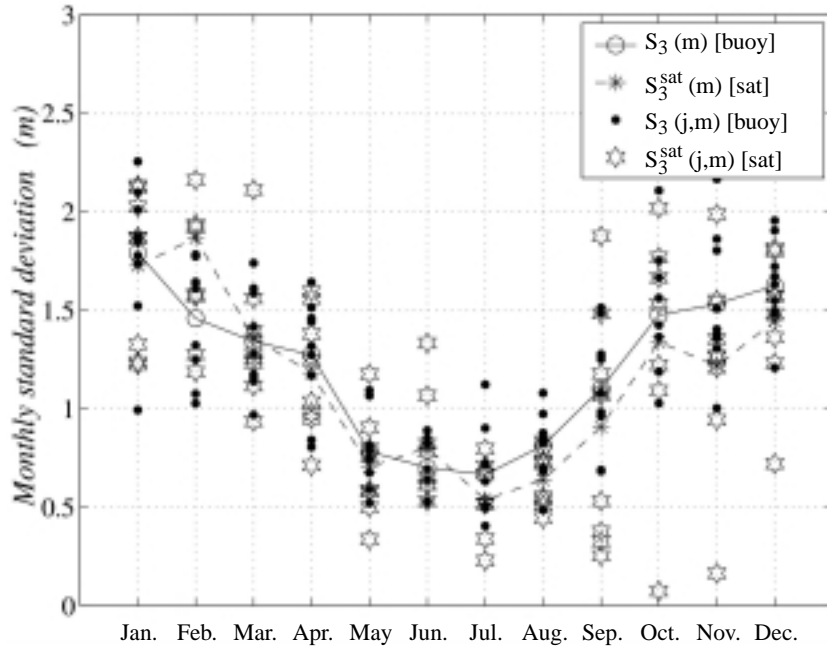
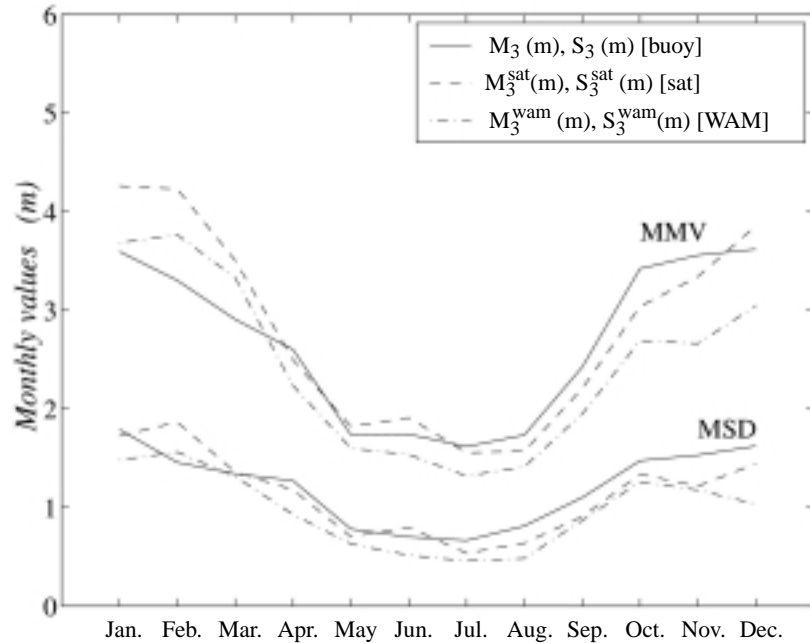


Figure 7—Comparison of monthly mean values and monthly standard deviations of  $H_s$  from buoy (continuous line) and satellite (dashed line) and hindcast (dashed-point line) data for Haltenbanken.



In order that the examined stochastic process is nearly Gaussian, we will work in this section with the time series  $X(\tau) = \log [H_s(\tau) + c]$  with  $c = 1m$ . Clearly, the whole analysis procedure presented above using the original time series  $H_s(\tau)$  must be repeated, using the log-transformed time series  $X(\tau)$ , in order to estimate the residual series  $W^{sat}(\tau)$  and  $W(\tau)$  of the transformed data.

Once the residual time series  $W^{sat}(\tau)$  and  $W(\tau)$  have been obtained, we can check their equivalence by comparing, for example, their spectral densities. Let us denote by  $S_w^{sat}(f)$  the spectral density of  $W^{sat}(\tau)$ , and by  $S_w(f)$  the corresponding one for  $W(\tau)$ . Figure 9 shows the two spectral densities  $S_w^{sat}(f)$  and  $S_w(f)$ , calculated using various yearly segments of  $W^{sat}(\tau)$  and  $W(\tau)$ , respectively. It can be seen that, even in the case of a one-yearly segment, there is good agreement between the various versions of the spectral density. This finding leads to the conclusion that the one-year buoy measurements, deseasonalized by means of the satellite seasonal patterns, describe well the state-to-state correlation structure of the  $\alpha$ -hourly time series.



Figure 8—Comparison of monthly mean values and monthly standard deviations of  $H_s$  from buoy (continuous line) and satellite (dashed line) data for the NOAA buoy 41001 site.

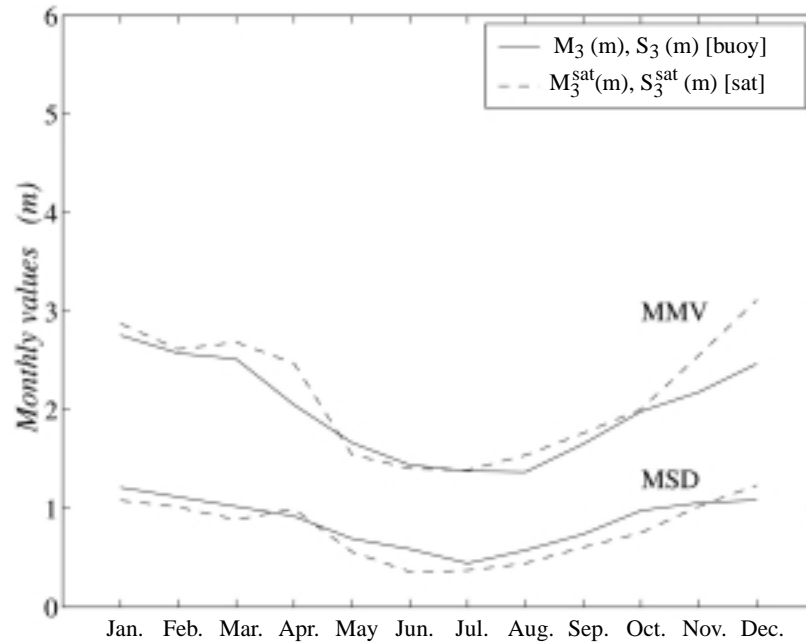
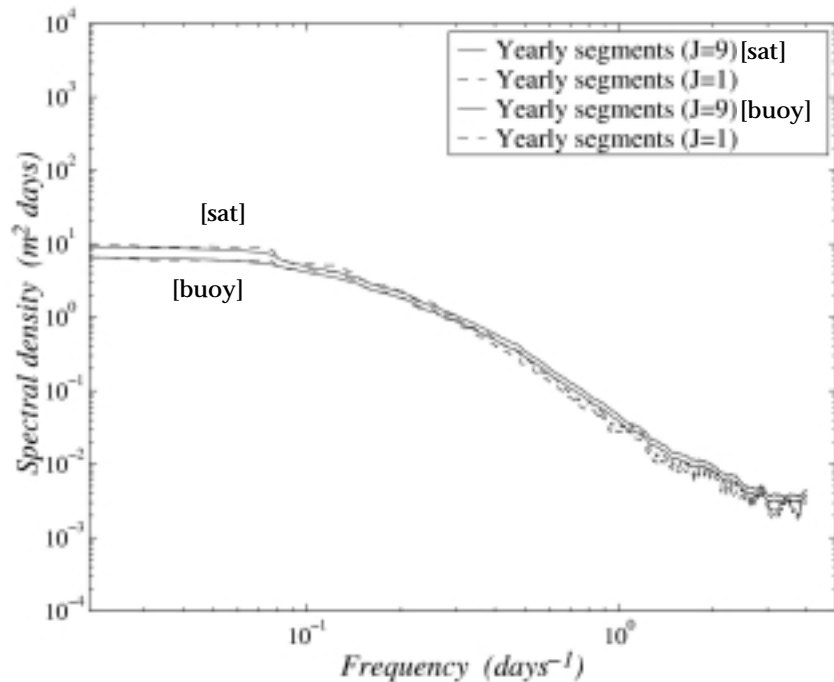


Figure 9—Spectral density functions  $S_w^{\text{sat}}(f)$  and  $S_w(f)$  calculated by analysing nine-year (continuous line) and one-year (dashed line) measurements. Site: Haltenbanken.

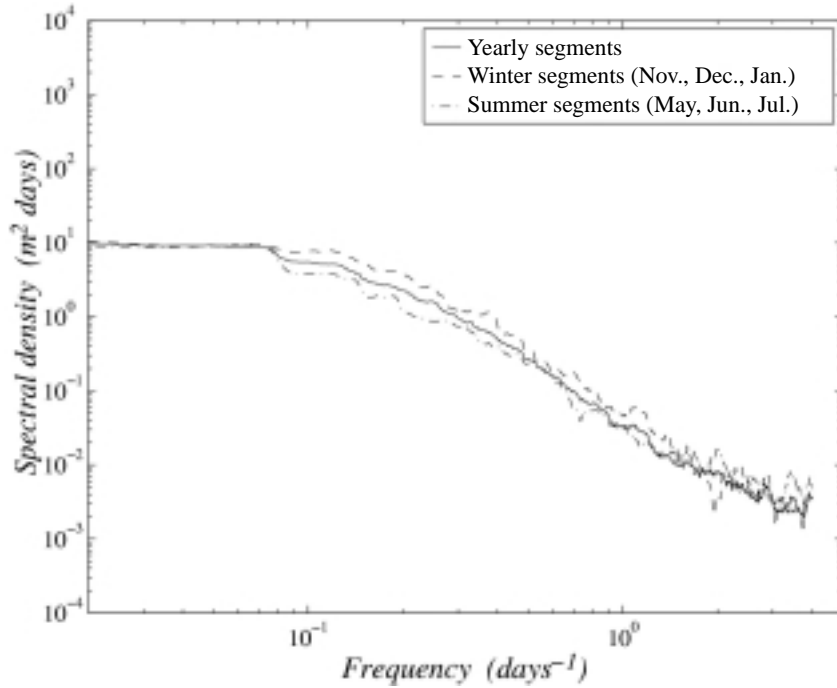


Next, the stationarity of  $W^{\text{sat}}(\tau)$  is examined. In Figure 10, the spectral density  $S_w^{\text{sat}}(f)$  is shown, estimated using various seasonal segments (yearly, winter and summer). The three versions exhibit a remarkable seasonal independence, confirming that the residual part  $W^{\text{sat}}(\tau)$  can be modelled as a stationary stochastic process.

Furthermore,  $W^{\text{sat}}(\tau)$  is given the structure of an autoregressive moving average model of order (2,2) (ARMA (2,2) model), the coefficients of which are estimated by least-square fitting to the raw spectral density (see, for example, Priestley, 1981 or Spanos, 1983). It is interesting to note that the ARMA coefficients, estimated either by means of a one-year segment, or by means of all nine-yearly segments, are very similar.

The ARMA modelling can be further exploited for simulation purposes. By generating zero-mean uncorrelated normal variates and using the estimated ARMA coefficients, a family of realizations of the same stationary stochastic

Figure 10—Spectral density function  $S_w^{\text{sat}}(f)$  calculated by analysing yearly (continuous line) winter (dashed line) and summer (dashed-point line) segments. Site: Haltenbanken.



process can be produced. Then, using Equation (2), we obtain realizations of the initial time series  $X(\tau)$ .

In Figures 11 and 12,  $S_w^{\text{sat}}(f)$  of the initial  $W^{\text{sat}}(\tau)$  is compared with  $S_w^{\text{sat}}(f)$  of the new  $W^{\text{sat}}(\tau)$ , which has been produced by simulation. Their agreement is found to be very good.

## 7. DISCUSSION AND CONCLUSIONS

In connection with various engineering applications, the long-term time series of significant wave height  $H_s$  (as well as various other wave and environmental parameters) can be considered as random series spanning at least three well-separated time scales: the sea-state duration (of the order of some hours), the yearly period (resolving the mean seasonal pattern), and a multi-year long time scale associated with possible long-term climatic trends.

Systematic long-term measurements covering all these scales are very impractical (time-consuming and expensive) and thus very rare. In fact, the evolution patterns of an environmental quantity in such different time scales may

Figure 11—Comparison of the initial residual time series  $W(\tau)$  and the corresponding simulated series. Site: Haltenbanken.

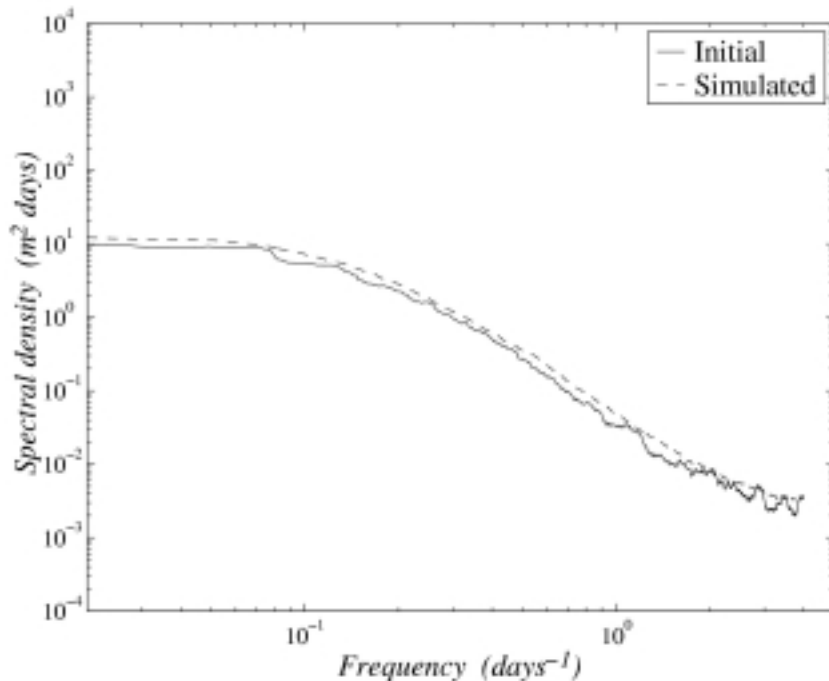
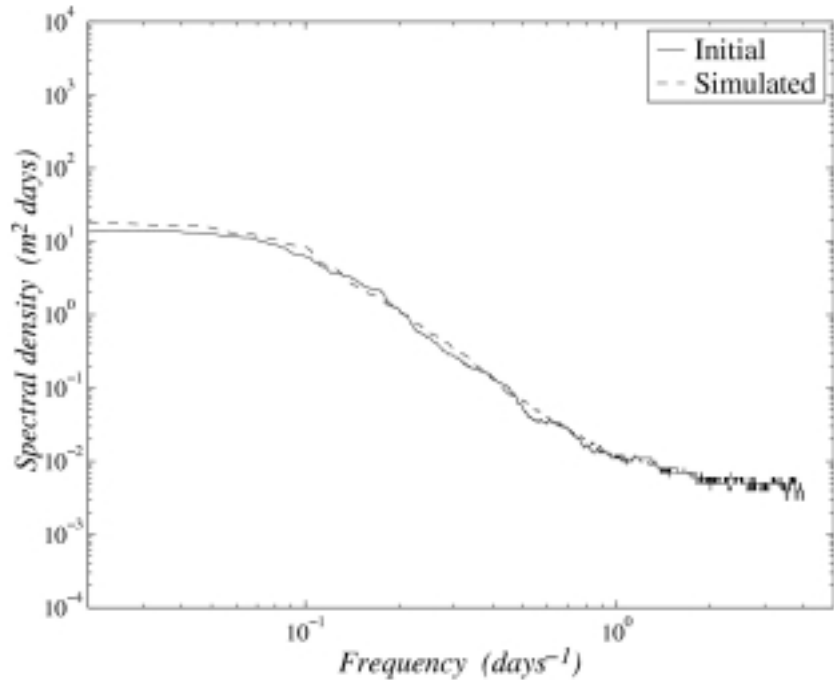


Figure 12—Comparison of the initial residual time series  $W(\tau)$  and the corresponding simulated series. Site: NOAA buoy 41001.



be studied by different scientific or engineering disciplines, using entirely different types of devices and models.

Different kinds of measurements, from different sources, each one resolving a different scale, are available nowadays. In the present work we have established that a restricted period of buoy measurements can resolve the state-to-state correlation structure (a continuum of scales associated with various weather patterns), while an appropriate spatial averaging of satellite altimeter measurements can describe the mean seasonal pattern and the seasonal variability. The key point is that we can use the seasonal pattern, as obtained from appropriately defined satellite monthly values, in order to deseasonalize the buoy measurements. Then, by analysing the deseasonalized buoy measurements (which can be assumed to be a stationary stochastic process; see, for example, Athanassoulis and Stefanakos, 1995), we can estimate the correlation structure associated with the state-to-state scale by calculating the corresponding autocorrelation (or spectral density) function.

If we have at our disposal data (from any source) or even indications about a possible long-term trend, we can extend the scope of this methodology by using the decomposition (1). In this case, both the satellite and the buoy measurements first will be detrended by subtracting  $\bar{X}_T(\tau)$ , and then the detrended time series will be treated as previously.

In applying the models (1) or (2) (having determined their parameters using data from different sources), we should bear in mind that, in principle, they contain exactly those characters that have been resolved in the stage of the analysis procedure. For example, intermediate scale phenomena (e.g. energetic frontal passages) not complying with the constitutive assumptions of our model are not included therein. There are, however, various benefits in using a carefully estimated model like (1) or (2), instead of a unique measured sample. For example, the model is free from gaps (missing values), it enables the performance of sensitivity studies either by obtaining a population of realizations (by using various independent identically distributed (iid) samples of the generating random sequence) or by varying the parameters of the model, and, also, it gives us the ability to treat more complex problems by combining the present model with other ones.

Among various possible generalizations of the models (1) and (2) and their applications, the following two seem to be the most interesting ones. First, the generalization towards multivariate data, e.g.  $\vec{X}(\tau) = (H_s(\tau), T_m(\tau))$  or  $\vec{X}(\tau) = (H_s(\tau), U_{wind}(\tau))$ , where  $T_m$  is the mean wave period and  $U_{wind}$  is the wind speed. The possibility of such an extension is also related to the quality and accuracy of the

satellite measurements for the additional parameters. Second, the generalization towards the inclusion of other phenomena evolving in different scales. For example, finer scale phenomena may be modelled by pulse-like processes (see, for example, Lopatoukhin *et al.*, 2000, Lopatoukhin *et al.*, 2001), while longer-scale phenomena might be included by introducing additional (longer) periods in the cyclostationary model (1).

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## REFERENCES

- Athanassoulis, G.A. and Ch.N. Stefanakos, 1995: A non-stationary stochastic model for long-term time series of significant wave height. *J. Geophys. Res.*, 100(C8), 16149 -16162.
- Athanassoulis, G.A. and Ch.N. Stefanakos, 1998: Missing-value completion of nonstationary time series of wave data. 17th International Conference on Offshore Mechanics and Arctic Engineering (OMAE) (Lisbon, Portugal).
- Barstow, S.F. and H.E. Krogstad, 1993: Analysis of extreme waves and recent storms in the Norwegian Sea. Climatic Trends and Future Offshore Design and Operation Criteria Workshop (Reykjavik, Iceland).
- Carter, D.J.T., 1999: Variability and trends in the wave climate of the North Atlantic: A review. 9th International Offshore and Polar Engineering Conference (ISOPE) (Brest, France).
- Krogstad, H.E. and S.F. Barstow, 1999: Satellite wave measurements for coastal engineering applications. *Coastal Engineering*, 37, 283-307.
- Lopatoukhin, L.J., I.V. Lavrenov, V.A. Rozhkov, A.V. Boukhanovsky, A.B. Degtyarev, 2000: Wind and wave climate of oil fields in the Barents, Pechora, and Kara Seas. 6th International Conference on Ships and Marine Structures in Cold Regions (ICETECH) (St. Petersburg, Russia, 475-482) (in Russian, English abstract).
- Lopatoukhin, L.J., V.A. Rozhkov, V.E. Ryabinin, V. Swail, A.V. Boukhanovsky, A.B. Degtyarev, 2001: *Estimation of Extreme Wind Wave Heights*. WMO/TD-No. 1041.
- Monaldo, F., 1988: Expected differences between buoy and radar altimeter estimates of wind speed and significant wave height and their implications on buoy-altimeter comparisons. *J. Geophys. Res.*, 93(C3), 2285-2302.
- Monaldo, F., 1990: Corrected spectra of wind speed and significant wave height. *J. Geophys. Res.*, 95(C3), 3399-3402.
- Priestley, M.B., 1981: *Spectral Analysis and Time Series*. Academic Press, 890 pp.
- Spanos, P-T.D., 1983: ARMA algorithms for ocean wave modeling. Trans. of the ASME, *Journal of Energy Resources Technology*, 105, 300-309.
- Stefanakos, Ch.N., 1999: Nonstationary stochastic modelling of time series with applications to environmental data. Ph.D. Thesis, National Technical University of Athens, Greece, 200 pp. (Supervisor: G.A. Athanassoulis).
- Tournadre, J., 1993: Time and space scales of significant wave heights. *J. Geophys. Res.*, 98(C3), 4727-4738.
- Tournadre, J. and R. Ezraty, 1990: Local climatology of wind and sea state by means of satellite radar altimeter measurements. *J. Geophys. Res.*, 95(C10), 18255-18268.
- The WASA Group, 1998: Changing waves and storms in the Northeastern Atlantic. *Bulletin of the American Meteorological Society*, 79(5), 741-760.